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"The Law-Decision Game"

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The Law-Decision Game

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Abstract

We consider a basic model of law formation with limited deterrence where a fully rational law-maker produces law strategically anticipating rational decisions that the law induces. In equilibrium, optimal law can induce optimal choices that are cyclic. Cyclic choices arise when there is a specific preference conflict between the law-maker and the decision-maker and maximum penalties can deter some but not all actions. Moreover, it is only when the law induces cyclic choices that the law and the choices it produces reveal preferences of law-makers and decision-makers. However, it may be possible to determine optimal law according to preferences that are unknown and cannot be inferred from choice.

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1 Introduction

Deterrence is a major function of the law. Deterrence is achieved by discouraging undesirable actions by the establishment of penalties. If penalties are sufficiently severe, then, at least in theory, a law-maker can incentivize a decision-maker to select the law maker's preferred outcome. But when penalties are limited the problem of what actions to penalize becomes more difficult. We show that conflicting law-makers and decision-makers preferences and limited deterrence creates incentives for law-makers to create law that is not *proportional* (when the law is proportional, the difference in penalties between pairs of alternatives depends upon unchosen options). A law that is not proportional may induce rational decision-makers to make cyclic choices. In addition, law that is not proportional and cyclic choices are limited to a specific conflict in preferences that we refer to as the teen drinking dilemma. Outside the teen drinking dilemma, decision makers choices are not anomalous and, hence, satisfy the weak axiom of revealed preference (WARP). We also show that the structure of optimal law varies non-monotonically with the penalty limit. In the teen drinking dilemma, optimal law may be proportional when limit penalties are either sufficiently large or small enough, but optimal law is not proportional when limit penalties are at intermediary levels. In contrast with the case of a single rational decision-maker, an outsider who observes optimally chosen law and optimal choices the law induces cannot infer preferences, except when choices are cyclic. This holds even though all agents are fully rational. However, in some cases, even if the law-makers preferences are unknown and cannot be inferred from choice, it may be possible to determine optimal law according to the law-makers preferences.

We consider a basic model in which a decision-maker (Dee or he) chooses an alternative from subsets of the available alternatives (issues). A law-maker, (Lee or she), assigns penalties up to some maximum limit to each alternative in each issue. We assume that Dee observes Lee's issue-specific penalties and chooses the option that maximizes his utility in each issue. Lee, correctly anticipating Dee's choices, chooses the law optimally according to her preferences. We refer to this game as the *law-decision game*.

1.1 The Teen Drinking Dilemma

A parent (Lee) must decide whether or not to permit her underage teenager (Dee) to drink. Lee prefers that Dee does not drink, but faces a dilemma if Dee might choose between not drinking at all (x), drinking at home (y) and drinking and driving (z). The question of whether drinking at home should be permitted as a way to head off drinking and driving is quite controversial.[1] Lee prefers that Dee does not drink, but she prefers that Dee drink at home rather than drink and drive. Dee's preferences are the opposite of Lee's preferences.

[1] See, for example, these articles about parents who were prosecuted for not stopping teen drinking at their house. Downloaded on 1-16-18 at https://www.washingtonpost.com/news/wonk/wp/2016/05/18/where-teenagers-can-legally-drink-in-the-u-s-yes-really/?utm_term=.8ec111282f96

He most prefers drinking and driving, his second choice is to drink at home and his least preferred option is not to drink at all. Lee must decide what penalties, if any, she will impose on Dee for each of his choices.

Assume first that Lee can either punish or not punish Dee depending on Dee's choice and the alternatives available to him (the issue). Lee has a single punishment available to her. If used, this punishment delivers a given disutility to Dee. Given the issue, if Lee does not punish Dee for taking some alternative, then we say that Lee makes this alternative to be legal in that issue. If Lee punishes Dee for taking some alternative, then we say that Lee makes this alternative to be illegal in that issue.

The teen drinking dilemma arises when the punishment available to Lee is insufficient to deter drinking and driving in all issues. If Dee cannot drink and drive, then Lee's punishment is sufficient to deter Dee from drinking at home. But when drinking and driving is a possibility, if Lee punishes drinking at home, then Dee chooses to drink and drive. If Lee does not punish drinking at home, but does punish drinking and driving, then Dee chooses to drink at home. Thus, Lee's optimal law is to punish drinking at home when drinking and driving is not available to Dee and to not punish drinking at home when drinking and driving is available to Dee. Given this law, it is optimal for Dee to not drink when the only alternative is to drink at home. It is also optimal for Dee to drink at home when the alternatives are to not drink and to drink and drive.

In the teen drinking dilemma, Dee's choices are anomalous. Dee chooses x in $\{x, y\}$ and y in $\{x, y, z\}$. Lee's optimally law is also anomalous. Lee rejects y as illegal in $\{x, y\}$, but Lee chooses y as legal in $\{x, y, z\}$. These choices violate WARP (the weak axiom of revealed preferences, see Samuelson (1938)).

In the example above, Lee has a single available punishment and, hence, Lee is limited to either punish or not punish Dee. Assume now that Lee can choose any punishment that delivers a disutility in $[0,\ D]$ to Dee, where $D > 0$ is the maximal punishment. We now show that optimal law can always be ordered by Lee's utility function. That is, regardless of the issue under which options are chosen, regardless of Dee's and Lee's preferences, and regardless of the maximal punishment D, options that Lee likes better are punished less severely than options that Lee likes less. Even in the teen drinking dilemma, optimal law can be ordered. Dee's optimal choices, however, remain anomalous. Thus, Dee's optimal choices can be anomalous even under ordered law.

We show that Dee's optimal choices can only be anomalous when the law does not satisfy a key property that we refer to as proportionality. When the law is proportional, the difference in penalties between pairs of alternatives do not depend upon the issue under which these options are taken. In the teen drinking dilemma, optimal law is not proportional and induces anomalous choices. In the Optimal Law Theorem, we show that outside of the teen drinking dilemma, optimal law can be proportional and the optimal choices induced by law are not anomalous. Thus, the teen drinking dilemma characterizes the conditions under which optimal law can be proportional and induce optimal choices that are anomalous. A corollary of the Optimal Law Theorem shows that the structure of the optimal law can vary non-monotonically with the limit D on penalties.

Optimal law can be proportional when the limit on penalties is either small or large, but when limit penalty is at intermediary levels, optimal law cannot be proportional. An additional corollary of the Optimal Law Theorem shows that there is no equilibrium of the law-decision game where Dee's choices are anomalous, but not cyclic. Thus, the law-decision game has empirical content and the model can be rejected in a straightforward way.

When the law is ordered by Lee's preferences, there is a direct connection between the law and the preferences of the law-maker. Options that Lee likes less are punished more. When the law is proportional, the differences in magnitudes of penalties between options are unambiguous. That is, penalty differences do not depend upon the issue under which these options were chosen. An optimal law that is ordered by Lee's preferences always exists and, outside of the teen drinking dilemma, an optimal law that is proportional also exists. However, we show an example that is outside of the teen drinking dilemma and yet there is no optimal law that is both proportional and ordered by Lee's preferences. Thus, these two basic structures of laws (order by Lee's preferences and proportionality) can be in conflict with each other. It may not be possible to obtain them simultaneously in optimal law.

In the Spirit of the Law Theorem, we assume that an outsider observes the law and the decisions it induces. The observer only knows or assumes that Lee and Dee are fully rational and, hence, make optimal decisions. We show that the law and the choices made under the law reveal the law-maker's preferences if and only if observed choices are cyclic. This results holds under full rationality and stand in contrast with the central result in rational decision theory where choices reveal preferences if and only if observed choices are *not* anomalous.

The Spirit of the Law Theorem and the Optimal Law Theorem show that law and decisions only reveal preferences in the special case of the teen drinking dilemma when optimal decisions are anomalous. However, consider now a judge who observes the law and the decisions incentivized by the law. The judge does not know the preferences of the law-maker and the preferences of the decision-maker. The judge also does not know the penalty limit. We show an example outside of the teen drinking dilemma where the judge cannot infer preferences from choice. And yet, in this example, the judge might be able to determine an optimal law in a new issue where the judge has not observed neither law or the decisions that the law induces in that issue. Hence, the judge might be able to determine that certain laws are optimal according to preferences that the judge does not know and cannot find out what they are.

1.2 Organization of the paper

A brief literature review is in section 1.3. The basic model is in section 2 along with a formalization of the teen drinking dilemma and other concepts such as ordered law and proportional law. In Section 3, we show that there is always an equilibrium where an optimal law is ordered by Lee's preferences. In section 4, we show that there exists an optimal law that is proportional if and only if a teen drinking dilemma cannot be formed. Hence, optimal choice induced by optimal law is anomalous if and only if a teen drinking dilemma can be formed. Section 4.1 also shows the Optimal Law Theorem. This theorem puts all the results together to characterize the teen drinking dilemma as a key demarcating condition on optimal law and optimal choice induced by law. In section 4.2, we show that the law-decision game has empirical content. In section 4.3, we show that the structure of optimal law may vary non-monotonically with the penalty limit. In section 5 we ask what law and choices alone can reveal about the law-maker's preferences. In section 5, the Spirit of the Law Theorem shows that it is only when optimal law is not proportional and optimal choices are cyclic that inferences about preferences can be drawn. Thus, law and decisions only reveal law-makers preferences in the special case of the teen drinking dilemma where optimal choices are anomalous. Section 5.1 shows an example where a judge can determine optimal law without knowing or being able to infer the law-maker's preferences. In section 8, we adapt our normative results on law formation to show how the teen drinking dilemma can produce positive theories about actual law. Future work is discussed in section 7. Section 8 concludes.

1.3 Related literatures

The logic of deterrence that we refer to in this paper is closely related to the phenomenon of marginal deterrence, described at least as far back as the 18th century by the criminologist Cesare Beccaria, and "officially named" by Stigler (1970). If, for instance, one punishes a certain crime harshly, one thereby may diminish one's ability to deter its completion: on the margin, the completion has become cheap. The idea is nicely conveyed by the English proverb "As good be hanged for a sheep as a lamb." This literature is too large to be fully reviewed here, but see, among several contributions, Bond and Hagerty (2010), DeMarzo et al. (1998), Detotto, McCannon, and Vannini (2015), Friedman and Sjostrom (1993), Kramer (1990), Mookherjee and Png (1994), Shavell (1992), Wilde (1992). The choice-theoretic aspect of the marginal deterrence phenomenon on which we focus here has not, however, been noticed in the literature. For instance, as far as we know, the marginal deterrence literature has not yet identified conditions under which optimal law induces anomalous choices, conditions under which preferences are revealed by choice, and determined whether optimal law can be ordered by the law-maker's preferences.

The bounded-rationality literature studies conflicting motivations that lead to anomalous choices in individual decision making. See, among many contributions, Ambrus and Rozen (2008), Apesteguia and Ballester (2008), Berheim and Rangel (2009), Chambers and Hayashi (2012), de Clippel and Eliaz (2012), de Clippel and Rozen (2012), Dietrich and List (2013), Eliaz, Richter and Rubinstein (2011), Eliaz and Spiegler (2011), Fudenberg and Levine (2006), Green and Hojman (2007), Heller (2012), Houy (2007), Houy and Tademuma (2009), Kalai, Rubinstein and Spiegler (2002), Lleras et. al., (2012), Lehrer and Teper (2011), Masatlioglu and Nakajima (2007), Masatlioglu and Ok (2005), Manzini and Mariotti (2007, 2012a, 2012b), Masatlioglu, Nakajima and Ozbay (2012), Ok, Ortoleva, and Riella (2008), Salant and Rubinstein (2006), (2006a), (2012), Spiegler (2002), Tyson (2013). Anomalous individual choices are typically associated with bounded rationality, and not with full rationality as in this paper, but bounded rationality and rational deterrence models are not unrelated. Consider models of categorization constraints (Manzini and Mariotti (2007, 2012a)), attention constraints (Lleras et. al., (2012)) or psychological constraints (Cherepanov et. al., (2013a)). In these decision-theoretic models, there are constraints added to feasibility constraints. In our model, the law is an additional constraint. While the law can be broken and is, in our model, optimally chosen by another agent, the broad idea of restrictions (beyond feasibility) shows some connection between decision-theoretic models of bounded rationality and the fully rational multi-agent model of deterrence in this paper.

Following the seminal work of Arrow (1950), incompatibilities between social and individual choices continues to be explored in modern research (see, among many contributions, Eliaz, Ray and Razin (2006), Mongin (1995), Jackson and Yariv (2015) and Zuber (2011)). In our model, Lee ranks laws by the outcomes it produces. It is the law itself that can be not proportional and it is the choices that the law induces that can anomalous. In this sense, our results do not have a clear counterpart in the social choice literature. Katz and Sandroni (2017) show that actual laws can induce law-abiding decision-makers to make cyclic choices. However, the law is not optimally chosen in their work. Hence, there is no reference to the teen drinking dilemma (or to any result in this paper) in Katz and Sandroni (2017). Moreover, in this paper, if Dee were to always be law-abiding, then his choices would always be ordered and never cyclic. Unlike in Katz and Sandroni (2017), our results require the assumption that Dee may break the law. Finally, as mentioned in the introduction, results are different when there is a single penalty or when there are multiple penalties. Unlike this paper, Katz and Sandroni (2017) consider the case of a single penalty.

The literature on revealed preferences is quite large (see Echenique (2020) and Varian (2006) for a review) and is generally based on a decision-theoretic framework. The Spirit of the Law Theorem and our results on the empirical content of the law-decision game are part of the literature of revealed preference in a game theoretic framework (see, among several contributions, Cason and Plott (2014), Chambers et al. (2017), Freer and Martinelli (2016), Ray and Zhou (2001), and Sprumont (2000)). However, the results in this paper are tailored to the specific law-decision game. They are not about general classes of games. Finally, the law-decision game can be seen as a principal-agent model where law-maker is the principal who uses penalties to incentivize the agent (the decision-maker) instead of rewards. The choice-theoretic focus of this paper, based on the idea of issues, makes the analysis in this paper to differ from the analysis standard principal-agent models. In particular, the results in this paper do not require hidden actions. All actions of the decision-maker are observed by the law-maker in this paper. We refer the reader to Laffont and Martimort (2001) for a review of the principal-agent model.

2 Basic Model

Let A be a finite set of alternatives. An *issue* B is a subset of A and let $\mathcal{B}$ be the set of issues. The *law* specifies penalties for choosing a given alternative in an issue. Formally, the *law* is a *penalty function* $P : \mathcal{B} \times \mathcal{A} \longrightarrow [0, +\infty)$ that maps the set of issues and alternatives in each issue into a penalty. For any issue $B \in \mathcal{B}$, the penalty for choosing alternative $x \in B$ in issue B is $P(B, x)$. Let $\mathcal{P}$ be the set of all penalty functions. The law is *D-limited* if the maximum penalty that can be imposed is $D > 0$. Let $\mathcal{P}_D = \{P \in \mathcal{P} \mid P(B, x) \in [0, D], \forall B \in \mathcal{B} \text{ and } x \in B\}$ be the set D-limited penalty functions. We may refer to $D > 0$ as the maximal penalty or the penalty limit.

2.1 The Law Maker and the Decision Maker

A law maker, named Lee, has a utility function $V : A \to \mathcal{R}$ such that $V(x) \neq V(y)$ if $x \neq y$. For any issue $B \in \mathcal{B}$, let $l_B = \max_{x \in B} V(x)$ be Lee's preferred alternative in issue B. A decision maker, named Dee, has a utility function $u : A \to \mathcal{R}$ that specifies his underlying preferences over alternatives. We assume that $u(x) \neq u(y)$ if $x \neq y$. So, indifference is ruled out. Let the payoff function U_P be as follows: For any $B \in \mathcal{B}$ and $x \in B$,

$$U_P(B, x) = u(x) - P(B, x).$$

That is, Dee's payoff U_P for choosing x in issue B depends upon his utility u as well as the penalty Lee imposes for choosing x in issue B.

2.2 The Law-Decision Game

Let a *choice function* C be a mapping that takes an issue B, as input, and returns, as output, an element $C(B)$ of B. Thus, a choice function is a mapping $C : \mathcal{B} \longrightarrow A$ such that $C(B) \in B$. Let $\mathcal{C}$ be the set of all choice functions. In the law-decision game, a law-maker (Lee) first chooses a law $P \in \mathcal{P}_D$. Given law $P \in \mathcal{P}_D$, a decision-maker (Dee) chooses a choice function C_P. In issue $B \in \mathcal{B}$, Dee's payoff is $U_P(B, C_P(B))$ and Lee's payoff is $V(C_P(B))$. Hereafter, we fix Dee's utility function u, Lee's utility function V and the penalty limit $D > 0$.

2.3 Equilibrium of the Law-Decision Game

Definition 1 *Given the law $P \in \mathcal{P}_D$, the choice function C_P is **optimal** if for every issue $B \in \mathcal{B}$,*

$$U_P(B, C_P(B)) \geq U_P(B, x) \text{ for all } x \in B$$

and, if there exists an $x \in B$ such that $x \neq C_P(B)$ and $U_P(B, C_P(B)) = U_P(B, x)$, then $V(C_P(B)) > V(x)$.

So, Dee optimally chooses the alternative in each issue that gives him the highest payoff, conditional on his utility function and the penalties Lee imposes.

If, taking penalties into account, Dee is indifferent between two alternatives, we assume that he chooses the option that Lee prefers. This assumption simplifies the exposition because the optimal choice function is unique.

Definition 2 *A **choice strategy** is a mapping S: $\mathcal{P}_D \to \mathcal{C}$ that takes the law P as input and returns a choice function $S(P)$ $(= C_P)$ as output. A choice strategy S is **optimal** if the choice function $S(P)$ $(= C_P)$ is optimal for every law $P \in \mathcal{P}_D$.*

So, a choice strategy determines Dee's choice function given each law. A choice strategy is optimal when it always returns an optimal choice function given each law. Let $\mathcal{S}$ be the set of choice strategies.

The law is optimal if, given Dee's optimal choice strategy, the law produces the best possible outcome in each issue from Lee's perspective.

Definition 3 *Given Dee's choice strategy $S \in \mathcal{S}$, a D-limited law $P^* \in \mathcal{P}_D$ is **optimal** if for every issue $B \in \mathcal{B}$,*

$$V(C_{P^*}(B)) \geq V(C_P(B)) \text{ for all } P \in \mathcal{P}_D,\ C_{P^*} = S(P^*),\ C_P = S(P).$$

A D-limited law P^* is optimal if no alternative D-limited law P induces Dee to choose an alternative that Lee prefers in any issue $B \in \mathcal{B}$. When there is no risk of confusion, we may refer to a $D-$limited law as simply a law.

Definition 4 *A pair $(S,\ P^*) \in \mathcal{S}x\mathcal{P}_D$ is an **equilibrium of the law-decision game** if $S \in \mathcal{S}$ is an optimal choice strategy and given $S \in \mathcal{S}$, $P^* \in \mathcal{P}_D$ is an optimal D-limited law.*

In equilibrium, Dee chooses the best available alternative in any issue, given the law. Lee correctly anticipates Dee's choice function contingent on any law and chooses the D-limited law that optimizes her utility function. Given an equilibrium $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$, $S(P^*) = C_{P^*} \in \mathcal{C}$ is the *optimal choice function* and $P^* \in \mathcal{P}_D$ is the *optimal law*.

A law might achieve a better outcome than other law in an issue and a worse outcome in some other issue. This may seemingly create a trade-off. However, an optimal law always exists because penalties can be issue-specific. Any issue in this model is finite and there are finitely many issues. Remark 1 is an immediate consequence of the assumption that A (the set of all options) is finite.

Remark 1 An equilibrium of the law-decision game exists.

Remark 2 is also immediate.

Remark 2 Fix u, V and $D > 0$. In equilibrium, there can be multiple optimal laws $P^* \in \mathcal{P}_D$, but the optimal choice function $C_{P^*} \in \mathcal{C}$ is unique. That is, in equilibrium, C_{P^*} is the same optimal choice function for different optimal laws $P^* \in \mathcal{P}_D$.

Optimal law may not be unique. Assume that Lee and Dee have identical preferences. Then, Lee might impose no penalties and Dee nevertheless always chooses Lee's preferred alternative. Alternatively, Lee could maximally punish every alternative and get the same result. However, Dee's optimal choice function is uniquely determined given any law. Thus, uniqueness of optimal choice function in equilibrium now follows directly from the assumption that Lee is not indifferent between different options.

2.4 The Teen Drinking Dilemma

We now define the special configuration of the teen drinking dilemma. First, consider three alternatives over which Lee and Dee have opposing preferences.

Definition 5 *A set $T = \{x, y, z\} \in \mathcal{B}$ is a **triple with opposing preferences***

$$\text{if } V(x) > V(y) > V(z) \text{ and } u(z) > u(y) > u(x).$$

Let the set $\mathcal{T} \subseteq \mathcal{B}$ be the set of all triples with opposing preferences.

In teen drinking dilemma, the limit penalty is not large enough to ensure that Dee always chooses the option that Lee prefers and the limit penalty is not low enough so that Dee always chooses the option that Dee prefers. The limit penalty is large enough so that there are penalties that will induce Dee to choose x over y, and also to induce Dee to choose y over z even though Dee prefers y over x and z over y. However, the limit penalty is not large enough to induce Dee to choose Lee's preferred option (x) over z. Let I_T be the interval

$$(\max\{u(y) - u(x), u(z) - u(y)\}, u(z) - u(x)).$$

There can be multiple intervals I_T because there can be multiple triples with opposing preferences. Let $I_{\mathcal{T}} = \cup_{T \in \mathcal{T}} I_T$ be the union of all intervals I_T.

Definition 6 *A **teen drinking dilemma** occurs when there exists a triple with opposing preferences ($\mathcal{T} \neq \varnothing$) and the maximal penalty D is such that $D \in I_{\mathcal{T}}$.*

A teen drinking dilemma occurs when Dee and Lee have opposing preferences over three options and the maximal penalty is at intermediary range(s).

2.5 Anomalous Optimal Choice

Definition 7 *A choice function C is* anomalous *if there is a pair of alternatives x, $y \in A$ and issues $B \in \mathcal{B}$ and $B' \in \mathcal{B}$ such that $\{x, y\} \subseteq B \subset B'$, $C(B) = x$ and $C(B') = y$. A choice function C is* cyclic *if there are three alternatives x, y, $z \in A$ with $C(\{x, y\}) = x$, $C(\{y, z\}) = y$ and $C(\{x, z\}) = z$.*

That is, a choice function is anomalous if it violates WARP. It is straightforward to show that in the teen drinking dilemma, Dee optimally chooses a cyclic choice function. Formally,

Remark 3 In the teen drinking dilemma, in equilibrium, the optimal choice function C_{P^*} is cyclic and therefore anomalous.

Optimal choices in the teen drinking dilemma are cyclic because between x and y, Lee prefers x over y and can induce Dee to choose x by penalizing y enough (and not penalizing x). Between y and z, Lee prefers y over z and can induce Dee to choose y by penalizing z enough (and not penalizing y). Between x and z, Dee prefers z over x and Lee cannot induce Dee to choose x even if she maximally penalizes z and does not penalize x.

2.6 Basic Structures of the Law

Definition 8 *A law $P \in \mathcal{P}$ is* ***ordered by utility function*** *$o : A \to \mathbb{R}$ if for every $B \in \mathcal{B}$ and x, $y \in B$,*

$$P(B,x) \geq P(B,y) \Leftrightarrow o(x) \leq o(y) \text{ and } P(B,x) > P(B,y) \Leftrightarrow o(x) < o(y).$$

A law P is ***ordered*** *if there exists a utility function o that orders P. A law $P \in \mathcal{P}$ is* ***ordered by Lee's utility function*** *if the law is ordered by $V : A \to \mathbb{R}$.*

If the law is ordered by an utility function o, then alternatives with lower utility are always more penalized than alternatives with higher utility. If the law is ordered by Lee's utility function, then options that Lee likes less are more penalized than alternatives that Lee likes more.

Optimal law may or may not be ordered. As we show, to be ordered by Lee's utility function does not suffice to induce optimal choices to be non-anomalous. The next basic structure of law (proportionality), however, does suffice to induce optimal choices to be non-anomalous.

Definition 9 *A law $P \in \mathcal{P}$ is* ***proportional*** *if for any pair of alternatives x, $y \in A$ and for any issue $B \in \mathcal{B}$ such that $\{x,y\} \subset B$,*

$$P(B,x) - P(B,y) = P(A,x) - P(A,y).$$

A law is proportional if the difference in penalties between any two alternatives is the same across issues. A special case of proportional law is context-independent law.

Definition 10 *A law $P \in \mathcal{P}$ is* ***context-independent*** *if there is a function $p : A \longrightarrow [0, D]$ such that for any issue $B \in \mathcal{B}$ and any option $x \in B$,*

$$P(B,x) = p(x).$$

A law is context-independent if penalties depends on actions taken, but not on the issue under which the choice was made. It is immediate that context-independent are proportional. Remarks 4 and 5 below are also straightforward:

Remark 4 If a law $P \in \mathcal{P}$ is context-independent, then it is proportional. If a law is proportional, then it is ordered.

Remark 4 shows that the requirement that the law be proportional is stronger than the requirement that the law be ordered. The proof of remark 4 is based on the following idea. Consider a proportional law P and let z be any option. Define $O(x,y) = P(A,x) - P(A,y)$ for any pair of options x and y, and let $o(x) = O(z,x)$. So, for any issue $B \in \mathcal{B}$,

$$o(x) < o(y) \Leftrightarrow O(z,x) < O(z,y) \Leftrightarrow P(A,y) < P(A,x) \Leftrightarrow P(B,y) < P(B,x).$$

Thus, P is ordered.

Remark 5 If a law $P \in \mathcal{P}$ is proportional, then the optimal choice C_P induced by the law P is not anomalous.

Remark 5 shows that proportional law imposes a great deal of structure on the law and on the choices that the law induces. Proportionality suffices to ensure that optimal choices induced by the law satisfy WARP. The proof of remark 5 is based on the idea that if x is chosen over y on an issue B, then $u(x) - P(B,x) \geq u(y) - P(B,y)$. Analogously, if y is chosen over x on an issue B', then $u(y) - P(B',y) \geq u(x) - P(B',y)$. These two inequalities cannot hold simultaneously when $P(B,x) - P(B,y) = P(B',x) - P(B',y)$ and one of these inequalities is strict. Finally, one of these two inequalities must be strict when $V(x) \neq V(y)$.

As a direct corollary of remark 5, in the teen drinking dilemma, optimal law cannot be proportional because optimal choices are cyclic. Formally,

Corollary 1 Consider the teen drinking dilemma. In equilibrium, any $D-$limited optimal law $P^* \in \mathcal{P}_D$ is not proportional.

By remark 4, all proportional laws are ordered, but a proportional law need not be ordered *by Lee's utility function*. Moreover, as we show in the example below, an optimal law that is both proportional and ordered by Lee's utility function may not exist, even if a teen drinking dilemma cannot be formed.

Example 1 A optimal law that is both proportional and ordered by Lee's utility function may fail to exist, even if a teen drinking dilemma cannot be formed.

Example 1 as follows: There are four alternatives: $g0$, $g1$, $b0$ and $b1$. Lee and Dee agree that both $g0$ and $g1$ are better than either $b0$ and $b1$. But they disagree about about which of the g's and b's alternatives are better. Dee's utility function is such that

$$\begin{aligned} u(g0) &> u(g1) + D0.5 > u(g1) > u(g0) - D > u(g1) - D > \\ u(b0) &> u(b1) + 0.5D > u(b1) > u(b0) - D. \end{aligned}$$

Lee's utility function is such that

$$V(g1) > V(g0) > V(b1) > V(b0).$$

Notice that D is sufficiently large so that Lee can always induce Dee to choose her most preferred alternative in any issue. Hence, a teen drinking dilemma cannot be formed. Thus, in any optimal law P^*,

$$\begin{aligned} P^*(\{g0,g1\},g0) - P^*(\{g0,g1\},g1) &\geq u(g0) - u(g1) \text{ and} \\ P^*(\{b0,b1\},b0) - P^*(\{g0,g1\},b1) &\geq u(b0) - u(b1) \end{aligned}$$

because otherwise Dee chooses $g0$ over $g1$ when Lee can induce Dee to choose $g1$ over $g0$ as Lee prefers or Dee chooses $b0$ over $b1$ when Lee can induce Dee to choose $b1$ over $b0$ as Lee prefers. Thus, in any optimal proportional law P^*, we must have

$$\begin{aligned} P^*(A,g0) - P^*(A,g1) &\geq u(g0) - u(g1) \text{ and} \\ P^*(A,b0) - P^*(A,b1) &\geq u(b0) - u(b1). \end{aligned}$$

If P^* is ordered by V, then $P^*(A,g0) < P^*(A,b1)$. So,

$$P^*(A,b0) \geq u(b0)-u(b1)+P^*(A,b1) > u(b0)-u(b1)+u(g0)-u(g1)+P^*(A,g1)$$

which is impossible because

$$u(b0) - u(b1) + u(g0) - u(g1) > D.$$

3 Ordered Law

We now show that optimal law can always be ordered by Lee's utility function.

Ordered Law Proposition Fix Lee's utility function V, Dee's utility function u and the limit penalty $D > 0$. Consider any optimal choice strategy $S \in \mathcal{S}$. Generically on any $D > 0$, there exists a D-limited law P^* such that $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is an equilibrium of the law-decision game and P^* is ordered by Lee's utility function V.[2]

The ordered Law Proposition shows that there is an equilibrium where the optimal law is ordered by Lee's utility function. Regardless of what Dee's and Lee's preferences might be, regardless of the issue under which Dee makes his choice, and generically on what the maximal penalty might be, there exists an ordered optimal law. By definition, under ordered law, penalties are smaller on alternatives that Lee likes more and penalties are higher on alternatives that Lee likes less. In ordered law, there is a direct connection between penalties and the preferences of the law-maker. This direct connection between penalties and Lee's preferences can exist in an optimal law even in the teen drinking dilemma.

The ordered law proposition is an existence result. It does not show that optimal law must be ordered. It shows that optimal law can be ordered. As we

[2] Here, generically means that we rule out finitely many points in the continuum $(0, \infty)$ of possible maximum penalties D.

show formally below, given that ordered optimal law always exists, and in the teen drinking dilemma optimal choices are cyclic for any optimal law, it follows that optimal choices can be cyclic even if optimal law is ordered.

The intuition of the ordered law proposition is that in every issue B, there is a subset $\bar{B}$ of options that Dee can induce Dee to take with penalties. The two critical alternatives for optimal law are $l_{\bar{B}}$ (the option that Lee's prefers among those that she can induce Dee to take) and d_B (Dee's preferred alternative). By definition, Lee cannot induce Dee to take any alternative that Lee prefers over $l_{\bar{B}}$. Thus, Lee may assign a minimal but positive punishment to $l_{\bar{B}}$ and punish everything she likes better than $l_{\bar{B}}$ also minimally but with decreasing punishments as Lee's utility increases. Similarly, Lee must punish d_B sufficiently to deter Dee from choosing d_B over $l_{\bar{B}}$. As long as Lee does not have to punish d_B maximally, she may punish alternatives that Lee likes less than $l_{\bar{B}}$ at a sufficiently high levels (to disincentivize Dee from taking these options) in a manner that is consistent with her preferences. This ensures that the law is ordered by Lee's preferences while maintaining optimality.

Remark 3 and the ordered law proposition, taken together, produce the following corollary:

Corollary 2 In the teen drinking dilemma, in equilibrium, there exists an optimal law that is ordered by Lee's utility function. The optimal choice function is cyclic and therefore anomalous.

4 Proportional Law

We now show that outside the teen drinking dilemma, there is an optimal proportional law.

Proportional Law Proposition Assume that Dee's and Lee's utility functions u and V and the maximal penalty D do not form a teen drinking dilemma. Then, there is an equilibrium where the optimal law P^* is proportional.

The proportional law proposition is also an existence result. It does not show that optimal law must be proportional even outside of the teen drinking dilemma. It shows that, outside of the teen drinking dilemma, optimal law can be proportional. Given that, in equilibrium, the optimal choice function is unique, the existence of a proportional law implies that the optimal choice function must not be anomalous. Therefore, outside of the teen drinking dilemma, the optimal choice function must not be anomalous.

The proof of the proportional law proposition is constructive and shows that outside of the teen drinking dilemma, there exists an optimal law that is context-independent. The (context-independent) penalty p on an option z is defined as the maximal difference in Dee's utility $u(z) - u(y)$ among options y under two constraints. First, Lee likes y better than z (or $y = z$) and, second, that the difference $u(z) - u(y)$ does strictly exceed D. The intuition is that if Lee likes

y better than z, then, outside of the teen drinking dilemma, these penalties incentivize Dee to choose y over z even if Dee likes z better than y.

We can now summarize our first main results in the Optimal Law Theorem

Optimal Law Theorem Fix Dee's and Lee's utility functions u and V and the maximum penalty $D > 0$. The following statements are equivalent:

1. In equilibrium, the optimal choice function C_{P^*} is cyclic;
2. In equilibrium, the optimal choice function C_{P^*} is anomalous;
3. In equilibrium, any optimal $D-$limited law P^* is not proportional;
4. In equilibrium, any optimal $D-$limited law P^* is not context-independent;
5. A teen drinking dilemma can be formed.

The optimal law theorem is a characterization result. In the teen drinking dilemma, optimal law is not proportional and optimal choices are cyclic. If a teen drinking dilemma cannot be formed, then there exists an optimal law that is proportional and optimal choices induced by any optimal law are not anomalous.

4.1 Empirical Content of the Law-Decision Game

A direct corollary of the Optimal Law Theorem is that if Dee's optimal choices are anomalous, then they are cyclic. Formally,

Corollary 3 There is no equilibrium where the optimal choice function C_{P^*} is anomalous and not cyclic.

By corollary 3, the law-decision game has empirical content. The model can be tested and rejected if the decision-maker's choices are anomalous, but not cyclic. A full characterization of the empirical content of the law-decision game is outside the scope of this paper and left for future work.

4.2 Non-Monotonicity as the Limit Penalty Changes

By the Optimal Law Theorem, optimal law is not proportional if and only if a teen drinking dilemma can be formed. Assume that there exists at least one triple with opposing preferences T. Then, a teen drinking dilemma is formed when the limit penalty D is in an intermediary range D_T. If there are multiple triples with opposing preference, then a teen drinking dilemma can be formed when the limit penalty D is any of the ranges D_T, $T \in \mathcal{T}$. In particular, when the limit penalty is sufficiently low, there is no teen drinking dilemma and, in equilibrium, optimal law can be proportional and optimal choices are not anomalous. When the limit penalty is sufficiently high, there is no teen drinking dilemma and, in equilibrium, optimal law can be proportional and optimal choices are not anomalous. If there exists a single triple with opposing preferences T, then, when the limit penalty D is at the intermediary range D_T, all optimal laws are not proportional and optimal choices are anomalous. If there are multiple triple with opposing preferences, and at least two ranges D_T and $D_{T'}$, $T \in \mathcal{T}$ and $T' \in \mathcal{T}$, are disjoint, then as the limit penalty D varies, optimal laws may switch multiple times from ranges in which optimal choices are anomalous to ranges in which optimal choices are not anomalous. Formally,

Corollary 4 Fix Dee's and Lee's utility functions u and V and the maximum penalty $D > 0$. Assume that there is at least one triple of opposing preferences ($\mathcal{T} \neq \varnothing$). Then, in equilibrium, if D is in the union of intermediary ranges D_T, $T \in \mathcal{T}$, any optimal $D-$limited law P^* is not proportional and optimal choices are anomalous. If D is not in the union of intermediary ranges D_T, $T \in \mathcal{T}$, there exists a proportional and optimal $D-$limited law and optimal choices induced by any optimal law are not anomalous.

By Corollary 4, the structure of optimal law and optimal choices induced by optimal law change non-monotonically with the limit penalty D.

5 Revealed Preference and the Spirit of the Law

In decision theory, preferences are revealed from the decision maker's choices. In this model, the choices are the law (selected by Lee) and the choice function (selected by Dee). Suppose that an observer sees the law P and the choices made in each issue. The observer can infer that Dee prefers x to y whenever Dee chooses x over y and the penalty for x is at least as great as the penalty for y. On the other hand, if the penalty for y is greater than the penalty for x, the observer may be unsure whether Dee prefers x to y or y to x.

If the law is known to be ordered by Lee's preferences, then the law directly reveals Lee's preferences. However, assume that the observer does not know that the law is either ordered or proportional. The outside observer only knows that the observed law and choices are optimal. Then, even if the law is ordered, the observer may be unsure of whether the law is ordered by Lee's utility function or by some other utility function. Therefore, a natural question is what inferences can be made about Lee's preferences when an outsider only knows that law and choices are in equilibrium.

Definition 11 *Given a pair of options x, $y \in A$, a law $P^* \in \mathcal{P}$ and choice function C_{P^*}* ***reveal that Lee prefers*** *x* ***over*** *y if $V(x) > V(y)$ for any $D > 0$ and u and V such that P^* and C_{P^*} are an equilibrium optimal D-limited law and optimal choice function, respectively. P^* and C_{P^*}* ***reveal some of Lee's preferences*** *if for some pair of options x, $y \in A$, P^* and C_{P^*} reveal that Lee prefers x over y.*

That is, a law and a choice function reveal that Lee prefers x over y, $x \neq y$, if Lee must prefer x over y whenever the observed law and choice function are optimal. A law and the choice function reveal some of Lee's preferences if at least one alternative is revealed to be preferred by Lee to another option.

When choices are not anomalous, these choices might be consistent with Dee's preference order if differences in Dee's utilities overwhelm the penalties of the law. However, these choices might also follow from the law's penalties if these penalties overwhelm differences in Dee's utilities. As we show below, if choices are not anomalous, then Lee's preferences are not revealed by the law and the choices that the law induces. On the other hand, suppose that observed choices are cyclic. Then, the Optimal Law Theorem shows that there is a teen drinking dilemma. In this case, the observer can infer both Dee's and Lee's over the triple opposing preferences. So, it is precisely the conditions that make optimal law not proportional and optimal choices anomalous that allow Lee's preferences to be revealed. We formalize this idea in the Spirit of the Law Theorem.

Spirit of the Law Theorem Let $P^* \in \mathcal{P}$ and $C_{P^*} \in \mathcal{C}$ be an equilibrium law and choice function, respectively. P^* and C_{P^*} reveal some of Lee's preferences if and only if C_{P^*} is cyclic.

The spirit of the law theorem shows that if a teen drinking dilemma cannot be formed, then an outsider cannot infer Lee's preferences using observed law and observed choices that the law induces alone. On the other hand, if a teen drinking dilemma is formed, then the resulting choices induced by the law is cyclic and from these cyclic choices it is possible to infer both the decision-maker's and the law-maker's preferences.

The intuition of the Spirit of the Law Theorem is as follows: If C_{P^*} is cyclic, then there are options x, y, and z such that x is chosen over y, y is chosen over z, and z is chosen over x. It follows from the Optimal Law Theorem that x, y and z are in a teen drinking dilemma. Thus, Dee's choice in the issue $\{x, y, z\}$ is his preferred middle option and Dee's choice in the binary issue that excludes this middle option is Dee's preferred option. Lee ranks x, y and z in the opposite way that Dee ranks these options. Thus, Lee preferences over x, y and z are revealed. On the other hand, if the choice function is not cyclic, then the choice function must satisfy WARP. Otherwise, by the Optimal Law Theorem, the choice function could not be part of an equilibrium. Therefore, if the limit penalty is small enough, the choice function is consistent with Dee's preference order, regardless of what Lee's preferences might be. Thus, Lee's preferences are not revealed.

5.1 Towards a Choice-Theoretic Model of Jurisprudence

The Spirit of the Law theorem shows that the law-maker's preferences can only be inferred when the choices that the law induces are cyclic. However, we now show that a judge might be able to determine an optimal law in an unobserved issue, *even if that judge is unable to infer the law-makers' preferences.*

Consider the options x, y and z. In all binary choices, no punishment is given for x, a punishment p is given for y, and a punishment of $p' > p$ is given for z. In addition, in the binary choices, x is chosen over y, y is chosen over z and x is chosen over z. These choices are not cyclic. Thus, there is no teen drinking dilemma and the law-maker's preference are not revealed. Now consider the issue $\{x, y, z\}$. The judge has observed law or choice in this issue, but, nevertheless, the judge can determine that if the law and choice were optimal in the binary choices, then in the issue with all three options, the same law where x is not punished, y is punished at p, and z is punished at $p' > p$ remains optimal.

From the optimal law theorem, the judge knows that any optimal law must produce the choice of x in the issue $\{x, y, z\}$. Otherwise, observed choices would be anomalous and not cyclic and, therefore, not part of an equilibrium. However, under the proposed law, Dee optimally chooses x in $\{x, y, z\}$. If Dee were to choose y, then he would have chosen y in the binary choice $\{x, y\}$. If Dee were to choose z, then he would have chosen in the choice $\{x, z\}$. So the judge knows that the proposed law on the new issue $\{x, y, z\}$ must be optimal for Lee (because all optimal laws induce Dee to take the same option x in $\{x, y, z\}$)

The logic of the example above extends to an arbitrary set of options. Assume that all binary choices are not cyclic and in binary issues, each option received a specific and fixed penalty. If a judge assumes that the penalties and choices in these binary issues are optimal, then the judge can infer that the same penalties remain optimal in any issue. This example shows that several inferences can be made about what penalties are optimal, even if the judge cannot infer the preferences of the law-maker. A complete characterization of inferences about optimal law is outside the scope of this paper and left for future work.

6 Normative and Positive Theories of Law

The analysis in this paper is normative. We are fundamentally interested in the properties of optimal law and in the properties of optimal choice that the law induces. The teen drinking dilemma proved to be a demarcating case that differentiate when optimal law cannot be proportional and optimal choices are anomalous. In this section, we ask whether the teen drinking dilemma can be used to develop positive theories of the law as well. That is, we now ask whether the teen drinking dilemma can be used to theorize about why the law is how it is. Here we do not look for definitive answers. Instead, our purpose is more modest. We merely ask whether and in what conditions the teen drinking dilemma can reveal a possible logic that underlies actual law. For example, consider labor law. In the US strikes are legal. One reason why strikes might be legal is that law-makers prefer strikes to be legal as opposed to illegal, due, perhaps, for a concern with rights or fairness. However, the teen drinking dilemma might point out to a different theory. Strikes might be legal even if law-makers prefer strikes to be illegal, but the punishments available to a law-maker may not suffice to deter workers (or unions) from striking and, making strikes illegal might induce workers to take action that law-makers like even less than striking. In this section we show five areas of the law (labor law, contract modification, bankruptcy law, self-incrimination and underinclusive rules more generally) where the law can be profitably be seen through the systematic schema of the teen drinking dilemma. These examples are by no means the only areas of the law that can be potentially attributed to the same underlying logic as in the teen drinking dilemma.

1. Labor law

Under American labor law, workers are entitled to pressure a employer with strikes and other activities. This used to be considered coercion and a violation of the prohibition against "anti-competitive combinations." It was soon recognized that such laws did not do much to inhibit these activities and lead workers to engage in more aggressive behavior such as sabotage, intimidation, and outright violence. Labor law worked out a compromise whereby some amount of these activities are allowed, but only up to a point.

Expressed more schematically, consider the following options for the worker (or union): (x) no pressure on employers, (y) limited pressure on employers with actions such as strikes, (z) full pressure on employers by any means. The teen drinking dilemma occurs when the law-maker prefers x to y to z, workers (or unions) prefer z to y to x, and the law-maker cannot induce workers to choose x over z, but can induce workers to choose x over y, and can also induce workers to choose y over z. In this case, as long as workers have the option of taking measures against employers that are more aggressive than strikes, it might be optimal for the law-maker to make strikes legal even if the law-maker prefers the option of no pressure on employers over the option of limited pressure on employers. Thus, the feasibility of aggressive measures against employers might be critical for strikes to be legal and sometimes exercised. In the teen drinking dilemma, if workers did not have the option of taking aggressive measures against employers, strikes would be made illegal and punished severely enough so that strikers would not be exercised. The teen drinking dilemma is a potential explanation for the reasons underlying legality in several other areas of the law as we now show.

2. Contract modification

A contractor commits to performing a certain job at a fixed price. At some point, the contractor discovers that the work is more onerous than expected. That does not relieve him of the duty to perform the job, but he threatens to walk off the job unless he is granted some increase in his fee. This is a classically coercive threat. He is threatening to do something he has no right to do and seeking a benefit for not going through with the threat. This used to be considered an invalid modification of a contract under the preexisting duty rule. But early in the 20th century the rule changed and such modifications came to be sometimes considered valid. The reason is not hard to fathom. If the contractor walks off the job, he is liable for breach of contract, but this is not a potent deterrent because it is costly for the client to sue, and it is hard to meet the required burden of proof for the loss.

Expressed more schematically, consider these options for the contractor: (x) to fulfill the contract, (y) to exert a bit of coercion, and (z) to walk-off the unprofitable contract. Assume that the law-maker prefers x to y to z, and the contractor prefers z to y to x. Also assume that the law-maker cannot induce the contractor to perform the job (over not doing it, i.e., to choose x over z), but can induce the contractor to exert a bit of illicit coercion over walking off the contract (choose y over z) and has high enough penalties to induce the contractor to choose x over y if those were the only options. Then, this is a teen drinking dilemma.

3. Personal bankruptcy: The fresh start policy

The law gives debtors whose debts greatly exceed their assets the option to giving up all their current assets (except for certain designated exempt ones equivalent to "the shirt on one's back") and obtain a fresh start free of debts. Arguably a significant reason for this is that otherwise the debtor might just ditch his creditors altogether instead of making a partial payment to them. Expressed more schematically, consider the options of the debtor: (x) make all possible payments and stay in debt, (y) make partial payments and be debt-free, and (z) stiff all creditors. The teen drinking example now holds under the same law-maker and decision-maker preferences and capabilities (of the law-maker) as in the other examples over x, y and z.

4. Self-Incrimination

It is a crime to obstruct justice by tampering with evidence, intimidating witnesses, or lying under oath, but it is permissible to refuse to give testimony that might incriminate oneself. This could be interpreted as a teen drinking dilemma where the options of the defendant are: (x) no obstruction of the prosecution, (y) partial obstruction of the prosecution by refusing self-incrimination, and (z) obstruction of the prosecution by any means.

5. Underinclusive rules generally

Criminal law rules are often underinclusive, in the sense that they do not punish undesirable conduct such as insults, infliction of emotional distress, some forms of nonphysical abuse, softer forms of corruption and self-dealing, and criminal conduct obtained severe duress or even just temptation. All of these could be rationalized as instances of the teen drinking dilemma where the options of the citizen are: (x) no undesirable conduct, (y) some of the more restrained undesirable conduct, (z) use all violent criminal means if need be.

7 Future Work

The Law-Decision game is a step towards the development on a theory of optimal law formation and optimal choices induced by the law. These developments include a canonical model of law formation which shares some characteristics with the principal-agent model, but where punishments incentivize decision-makers. In future work, we expect to introduce hidden actions in the law-decision game and other features that are common in principal-agent models. In addition, the law-decision game permits preference revelation analysis in this specific game-theoretic context. In future work, we expect to fully characterize the empirical content of the law-decision game and the inferences that a judge can make about optimal law. Finally, the law-decision game permits a deeper analysis of which basic structures of the law can and cannot be obtained simultaneously in optimal law.

8 Conclusion

Even if law-makes and decision-makers are fully rational, an optimal law can be not proportional and the choices that the law induces can be cyclic. This, however, only occurs under the specific configuration of preferences and limit penalty for breaking the law of the teen drinking dilemma. Moreover, when the law induces cyclic choice, then the preferences of the law-maker and the decision-maker can be inferred from choice, but not otherwise. Finally, it may be possible to determine optimal law according to preferences of a law-maker, even if these preferences are unknown and cannot be inferred from choice.

9 Appendix: Proofs

For any issue $B \in \mathcal{B}$, let $d_B = \max_{x \in B} u(x)$ be Dee's *preferred alternative in issue* B. Given $D > 0$, for any issue $B \in \mathcal{B}$, let

$$\bar{B}(= \bar{B}_D) = \{x \in B |\ u(x) \geq u(y) - D,\ \forall y \in B,\ y \neq x\}$$

be the *legally feasible subset of* B. That is, $\bar{B} \subseteq B$ is the subset of alternatives in B that Dee might be induced to choose when all other alternatives are given the maximum punishment D. It is easy to see that $\bar{B} \neq \emptyset$. This follows because Dee's preferred alternative d_B in B is always legally feasible since $u(d_B) \geq u(y)$, $\forall y \in B$. Since $\bar{B} \neq \emptyset$, it follows that there is an alternative $l_{\bar{B}} \in \bar{B}$ that is Lee's *most preferred legally feasible alternative in* B. That is,

$$V(l_{\bar{B}}) > V(x) \text{ for all } x \in \bar{B},\ x \neq l_{\bar{B}}.$$

We now show two simple claims. Lemma 1 shows that, in equilibrium, for any D-limited law and optimal choice function and for any issue, Dee always chooses an alternative from the legally feasible subset of alternatives. This must be the case because Dee prefers his most preferred alternative even when it is punished maximally to any alternative that is not legally feasible, even if all of them are not punished.

Lemma 1 For any D-limited law $P \in \mathcal{P}_D$, the optimal choice function C_P is such that

$$C_P(B) \in \bar{B} \text{ for any } B \in \mathcal{B}.$$

Proof of Lemma 1: Suppose there exists a law $P \in \mathcal{P}_D$ and issue $B \in \mathcal{B}$ such that $C_P(B) = a$ where $a \in B$, $a \notin \bar{B}$. Given that $a \notin \bar{B}$ and $d_B \in B$, $u(d_B) - D > u(a)$. Since $U_P(B, d_B) = u(d_B) - P(B, d_B) \geq u(d_B) - D$ and $u(a) \geq U_P(B, a)$ it follows that $U_P(B, d_B) > U_P(B, a)$. Given that $a \in B$ and $d_B \in B$, C_P is not an optimal choice function.■

Lemma 2 shows that if the law induces Dee to choose Lee's most preferred legally feasible alternative in each issue then the law is optimal. This follows because all alternatives that D-limited laws induce are legally feasible.

Lemma 2 For any D-limited law $P \in \mathcal{P}_D$, let C_P be the optimal choice function induced by P. Let $S(P) = C_P$ for all $P \in \mathcal{P}_D$. If $C_{P^*}(B) = l_{\bar{B}}$ for some law $P^* \in \mathcal{P}_D$ and any issue $B \in \mathcal{B}$, then $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is an equilibrium of the law-decision game.

Proof of Lemma 2: By Lemma 1, for any $B \in \mathcal{B}$, $C_{P^*}(B) \in \bar{B}$ and $C_P(B) \in \bar{B}$. Since $l_{\bar{B}} = arg \max_{a \in \bar{B}_D} V(a)$, it follows that for every issue $B \in \mathcal{B}$. $V(C_{P^*}(B)) \geq V(C_P(B))$ for all $P \in \mathcal{P}_D$. Thus, given that $S(P) = C_P$, P^* is optimal for Lee. By definition, given any law $P \in \mathcal{P}_D$, $S(P) = C_P$ is optimal for Dee.■

Conversely, in equilibrium, any optimal choice function selects Lee's most preferred legally feasible alternative in each issue.

Lemma 3 For any D-limited law $P \in \mathcal{P}_D$, let C_P be the optimal choice function induced by P. In any equilibrium $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$, $S(P) = C_P$ for all $P \in \mathcal{P}_D$, and $C_{P^*}(B) = l_{\bar{B}}$ for any issue $B \in \mathcal{B}$.

Proof of Lemma 3: $S(P) = C_P$ for all $P \in \mathcal{P}_D$ by definition. By Lemma 1, $C_P(B) \in \bar{B}$ for any $B \in \mathcal{B}$. Assume, by contradiction, that $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is an equilibrium and for some issue $B_1 \in \mathcal{B}$, $C_{P^*}(\bar{B}_1) \neq l_{\bar{B}_1}$. Then, $C_{P^*}(B) = a$, where $a \in \bar{B}_1$, $a \neq l_{\bar{B}_1}$. By definition of $l_{\bar{B}_1}$, $V(l_{\bar{B}_1}) > V(a)$. Let P be a law such that $P(B_1, l_{\bar{B}_1}) = 0$ and $P(B_1, x) = D$ for all $y \in B_1$, $y \neq l_{\bar{B}_1}$. Then, $C_P(\bar{B}_1) = l_{\bar{B}_1}$. This follows because, by definition, $l_{\bar{B}_1} \in \bar{B}_1$ and $u(l_{\bar{B}_1}) \geq u(y) - D\ \forall y \in B,\ y \neq x$. Thus, $V(C_P(\bar{B}_1)) > V(C_{P^*}(\bar{B}_1))$. Hence, $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is not an equilibrium. A contradiction.■

We can now characterize D-limited optimal laws.

Proposition 1 For any D-limited law $P \in \mathcal{P}_D$, let C_P be the optimal choice function. Let $S \in \mathcal{S}$ be such that $S(P) = C_P$ for all $P \in \mathcal{P}_D$. Let $P^* \in \mathcal{P}_D$. Then, the following is equivalent:

1. $P^*(B, x) \in [\max\{0,\ u(x) - u(l_{\bar{B}}) + P^*(B, l_{\bar{B}})\},\ D]$ for all $B \in \mathcal{B}$, $x \in \bar{B}$, $x \neq l_{\bar{B}}$.
2. $C_{P^*}(B) = l_{\bar{B}}$ for all issues $B \in \mathcal{B}$.
3. $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is an equilibrium.

Proposition 1 shows that, in optimal law, penalties must induce Dee to choose Lee's most preferred legally feasible option.

Proof of Proposition 1 The equivalence between 2 and 3 follows from Lemmas 2 and 3.

We now show that 1. implies 2. Let $P^* \in \mathcal{P}_D$ satisfy the conditions in 1. By Lemma 1 and $P^* \in \mathcal{P}_D$ it follows that $C_{P^*}(B) \in \bar{B}$. Let $x \in \bar{B}$ such that

$x \neq l_{\bar{B}}$. Then, $V(l_{\bar{B}}) > V(x)$ by the definition of $l_{\bar{B}}$ and $P^*(B,x) \geq u(x) - u(l_{\bar{B}}) + P^*(B,l_{\bar{B}})$ by assumption. Thus, $u(l_{\bar{B}}) - P^*(B,l_{\bar{B}}) \geq u(x) - P^*(B,x)$. Hence, $C_{P^*}(B) = l_{\bar{B}}$.

We now show that 2. implies 1. Assume that for some $x \in B$, $x \neq l_{\bar{B}}$, $P^*(B,x) \notin [\max\{0,(u(x) - u(l_{\bar{B}})) + P(B,l_{\bar{B}})\},\ D]$. Given that $P^*(B,x) \in [0,D]$, $P(B,x) < u(x) - u(l_{\bar{B}}) + P(B,l_{\bar{B}})$. Thus, $u(l_{\bar{B}}) - P^*(B,l_{\bar{B}}) < u(x) - P^*(B,x)$. It follows that $C_{P^*}(B) \neq l_{\bar{B}}$.■

Proof of the Ordered Law Proposition: Assume that for all $B \in \mathcal{B}$, $D \neq u(d_B) - u(l_{\bar{B}})$. Given that there are finitely many issues $B \in \mathcal{B}$, only finitely many possibilities on $(0,\infty)$ were ruled out. Given that $d_B \in \bar{B}$ and $l_{\bar{B}} \in \bar{B}$, $u(l_{\bar{B}}) \geq u(d_B) - D$. Given that $u(l_{\bar{B}}) \neq u(d_B) - D$, it follows that $u(l_{\bar{B}}) > u(d_B) - D$ or, equivalently, $D > u(d_B) - u(l_{\bar{B}})$. Let $\varepsilon_B > 0$ be small enough such that $D - 2\varepsilon_B > u(d_B) - u(l_{\bar{B}})$.

For any issue $B \in \mathcal{B}$ relabel the alternatives so that $B = \{b_1, b_2, ..., b_k\}$ such that $V(b_i) > V(b_j) \iff i < j$. That is, Lee prefers alternatives with lower indices. Let $b_{i^*} = l_{\bar{B}} \in \bar{B}$.

Let P^* be such that

$$
\begin{aligned}
P^*(B,b_{i^*}) &= \varepsilon_B; \\
P^*(B,b_i) &= D - \frac{\varepsilon_B}{i} \text{ if } i > i^*; \\
P^*(B,b_i) &= \varepsilon_B(1 - \frac{0.5}{i}) \text{ if } i < i^*.
\end{aligned}
$$

Given that $u(d_B) - u(l_{\bar{B}}) \geq 0$, $\varepsilon_B < 0.5D$. Thus, $\varepsilon_B < D - \varepsilon_B < D - \frac{\varepsilon_B}{i}$ if $i > 1$. Hence, $P^*(B,b_i)$ is strictly increasing in i. Finally, if $b_i \in \bar{B}$, $b_i \neq l_{\bar{B}}$, then $i > i^*$. Thus, $P^*(B,b_i) = D - \frac{\varepsilon_B}{i} \geq D - \varepsilon_B > u(d_B) - u(l_{\bar{B}}) + \varepsilon_B \geq u(b_i) - u(l_{\bar{B}}) + P^*(B,l_{\bar{B}})$. By Proposition 1, $(S,\ P^*) \in \mathcal{S}\text{x}\mathcal{P}_D$ is an equilibrium.■

Proof of Remark 3: Let $D \in I_{\mathcal{T}}$ so there is a triple with limited deterrence $T \in \mathcal{T}$ such that $u(t_3) - u(t_1) > D > \max(u(t_2) - u(t_1), u(t_3) - u(t_2))$. Let $B = \{t_1,t_2\}$, $B' = \{t_2,t_3\}$, $B'' = \{t_1,t_3\}$. Since $V(t_1) > V(t_2)$ and $D > u(t_2) - u(t_1)$ we have $l_{\bar{B}} = t_1$. Similarly $l_{\bar{B}'} = t_2$. But since $u(t_3) - u(t_1) > D$ we have $l_{\bar{B}''} = t_3$. By Proposition 1 it must be the case that $C_{P^*}(\{t_1,t_2\}) = t_1$, $C_{P^*}(\{t_2,t_3\}) = t_2$, $C_{P^*}(\{t_1,t_3\}) = t_3$ and $C_{P^*}(\{t_1,t_2,t_3\}) = t_2$. So, the optimal choice function is cyclic and anomalous.■

Proof of Remark 4: For any pair of alternatives $x,y \in A$, let $O(x,y) = P(A,x) - P(A,y)$. Given that P is proportional, $O(x,y) = P(B,x) - P(B,y)$ for any issue $B \in \mathcal{B}$. Let $z \in A$ be an arbitrary alternative in A. Let $o(z) = 0$ and $o(x) = O(z,x)$ if $x \neq z$. Then, for any issue $B \in \mathcal{B}$, and for any alternative $x \neq z$, $P(B,z) - P(B,x) > 0 \Leftrightarrow O(z,x) > 0 \Leftrightarrow o(x) - o(z) > 0$. For any pair of alternatives $x,y \in A$, $x \neq z$, $y \neq z$, and for any issue $B \in \mathcal{B}$, $P(B,x) - P(B,y) > 0 \Leftrightarrow P(B,x) - P(B,z) - (P(B,y) - P(B,z)) > 0 \Leftrightarrow O(z,x) - O(z,y) > 0 \Leftrightarrow o(x) - o(y) > 0$. The same inequalities hold if $>$ is replaced with $\geq$.■

Proof of Remark 5: Suppose that C_P is anomalous for some law $P \in \mathcal{P}$. Then there must exist issues B and B' such that $\{x,y\} \subseteq B \subset B'$, $C_P(B) = x$ and $C_P(B') = y$. Now $C_P(B) = x$ requires $u(x) - P(B,x) \geq u(y) -$

$P(B,y)$ while $C_P(B') = y$ requires $u(y) - P(B',y) \geq u(x) - P(B',x)$ and since $V(x) \neq V(y)$ one of the inequalities must be strict. If P is proportional, $P(B,x) - P(B,y) = P(B',x) - P(B',y)$. A contradiction.■

Proof of the Proportional Law Proposition: The proof is by construction. We first index the alternatives in A so that $A = \{a_1, a_2, ..., a_n\}$ where $V(a_i) > V(a_j)$ for any a_i, $a_j \in A$ so $i < j$ implies that Lee prefers alternative a_i to alternative a_j. Define a function $p : A \to [0, D]$ be such that

$$p(a_j) = \max_{i \leq j \text{ and } D \geq u(a_j) - u(a_i)} u(a_j) - u(a_i).$$

Note that $p(a_j) \geq 0$ by $j \leq j$ and $u(a_j) - u(a_j) = 0$. In addition, $D \geq p(a_j)$ by construction.

Lemma 4 Assume that Dee's and Lee's utility functions u and V and the maximal penalty D do not form a teen drinking dilemma. Then, for any $i \leq j$ such that $D \geq |u(a_j) - u(a_i)|$ it is the case that $u(a_i) - p(a_i) \geq u(a_j) - p(a_j)$.

Proof of Lemma 4. If $i = j$ the inequality holds trivially. So suppose $i < j$ and $D \geq |u(a_j) - u(a_i)|$. There are two cases to consider: (1) $u(a_j) > u(a_i)$; and (2) $u(a_j) < u(a_i)$.

We begin with case (1) and $u(a_j) > u(a_i)$. Since $i < j$ and $D \geq u(a_j) - u(a_i) > 0$ we must have $p(a_j) \geq u(a_j) - u(a_i)$. If $p(a_i) = 0$, then the inequality is satisfied. If $p(a_i) > 0$, then there exists a $k < i$ such that $D \geq u(a_i) - u(a_k) = p(a_i) > 0$. It follows that $u(a_j) > u(a_i) > u(a_k)$. By $k < i < j$, $V(a_k) > V(a_i) > V(a_j)$. Since $D \geq u(a_i) - u(a_k) > 0$, $D \geq u(a_j) - u(a_i) > 0$, and $D \notin I_{\mathcal{T}}$ we must have $D \geq u(a_j) - u(a_k) > 0$ and so $p(a_j) \geq u(a_j) - u(a_k)$. It follows that $u(a_i) - p(a_i) = u(a_k) \geq u(a_j) - p(a_j)$.

Consider case (2) $u(a_i) > u(a_j)$. If $p(a_i) = 0$, then the inequality holds trivially since $u(a_i) - p(a_i) = u(a_i) > u(a_j) \geq u(a_j) - p(a_j)$. If $p(a_i) > 0$, then as in case (1) there is a $k < i$ such that $D \geq u(a_i) - u(a_k) = p(a_i) > 0$. If $u(a_k) > u(a_j)$, then $u(a_i) - p(a_i) = u(a_k) \geq u(a_j) - p(a_j)$. If $u(a_j) > u(a_k)$, then since $D \geq u(a_i) - u(a_k)$ and $u(a_i) > u(a_j)$ it follows that $D > u(a_j) - u(a_k) > 0$. By $k < j$ and the definition of p, it follows that $p(a_j) \geq u(a_j) - u(a_k)$. Hence, $u(a_i) - p(a_i) = u(a_k) \geq u(a_j) - p(a_j)$.QED

Lemma 5 Assume that Dee's and Lee's utility functions u and V and the maximal penalty D do not form a teen drinking dilemma. Then, for any $B \in \mathcal{B}$ and $x \in B$,

1. if $V(l_{\bar{B}}) \geq V(x)$, then $u(l_{\bar{B}}) - p(l_{\bar{B}}) \geq u(x) - p(x)$ and
2. if $V(x) > V(l_{\bar{B}})$, then $u(l_{\bar{B}}) - p(l_{\bar{B}}) > u(x) - p(x)$.

Proof of Lemma 5: Consider the case $V(l_{\bar{B}}) \geq V(x)$. If $V(l_{\bar{B}}) = V(x)$ then $l_{\bar{B}} = x$ and $u(l_{\bar{B}}) - p(l_{\bar{B}}) = u(x) - p(x)$. Suppose $V(l_{\bar{B}}) > V(x)$. If $D \geq |u(l_{\bar{B}}) - u(x)|$, then the inequality holds by Lemma 4. Suppose that

$|u(l_{\bar{B}}) - u(x)| > D$. By $l_{\bar{B}} \in \bar{B}$, we cannot have $u(x) - D > u(l_{\bar{B}})$. Thus, $u(l_{\bar{B}}) - D > u(x)$. It now follows from $D \geq p(x)$ hat $u(l_{\bar{B}}) - p(l_{\bar{B}}) \geq u(l_{\bar{B}}) - D > u(x) \geq u(x) - p(x)$.

Consider the case $V(x) > V(\bar{l}_{\bar{B}})$. Since $l_{\bar{B}} = \arg\max_{y \in \bar{B}} V(y)$ it must be the case that $x \notin \bar{B}$. Hence, $u(d_B) - D > u(x)$. By $u(d_B) \geq u(d_{\bar{B}})$, $V(l_{\bar{B}}) \geq V(d_B)$ and $u(l_{\bar{B}}) \geq u(d_B) - D$ the conditions of Lemma 4 are satisfied and so $u(l_{\bar{B}}) - p(l_{\bar{B}}) \geq u(d_B) - p(d_B)$. But $u(d_B) - p(d_B) \geq u(d_B) - D > u(x) \geq u(x) - p(x)$. QED

The proof of the Proportional Law Proposition now follows directly from Lemma 5. Define P^* such that for any $B \in \mathcal{B}$ and $x \in B$, $P^*(B, x) = p(x)$. By Lemma 5 part 2, if $V(x) > V(l_{\bar{B}})$, then $u(l_{\bar{B}}) - p(l_{\bar{B}}) > u(x) - p(x)$. Hence, Dee strictly prefers $l_{\bar{B}}$ to x. Therefore, $V(l_{\bar{B}}) \geq V(C_{P^*}(B))$. By Lemma 5 part 1, $U_{P^*}(B, l_{\bar{B}}) \geq U_{P^*}(B, x)$ for all $x \in B$. Assume, by contradiction, that $l_{\bar{B}} \neq C_{P^*}(B)$. Then, either $U_{P^*}(B, l_{\bar{B}}) > U_{P^*}(B, C_{P^*}(B))$ or $U_{P^*}(B, l_{\bar{B}}) = U_{P^*}(B, C_{P^*}(B))$ and $V(l_{\bar{B}}) > V(C_{P^*}(B))$. A direct contradiction in both cases.■

Proof of the Spirit of the Law Theorem: Let P^* and C_{P^*} be the observed D-limited law and choice function, for some unobserved D such that $D \geq \max_{B \in \mathcal{B}, \text{ and } x \in B} P^*(B, x)$. If the observed optimal choice function C_{P^*} is cyclic then it is anomalous. That is, there is a set $B = \{x, y\}$ and a set $B' = \{x, y, z\}$ such that $C_{P^*}(B') \in B$ and $C_{P^*}(B) \neq C_{P^*}(B')$. Without loss of generality, let $C_{P^*}(B) = x$ and $C_{P^*}(B') = y$. By Proposition 1, x is Lee's most preferred legally feasible alternative in B (i.e., $x = l_{\bar{B}}$) and y is her most preferred legally feasible alternative in B' (i.e., $y = l_{\bar{B}'}$). Since $x \in B'$ and $y \in \bar{B}'$ then $u(y) \geq u(x) - D$ by the definition of legal feasibility. It follows that $y \in \bar{B}$ (i.e., y must also be legally feasible in B). But then $x = l_{\bar{B}}$ implies $V(x) > V(y)$ so it is revealed that Lee prefers x to y.

Conversely, assume that the observed choice function C_{P^*} is not anomalous. Define $\bar{D} = \max_{B \in \mathcal{B}, x \in B} P^*(B, x)$. Consider a utility function $\tilde{u}$ that rationalizes C_{P^*} (see Samuelson 1938). That is,

$$\tilde{u}(C_{P^*}(B)) > \tilde{u}(y) \text{ for every } y \in B,\ y \neq C_{P^*}(B).$$

Let u be an utility function that is ordinally the same as $\tilde{u}$ (i.e., $\tilde{u}(x) > \tilde{u}(y) \iff u(x) > u(y)$ for any two options x and y, $x \neq y$) and such that the utility difference between any two alternatives is greater than $\bar{D}$. That is, for any two options x and y, $x \neq y$, if $u(x) > u(y)$, then $u(x) - \bar{D} > u(y)$. The utility function u can be defined recursively as follows: Let $A = \{b_1, ..., b_n\}$ be such that $\tilde{u}(b_i) > \tilde{u}(b_j) \iff i > j$. That is, options with higher indices are preferred io options with lower indices. Let $u(b_1) = \tilde{u}(b_1)$. For any $k \in \{1, ..., n-1\}$, if $u(b_k)$ is defined, then $u(b_{k+1}) = u(b_k) + D + 1$.

Assume that Dee's utility function is u, the limit penalty is $\bar{D}$ and Lee's utility function is an arbitrary function V. It follows that there is only one option that is legally feasible in every issue and, therefore, that option is Dee's preferred legally feasible option. That is, $l_{\bar{B}} = d_B = C_{P^*}(B) = C_P(B)$ for any issue $B \in \mathcal{B}$, and for any law $P \in \mathcal{P}_{\bar{D}}$.

Let S be any choice strategy such that $S(P) = C_{P^*}$ for any law $P \in \mathcal{P}_{\bar{D}}$. By construction, (S, P^*) is an equilibrium for any utility function V. Thus, P^* and C_{P^*} do not reveal any of Lee's preferences.■

References

[1] Ambrus, A., and K. Rozen (2008) "Revealed Conflicting Preferences," mimeo.

[2] Apesteguia, J. and Ballester (2008) "A Characterization of Sequential Rationalizability," mimeo.

[3] Arrow, K. (1950), "A Difficulty in the Concept of Social Welfare," *Journal of Political Economy*, 58(4), 328-346.

[4] Bernheim, D. and A. Rangel (2009) "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics," *Quarterly Journal of Economics,* 124 (1), 51-104.

[5] Bond, P. and K. Hagerty (2010) "Preventing Crime Waves" *American Economic Journal: Microeconomics* 2, 138-159.

[6] Cason, T. and C. Plott (2014) "Misconceptions and Game Form Recognition: Challenges to Theories of Revealed Preference and Framing," *Journal of Political Economy* 122, 1235 - 1270.

[7] Chambers, C. and T. Hayashi (2012) "Choice and Individual Welfare," *Journal of Economic Theory*, 147-5, 1818–1849.

[8] Chambers, C., Echenique, F. and E. Shmaya (2017) "General Revealed Preference Theory," *Theoretical Economics*, 12, 493-511.

[9] Cherepanov, V., Feddersen, T., and A. Sandroni (2013) "Rationalization" *Theoretical Economics* 8-3, 775–800.

[10] de Clippel, G. and K. Eliaz (2012) "Reason-Based Choice: A Bargaining Rationale for the Attraction and Compromise Effects," *Theoretical Economics* 7-1, 125-162.

[11] de Clippel, G. and K. Rozen (2012) "Bounded Rationality and Limited Datasets: Testable Implications Identification, and Out-of-Sample Predictions," mimeo.

[12] DeMarzo, P., Fishman, M., and K. Hagerty (1998) "The Optimal Enforcement of Insider Trading Regulations," *Journal of Political Economy* 106, 602-632.

[13] Detotto C., McCannon, B., M. Vannini (2015) "Evidence of Marginal Deterrence: Kidnapping and Murder in Italy," *International Review of Law and Economics* 41, 63-67.

[14] Dietrich, F. and C. List (2013) "A Reason-based Theory of Rational Choice" *Nous*, 47-1, 1–201.

[15] Echenique, F. (2020) "New Developments in Revealed Preference Theory: Decisions under Risk, Uncertainty, and Intertemporal Choice," *Annual Review of Economics*,12, 299-316.

[16] Ehlers, L. and Y. Sprumont (2008) "Weakened Warp and Top-cycle Choice Rules," *Journal of Mathematical Economics*, 44, 87–94.

[17] Eliaz, K., M. Richter and A. Rubinstein (2011) "Choosing the Two Finalists," *Economic Theory*, 46, 211-219.

[18] Eliaz, K. and R. Spiegler (2011) "Consideration Sets and Competitive Marketing" *Review of Economic Studies*, 78-1, 235-262.

[19] Eliaz, K., D. Ray and R. Razin (2006) "Choice Shifts in Groups: A Decision-Theoretic Basis," *American Economic Review*, 96, 1321-1332.

[20] Freer, M. and C. Martinelli (2016) "A Representation Theorem for General Revealed Preference," George Mason Research Paper Series.

[21] Friedman, D., and W. Sjostrom (1993) "Hanged for a Sheep: the Economics of Marginal Deterrence" *Journal of Legal Studies* 22, 345-366.

[22] Fudenberg, D. and D. Levine (2006) "A Dual-Self Model of Impulse Control," *American Economic Review*, 96, 1449-1476.

[23] Green, J., and D. Hojman (2007) "Choice, Rationality and Welfare Measurement," Harvard University, Working Paper.

[24] Heller, Y. (2012) "Justifiable Choice," *Games and Economic Behavior*, 76-2, 375-390.

[25] Houy, N. (2007) "Rationality and Order-Dependent Sequential Rationality" *Theory and Decision*, 62, 119-134.

[26] Houy, N. and K. Tadenuma (2009), "Lexicographic Compositions of Multiple Criteria for Decision Making," *Journal of Economic Theory*, 144-4, 1770-1782.

[27] Jackson, M. and L. Yariv (2015) "Collective Dynamic Choice: The Necessity of Time Inconsistency," *American Economic Journal: Microeconomics*, 7-4, 150-178.

[28] Kalai, G., A. Rubinstein, and R. Spiegler (2002) "Rationalizing Choice Functions by Multiple Rationales," *Econometrica*, 70 (6), 2481-2488.

[29] Katz, L. and A. Sandroni (2017) "The Inevitability and Ubiquity of Cycling in All Feasible Legal Regimes: A Formal Proof," forthcoming *Journal of Legal Studies*.

[30] Kramer, S. (1990) "An Economic Analysis of Criminal Attempt: Marginal Deterrence and the Optimal Structure of Sanctions," *Journal of Criminal Law and Criminology* 81, 398-417.

[31] Laffont, J., and D. Martimort (2001) "*The Theory of Incentives: The Principal-Agent Model*," Princeton University Press.

[32] Lehrer, E. and R. Teper (2011) "Justifiable Preferences," *Journal of Economic Theory*, 146, 762-774.

[33] Lleras, J., Masatlioglu, Y., D. Nakajima and E. Ozbay (2010) "When More is Less: Limited Consideration," mimeo.

[34] Manzini, P. and M. Mariotti (2007) "Sequentially Rationalizable Choice" *American Economic Review* **97**-5, 1824-1839.

[35] Manzini, P. and M. Mariotti (2011) "Categorize then Choose: Boundedly Rational Choice and Welfare," *Journal of the European Economic Association,*10(5), 1141-1165.

[36] Manzini, P. and M. Mariotti (2012) "Choice by Lexicographic Semiorders," *Theoretical Economics* 7(1), 1-23.

[37] Masatlioglu, Y. and D. Nakajima (2007) "A Theory of Choice by Elimination," mimeo.

[38] Masatlioglu, Y. and E. Ok (2005) "Rational Choice with Status Quo Bias," *Journal of Economic Theory*, 121, 1-29.

[39] Masatlioglu, Y., D. Nakajima and E. Ozbay (2012) "Revealed Attention," *American Economic Review,* 102(5), 2138-2205.

[40] Mookherjee, D. and I. Png (1994) "Marginal Deterrence in Enforcement of Law" *Journal of Political Economy* 102, 1039-1066.

[41] Mongin (1995) "Consistent Bayesian Aggregation," *Journal of Economic Theory*, 66, 313-351.

[42] Ok, E., P. Ortoleva, and G. Riella (2008) "Rational Choice with Endogenous Reference Points," mimeo.

[43] Ray, I. and L. Zhou (2001) "Game Theory via Revealed Preferences," *Games and Economics Behavior* 37, 415-424.

[44] Salant, Y. and A. Rubinstein (2006) "Two Comments on the Principle of Revealed Preferences," mimeo.

[45] Salant, Y. and A. Rubinstein (2006a) "A Model of Choice from Lists," *Theoretical Economics*, 1, 3-17.

[46] Salant, Y. and A. Rubinstein (2012) "Eliciting Welfare Preferences from Behavioral Data Sets," *Review of Economic Studies*, 79-1, 375-387.

[47] Samuelson, P. (1938) "The Empirical Implications of Utility Analysis," *Econometrica*, 6 (4), 344-356.

[48] Sen, A. (1970) "The impossibility of a Paretian Liberal," *Journal of Political Economy*, 78, 152-157.

[49] Shavell, S. (1992) "A Note on Marginal Deterrence" *International Review of Law and Economics* 12, 345-355.

[50] Sprumont, Y. (2000) "On the Testable Implications of Collective Choice Theories," *Journal of Economic Theory*, 93, 205-232.

[51] Spiegler, R. (2002) "Equilibrium in Justifiable Strategies: A Model of Reason-Based Choice in Extensive-Form Games" (2002), *Review of Economic Studies* 69, 691-706.

[52] Stigler, G. (1970) "The optimum Enforcement of Laws," *Journal of Political Economy,* 78, 526-536.

[53] Tyson, C. (2013) "Behavioral Implications of Shortlisting Procedures," *Social Choice and Welfare*, 41- 4, 941–963.

[54] Varian H (2006) "Revealed Preference," *Samuelsonian Economics and the Twenty-First Century*, M Szenberg, L Ramrattan, AA Gottesman 99–115 Oxford, UK: Oxford Univ. Press

[55] Wilde, L. (1992) "Criminal Choice, Non-Monetary Sanctions and Marginal Deterrence: a Normative Analysis" *International Review of Law and Economics* 12, 333-344