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"A Theory of Non-Democratic Redistribution and Public Good Provision"

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Abstract

Two office-motivated politicians compete for power by making promises to citizens. Unlike in voting models, a citizens' political activity is observable and subject to retaliation if the citizen happens to back the wrong politician. In equilibrium, a citizen supports the politician that is expected to win the political contest, not the one who promises her the most. Consequently, the incentives for politicians to redistribute resources and provide public goods are different from those in voting models. Citizens who are better able to coordinate with others receive more resources in equilibrium. A transition to democracy must come about through a citizen-led process, not a politician-led one.

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1 Introduction

A core question in political economy is how political competition for votes shapes the allocation of resources. Votes, however, are not the only form of political activity: politicians also compete for public endorsements, for financial support, and for activists' backing (be it peaceful or not, including the use of force). Unlike voting, these forms of political activity are *not anonymous*, and citizens *may be individually punished or rewarded* for engaging in them.

This paper provides a model of resource allocation where two office-motivated politicians compete for the citizens' *observable* support. In this model, a citizen's political activity may trigger individual retaliation, and a politician's promise to a citizen looks like this:

> "I, a politician, promise you some future benefit, provided I will hold office, *and conditional on you having supported me politically.*"

Except for the last conditional clause, this would be the kind of promise that one sees in voting models. The last clause represents the novelty: an individual citizen will be retroactively penalized if she happens to have backed the wrong politician. This kind of individual retaliation is impossible in voting models because voting is anonymous. Without the protection of anonymity, however, a citizen is exposed to retaliation by politicians. The potential for retaliation makes this setting "non-democratic."

This paper addresses the following question: when individual retaliation is possible, how does political competition shape the allocation of resources? To fix ideas, consider a divide-the-dollar (i.e., purely redistributive) policy space: does political competition push politicians to allocate the dollar in an egalitarian fashion, or does it push them toward the unequal treatment of similarly situated citizens? The voting literature, operating under the assumption that the citizens' political activity is anonymous, is split: some papers find that different politicians have an incentive to create different favored and disfavored groups even among otherwise identical citizens;[1] other papers reach the opposite conclusion.[2] This paper re-examines this question in a different, i.e., non-democratic setting.

The model, from the citizens' perspective, works like this. Each citizen i receives two competing promises conditional on i's support: α_i (vs $\lambda\alpha_i$ if i fails to support) from

[1] The logic is that a politician does not want to waste resources courting groups that are too expensive to win because they are treated nicely by the other politician. See, e.g., Myerson (1993), and Groseclose and Snyder (1996).

[2] Lindbeck and Weibull (1987) find that each group is treated equally by both politicians.

the incumbent; and ω_i (vs $\lambda\omega_i$ if i fails to support) from the challenger. The factor $\lambda < 1$ captures the retaliation visited on citizen i by the winner of the political contest. Individually, citizen i's political behavior is assumed to have no impact on who is in office. Therefore, when choosing whom to (observably) support, citizen i's first-order concern is not comparing α_i with ω_i, which she cannot control but, rather, avoiding being hit with λ, which she can control. This calculus leads the citizen to support whomever is the political contest's expected winner, which she will guess using a (very precise) signal about the incumbent's vulnerability. In this model, a citizen doesn't seek to support the politician who offers her the most but, rather, she seeks to support the winner. This incentive gives rise to a coordination game which will be modeled as global game.

Now, to the main question: how will politicians distribute resources (the α_i's and the ω_i's) across citizens in this game? I identify three principles reflecting strategic considerations that do not arise in the voting literature. The first principle, which I call **incumbent must equalize**, has to do with how politicians must strategically allocate promises in order to win the coordination game, i.e., to maximize the probability that citizens coordinate on supporting her. This objective, I will show, pushes *both* politicians toward equal treatment of identical citizens. However, for the challenger this consideration is qualified by a second principle, which I call **challenger need not equalize**. This principle stems from an added modeling wrinkle: I assume that, before the outcome of the political contest is determined, the incumbent is able to penalize those who (overtly) support the challenger. This penalty, which captures the incumbent's monopoly over the state's repressive apparatus, is strategically different from the application of λ to α_i because it is applied even if the incumbent is eventually replaced. If this penalty is large enough, the challenger will be at such disadvantage that he will be forced to focus his resources (the ω_i's) on a few citizens – a revolutionary vanguard – at the expense of the rest. Therefore, this channel may lead to unequal treatment of identical citizens by the challenger only: in short, the challenger "need not equalize." The third and final principle, which I call **coordination premium**, arises when some citizens lack the necessary information to coordinate with others which, in the model, means that they are not in a position to know whether the conditions are ripe for regime change. These "out of the loop" citizens may be afraid to stick their neck out by (openly) supporting the challenger. Buying these citizens' support may be too expensive for the challenger; this, in turn, implies that the incumbent will not need to spend any resources on these citizens either. As a result, these citizens are neglected by both politicians, in contrast with citizens who have the necessary information to coordinate.

A historical example will serve to illustrate how these principles manifest in economic policy. A disclaimer: the example is discussed at a high level of generality and it is not meant to provide a rigorous historical analysis.

Illustrative example: the three principles at work

Consider Russia circa 1914, before the revolution. The economic policy offered by the incumbent, the Tsarist state, was a small-government, laissez faire model in which wealth flowed to a well-connected elite including rentiers and industrialists. The rural citizens, still a great majority of the population, did not benefit from this economic policy. Industrial workers fared somewhat better due, in part, to the state's intentional efforts to protect them through labor law.[a] The program of the challenger – I shall call it the Radical Left – entailed two phases. First, the brutal taxation of a minority of the rural population would provide the resources for rapid industrialization and agricultural mechanization. Subsequently, highly redistributive policies would benefit all the populace equally.

The model interprets these different economic policies as binding promises: the α_i's and ω_i's promised by politicians to citizens in order to obtain their support. Next, I make the case that these promises were consistent with the above-mentioned three principles.

In failing to protect the rural population but attempting to protect industrial workers, the Tsarist state's economic policy was consistent with the **coordination premium** principle: the rural population did not command a coordination premium because it was geographically dispersed and thus unable to coordinate in an uprising against the state; industrial workers, in contrast, earned the premium because, living and working as they did in close proximity to each other, they were in a position to coordinate. Furthermore, consistent with the **incumbent must equalize** principle, the state redistributed, to some degree, among citizens who had the ability to coordinate: in this case, from elites to industrial workers through labor protections. Consistent with the model, this redistribution was explicitly designed to prevent unrest.[b] That equalization between elites and factory workers was not full is also consistent with the model: a factory worker had less potential impact on regime change than an elite member – and the model will accommodate this heterogeneity.[c]
The Tsarist state's economic policy was successful in preventing a revolt – until conditions changed. When the rural population was conscripted into WWI military service, rural young men suddenly acquired the ability to coordinate their political

activity. In that moment, the Tsarist state's resource allocation came to violate the **coordination premium + incumbent must equalize** principles, which now required an equitable treatment of rural folks as well. The Tsarist policies were now vulnerable to a challenge.

The Radical Left's policies were perfectly suited to the new conditions. Consistent with the **coordination premium + incumbent must equalize** principles, the Left's program entailed egalitarian redistribution among all – except for a minority of farmers. But why would it make sense to disfavor some of the farmers? This is because of the **challenger need not equalize** principle: to compensate against the Tsarist states's power to repress overt dissent, the Radical Left had to focus all the available resources on a subset of the population at the expense of the rest: in the theory, any subset of the population would do. In practice, this fortunate subset happened to be factory workers and a majority (but not all) of the rural folks.

In this account, the Tsarist state's resource allocation was optimally adapted to hold on to power in an environment where the rural men could not coordinate against the regime. The Radical Left's resource allocation was optimally adjusted to gain power in an environment where the rural workers had (temporarily) gained the ability to coordinate, and yet the regime retained police power. The raising of a conscript army provided the Left with the coordination environment it needed for its policy to trigger regime change.[d]

After the Radical Left had won the contest for power and delivered on the promised ω_i's, including retaliation against individuals who had failed to support the new regime,[e] a new set of promises could be elaborated over time.

The theory treats this elaboration as the new incumbent (now, the Left) crafting his own α_i's. The **coordination premium + incumbent must equalize** principles require equal treatment of all citizens with the ability to coordinate. By that time, rural folks had gained the ability to coordinate even in peacetime because they had been rounded up in collective farms. Therefore, it was optimal for the incumbent Radical Left to promise equal treatment across both urban and rural citizens, without exception.

Summing up: the Tsarist state's resource allocation was optimally adapted to hold on to power in an environment where the rural men could not coordinate. The Radical Left's resource allocation was optimally adjusted to gain power in an environment where the rural workers had gained the ability to coordinate, and then evolved to hold on to power in that same environment.

[a]That industrial workers fared better than farmers is proved by strong migration to cities: see Bradley (2022). That the state intentionally protected industrial workers through several labor laws promulgated between 1882 and 1905 is shown by Rimlinger (1960).

[b]That labor protections were designed to prevent unrest is revealed by a specific legislative provision: fines of 100 rubles could be levied on employers who reduced wages illegally, but "*if the reduction had become the cause of labor unrest*, instead of a fine, the penalty was up to three months in prison for the employer and possibly the loss of the right to manage an industrial establishment for a period of two years." (Rimlinger 1960, p. 239, emphasis is mine).

[c]In the model, citizens with $i \in [\eta, 1]$ have less impact on regime change. The model makes the obvious, and directionally correct prediction that low-impact citizens receive fewer resources.

[d]Wildman (1970) describes how in 1917 the Radical Left took over the imperial army.

[e]The Red Terror (1918-22) is the time when the new regime's opponents, both past and present, were brutally dealt with.

In addition to illustrating the three principles, this historical example provides some hints about how the model applies to reality. First, when I say that a policy "is consistent" with the three principles, I do not mean that these principles are inviolable imperatives: rather, they are important (but not exclusive) considerations that, if respected, help an economic policy be successful politically. Other considerations – technological constraints, foreign affairs, the politician's personal greed – may conflict with the three principles, and, when they do, a tradeoff arises in terms of regime stability. Second, when circumstances change, the politicians cannot change their the α_i's and ω_i's sharply and suddenly. For example, once the conscript army was assembled, the Tsarist state could not sharply pivot and, after centuries of neglect, *credibly* promise major redistribution in favor of farmers. This is because the α_i's and ω_i's represent the holistic welfare level that citizen i *actually trusts* the politicians to deliver for her; this trust rests on political values and ideologies that are durable, not on political platforms that can be changed at the drop of a hat.[3] Third, although I have talked about distributing the α_i's and ω_i's as if they were liquid resources, economic theory suggests that redistribution comes at an efficiency cost, e.g., through the deadweight loss of taxation: more on this below.

The paper proceeds as follows. Section 2 presents the formal model. Section 3 characterizes the citizens' behavior for given promises, and Section 4 how politicians make promise in equilibrium, culminating in the first two principles (incumbent must equalize, challenger need not). Section 5 introduces uncoordinated citizens and establishes the third principle (coordination premium). A compare-and-contrast with anonymous voting

[3]To use a business analogy, the α_i's and ω_i's are more like corporate brands than product labels.

models is provided in Section 6: their mechanics are shown to be very different from the present model's.

The efficiency cost of redistribution is introduced in Section 7. Formally, I allow politicians to choose between two policies: one that produces higher aggregate welfare (sum of utilities) but takes away the politician's ability to target resources; and another one that allows the politician to redistribute freely but is less welfare-efficient. The welfare difference between the two policies represents the efficiency cost of redistribution. I find that the two politicians differ in their strategy, with the incumbent being more redistribution-oriented. Furthermore, I highlight the difference between the sources of inefficiency in the present model relative to anonymous voting models.

Finally, in Section 8, I briefly discuss transition from non-democracy to democracy. The discussion starts from the premise that this transition is desirable not only for economic reasons but also for the important freedoms (of speech, of thought, etc.) that democracy affords. The thought experiment is this: suppose that all citizens want to transition to democracy, will office-motivated politicians who compete for the citizens' support commit to enacting democracy? The answer, dismally, is no: democracy implies protection from political retaliation, and this tool is too valuable strategically for any politician to give up unilaterally. I conclude, then, that democratic transition must start from the people, not from the political elites. This finding is somewhat in contrast with the previous literature on franchise extension (Acemoglu and Robinson 2000, Lizzeri and Persico 2004).

The main contribution in this paper is to provide the first model of competitive redistribution that is functional to winning a coordination game among citizens. The general question is this: when we depart from anonymous voting as a mechanism for replacing the incumbent, but keep the same policy space (divide-the-dollar, or variants of it) as in redistributive voting models, what forces shape the policies that are chosen (targeted vs universalistic policies) and which citizens are rewarded? I find that benefits are targeted to citizens who have the ability to coordinate their political activity (**coordination premium**), which is novel because the question of coordination does not arise in voting models with two politicians.[4] I also find that a sufficiently repressive state apparatus places the challenger at such a disadvantage that he may be forced to "give up" on some potential supporters (**challenger need not equalize**) – another channel which is not

[4]Virtually all voting models with two politicians assume that each voter has a dominant strategy, which is to vote for the politician that offers her the greatest utility. Hence, the question of coordination is moot.

present in voting models. Finally, and most novel in my opinion, I find that the incumbent will propose egalitarian policies among those who have the ability to coordinate (**incumbent must equalize**).

The existing literature features models of non-democratic redistribution but, in these models, the coordination game is not contestable. In the classic model by McGuire and Olson (1996), for example, the incumbent faces no competition for power. More recent models, including Padro i Miquel (2007), De Mesquita and Smith (2010), and Francois et al. (2015), assume that incumbents do face competition; however, as in Baron and Ferejohn (1989), only the incumbent can create coalitions. As a consequence, these coalitions are built such that citizens never revolt, whereas in my model they do with positive probability. The fundamental difference is that, in these models, citizen coordination is not really contestable because only one politician at a time (the incumbent) has the power to propose agendas. In this same vein, Jia et al. (2021) provide a model where an autocrat seeks to maintain power by promising resources to two groups. As in my paper, the autocrat has the power to "claw back" some of its promises from the citizens that failed to support him, but their paper does not address political competition (the challenger's strategy is fixed exogenously).[5] My paper innovates on the existing literature by studying *competitive* (i.e., *contestable*) political redistribution *that is functional to winning a coordination game.*

The literature also features many papers that model citizen coordination as global games, as I do. In these papers the outcome of interest is typically the probability of incumbent replacement as a function of exogenous parameters.[6] But two papers, to my knowledge, focus on the provision of material incentives to citizens by a single politician to prevent (Bueno de Mesquita and Shadmehr 2023) or promote (Morris and Shadmehr 2023) revolutions. These papers do not consider redistributive policies nor, indeed, competition

[5]Also, Jia et al. (2021) does not nest anonymous voting as a special case, which is a contribution of the present paper. Somewhat similar to Jia et al. (2021), Besley and Kudamatsu (2007) study redistribution between two groups of voters. Roemer (1985) studies a redistributive game between a challenger and an incumbent whose promises are fixed exogenously; the focus is whether the challenger's promises are more egalitarian than the incumbent's. In none of these papers is redistribution functional to winning a coordination game. Less related are papers where the policy space is not redistributive. Within this class of papers, Myerson (2008) highlights the commitment problem that an autocrat faces in promising benefits; I have simply assumed away this commitment problem. Guriev and Treisman (2020) develop a theory where the incumbent autocrat survives if the media say good things about her, and so an autocrat will invest resources in state-controlled media. Bidner et al. (2015) focus on "minimal democracies" where incumbents step down after they lose elections, and they ask why incumbents do so even if they have the power to resist the transition. Acemoglu et al. (2008, 2010, 2012, 2015) study relatively unstructured environments where institutions are minimal, and derive the features of "stable" regime types.

[6]See, e.g., Shadmehr and Bernhardt (2011), Hollyer et al. (2015), and the literature cited therein.

between politicians. My paper innovates on this literature by looking at the coordination of citizens *as a motive for competitive political redistribution.*

2 Baseline model

Society is a mass one of identical citizens indexed by $i \in [0,1]$. Two office-motivated politicians, an incumbent (she) and a challenger (he), simultaneously make promises to citizens. Based on these promises, citizens simultaneously choose either $a_i = 0$ ("support incumbent") or $a_i = 1$ ("support challenger"). If enough citizens support the challenger, the incumbent will be replaced by the challenger. The incumbent (resp., the challenger) maximize the probability of retaining (resp., acquiring) power. The game is described next.

Stage 1: promises The incumbent promises $\alpha_i \geq 0$ to citizen i if the citizen supports her, and $\lambda\alpha_i$ otherwise. This promise is only kept if the incumbent retains power. The incumbent's promises must satisfy the resource constraint:

$$\int_0^1 \alpha_i \, di = B_1 > 0. \tag{1}$$

The parameter λ lies in $[0,1)$.

Simultaneously, the challenger promises $\omega_i \geq 0$ to individual i if the citizen supports him, and $\lambda\omega_i$ otherwise. This promise is kept only if the incumbent is ousted. The challenger's promises must satisfy the resource constraint:

$$\int_0^1 \omega_i \, di = B_2 > 0. \tag{2}$$

It is natural to assume that $B_1 = B_2$, meaning that both politicians redistribute the same-size pie and so no politician enjoys an advantage, but that will not be necessary for the analysis. The case $B_1 \neq B_2$ may capture the idea that politicians have different *valence*. Constraints (1) and (2) mean that the policy space is purely redistributive. Later, this policy space will be expanded to include a public good.

Stage 2: imponderables A realization θ is drawn from a Uniform distribution with support $\left[\underline{\theta}, \overline{\theta}\right]$. This random draw represents the accidents of history that determine the incumbent's vulnerability at the time when she is challenged. I assume $\underline{\theta} < 0 < 1 < \overline{\theta}$. Citizen i observes a private signal $z_i = \theta + \sigma\varepsilon_i$, where ε_i is i.i.d. independent of θ and has support $[-1/2, 1/2]$, and $\sigma \in (0, 1]$ is a scaling factor that determines the precision of i's signal.

Henceforth, I will focus on the limit as $\sigma \to 0$, that is, on the case of very precise signals. Later, I will explore the case where some citizens do not observe z_i.

Stage 3: citizens take action Every citizen i contemplates z_i and the entire vectors of promises $\boldsymbol{\alpha} = \{\alpha_i\}$ and $\boldsymbol{\omega} = \{\omega_i\}$. Then, all citizens simultaneously choose $a_i \in \{0, 1\}$. Citizen i's payoff is as follows:

	Incumbent replaced	**Incumbent survives**	
$a_i = 1$ (support challenger)	$\omega_i - k$	$\lambda\alpha_i - k$	(3)
$a_i = 0$ (support incumbent)	$\lambda\omega_i$	α_i	

The parameter $k \geq 0$ represents the penalty that the incumbent imposes through the state's repressive apparatus on those who support the challenger. This penalty applies whether or not the incumbent survives.

Stage 4: regime change The incumbent is replaced if:

$$a = \int_0^\eta a_i \, di \geq 1 - \theta. \tag{4}$$

The incumbent's vulnerability is increasing in θ. When choosing promises $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$, the incumbent (the challenger) minimizes (maximizes) the probability that event (4) happens.

The number a measures the political support for the challenger. Citizens with $i \in [0, \eta]$ are said to have political voice: if more than $1 - \theta$ among them choose $a_i = 1$, the incumbent is replaced. Citizens with $i \notin [0, \eta]$ are politically voiceless: their actions do not affect regime change.

2.1 Discussion of modeling assumptions

Continuum of citizens Modeling citizens as a continuum allows me to use the law of large numbers, as in Myerson (1993). The index i could refer to a citizen or to an identifiable group of citizens. For example, i could represent "factory workers" or "workers in a given factory." For expositional brevity I will henceforth refer to i as a citizen.

What a_i represents The action $a_i = 1$ represents taking an observable stand against the regime, including by speaking out or protesting: it is citizen i's contribution to removing the incumbent. The action $a_i = 0$ is citizen i's contribution to keeping the incumbent in office: typically, this means not speaking out and keeping one's head down.

Abstention There is no third action besides $a_i = 0$ or 1: the model does not allow citizens to "sit out" the contest. This assumption implies that failing to support either politician, i.e., "being apolitical," is functionally the same as supporting the incumbent.

What λ represents The parameter $\lambda < 1$ captures political retaliation by the winner of the political contest. It is the fraction of a citizen's economic status that she is allowed to retain after having supported the "wrong" politician. For example, an autocrat who, after surviving a power struggle, allows the "disloyal" citizens to be discriminated against in the workplace, imposes a cost λ on them. When λ is close to zero, citizens are highly vulnerable to political retribution: such a system might be called illiberal because every citizen's welfare is conditioned on her political activity. Conversely, when λ is close to 1 a citizen's political activity has almost no effect on her economic status. Citizens can avoid paying $(1 - \lambda)$ by supporting the winner of the political contest. The case $\lambda = 1$ will be treated separately in Section 6.

The assumption that λ is the same for both politicians is without loss of generality: in the analysis, the parameters $(1 - \lambda)$ and B_j play the same role, so the equilibrium of a game where politician j has greater ability to retaliate is the same as that in which the politician has a larger budget B_j.[7]

What k represents In contrast to λ, which captures ex post retaliation, k captures the incumbent's repression of dissent *during the contest for power*. What I mean by "during" is that k is applied even if the incumbent ends up losing the contest or, put differently, the penalty k is not voided if the challenger ends up winning: refer to the payoff matrix (3). In this sense, unlike λ, the penalty k is applied unconditionally on the outcome of the political contest.

[7]This is shown in the proof of Proposition 1.

In non-democratic regimes, k may be large: loss of job, beatings, imprisonment, or worse. When $k > 0$ the challenger has a disadvantage, so the model is asymmetric. If k is large, this disadvantage will prompt the challenger to make inequitable promises to citizens (refer to the discussion following Corollary 1).

What η represents The set $[0, \eta]$ represents the fraction of citizens who, collectively, have the power to replace the incumbent. In military coups, for example, the set $[0, \eta]$ could represent those citizens who have weapons – the military.

What θ represents The realization θ represents the incumbent's capacity to successfully cope with a given amount of dissent. If θ is low, the incumbent may survive even though many citizens oppose her. For example, a cunning chief of police or interior minister may be captured by a low θ. It is important for the analysis that θ is realized after the promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$ have been committed to.

What α and ω represent Because θ is realized after the promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$ have been committed to, the promises must be interpreted not as *tactical* redistribution that can easily be adjusted within the space of, say, months but, rather, as strategic commitments that are difficult to substantially alter in the short run. Refer to footnote 3 and the discussion preceding it.

Coordination game For any constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, the payoff matrix (3) reveals that every citizen who is promised little by the challenger (specifically, $\omega_i \leq k/(1-\lambda)$) has a dominant strategy to support the incumbent; the rest of the citizens are locked in a *coordination game*. The two politicians set $\boldsymbol{\alpha}$ and $\boldsymbol{\omega}$ competitively to win this coordination game. In equilibrium, citizens who don't have a dominant strategy will coordinate their actions perfectly for almost all values of θ.[8] As a consequence, there are no failed challenges in equilibrium.

What politicians maximize Incumbent replacement is determined by condition (4). Before θ is realized, the incumbent (resp., the challenger) chooses her/his promises so as to minimize (resp., maximize) the ex ante probability that condition (4) holds. This amounts to minimizing (resp., maximizing) the left-hand side of (4).

[8] See Section 3.

Role played by the prior distribution over θ The prior distribution over θ does not affect the equilibrium behavior of citizens: indeed, equation (5) below does not depend on $\underline{\theta}$ or $\overline{\theta}$. Intuitively, citizens do not need to rely on the prior over θ because, before acting, they observe an extremely precise signal about θ that supersedes the prior. By manipulating the support $\left[\underline{\theta}, \overline{\theta}\right]$ from which θ is drawn one can, without changing the citizens' behavior, obtain settings where, mechanically, the incumbent has either a very large (if $\underline{\theta}$ is a very large negative number so condition (4) almost never holds) or a very small (if $\overline{\theta}$ is positive and very large so condition (4) almost always holds) ex ante probability of retaining power, and the contest's outcome is almost predetermined. But, even in such circumstances, since the politicians focus exclusively on the event when the citizens' actions make a difference, condition (4) drives the politicians' promises entirely. This observation shows that the width and location of the interval $\left[\underline{\theta}, \overline{\theta}\right]$ are strategically inconsequential, meaning that it will not affect the equilibrium resource allocation, although they will affect the ex ante probability that the incumbent is replaced.

Contest observability While, in the model, the incumbent runs the risk of being replaced, this risk may not be manifest to an outside observer. This is because citizens coordinate perfectly in equilibrium, so whenever the realized θ is low *all* citizens do not take action (i.e., select $a_i = 0$). Thus, failed challenge are never observed in equilibrium, even though the incumbent is in fact at risk.

How the model can nest anonymous voting If a_i is interpreted as anonymous voting, rewards and punishment cannot be conditioned on a_i; so the voting interpretation requires $k = 0$ and $\lambda = 1$. I will study this case in Section 6.

3 Citizens' behavior for given politicians' promises

This section solves for the citizen's equilibrium behavior given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. The main result is that all citizens will coordinate on supporting the incumbent if θ, which they observe almost perfectly, satisfies the inequality (5). Else, all citizens coordinate on supporting the challenger. The right-hand side of (5) will be called the *vulnerability index.*

Citizens' equilibrium behavior Given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, citizens are engaged in a "global game" with potentially heterogeneous players. Heterogeneity is the norm rather than the exception: it arises whenever any two citizens receive different promises, i.e., whenever $(\alpha_i, \omega_i) \neq (\alpha_j, \omega_j)$.

A strategy for citizen i is a mapping from her signal z_i into $\{0, 1\}$. In a similar setting, Sakovics and Steiner (2012) show that, in equilibrium, individual i supports the challenger if and only if her signal z_i exceeds a personal threshold z_i^*. Conveniently, as $\sigma \to 0$, all the thresholds z_i^* converge to a common limit θ^*.[9] The fact that the limit threshold is independent of i *even when different citizens receive different promises*, while not obvious, is intuitive: as signals become sufficiently precise, any player using a personal threshold different from θ^* knows that, around her threshold, she is miscoordinating with everyone else, which she does not want because her action alone cannot change the outcome but miscoordination will saddle her with λ.

The fact that all citizens have the same limit threshold implies that citizens never miscoordinate in equilibrium as $\sigma \to 0$. In the limit, moreover, since i's strategy is independent of α_i and ω_i, an individual citizen's equilibrium behavior is *independent of the promises that the individual received* and depends, instead, on the *profile of societal promise*s $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. This property reflects the fact that citizen i's actions are not dictated by comparing α_i with ω_i but, rather, by avoiding being hit with λ, which leads the citizens to support whomever is the political contest's expected winner. Therefore, in this model individual behavior is dictated *solely by the collective perception of regime stability.*

The above remarks apply to all citizens *except* those who are promised little by the challenger. Specifically, citizens who are promised less than $k/(1-\lambda)$ have a dominant strategy to support the incumbent – refer to the dominant strategy property discussed at page 12.

Equilibrium probability of replacement: the vulnerability index The next lemma, which is adapted from Sakovics and Steiner (2012), is key to the rest of the paper.[10] For ease of exposition, the lemma is stated in the case $(\eta, \lambda) = (1, 0)$; I will remove this restriction in Sections 4 and ff.

Lemma 1 (key lemma: vulnerability index) *Suppose* $(\eta, \lambda) = (1, 0)$. *Given a constellation of promises* $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, *as* $\sigma \to 0$ *the equilibrium condition for incumbent survival*

[9] Sakovics and Steiner (2012), Proposition 1.

[10] The adaptation is required to account for the fact that, if ω_i is small enough, it is a dominant strategy for citizen i to support the incumbent if $k > 0$, a circumstance which violates a maintained assumption in Sakovics and Steiner (2012).

converges to:

$$1-\theta > \underbrace{\int_0^1 \frac{\omega_i - k}{\omega_i + \alpha_i} \cdot \mathbf{1}\left[\omega_i \geq k\right] \; di}_{\text{incumbent vulnerability index}} . \tag{5}$$

Proof. See the appendix. ■

The right hand side of (5) is the formula that characterizes $1-\theta^*$ where, per the above discussion, θ^* is the limit threshold to which all the personal thresholds z_i^* converge. The right hand side of (5) will be called the *vulnerability index.* The vulnerability index represents the collective perception of regime stability for any given constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. Henceforth I will assume that the incumbent (resp., the challenger) minimize (resp., maximize) the vulnerability index, which is the same as minimizing (maximizing) the probability that event (4) happens.

The vulnerability index has the expected properties. The integrand is between zero and one and thus so, too, is the index; this implies that, regardless of the promise profiles $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, the incumbent survives when $\theta < 0$ and is ousted when $\theta > 1$. The indicator function $\mathbf{1}[\cdot]$ ensures that the mass of citizens who are promised $\omega_i \leq k$ do not contribute to the index, regardless of α_i: this reflects the fact that, for them, supporting the challenger is a dominated strategy. In the region $\omega_i \geq k$, the integrand is increasing in ω_i and decreasing in α_i. These properties are intuitive: the incumbent is less vulnerable when the incumbent's promises are more generous and the challenger's promises are less generous. As expected, the index is nonincreasing in k, meaning that incumbent replacement is less likely when the cost of supporting the challenger is high.

A rough intuition for the functional form in (5) is the following. Set $\lambda = 0$ in the payoff matrix (3). At her personal threshold z_i^*, citizen i must be indifferent between supporting either politician. Hence, the citizen's expectation of incumbent survival p_i must solve:

$$p_i \cdot \Delta_\alpha + (1-p_i) \cdot \Delta_\omega = 0,$$

where $\Delta_\alpha = -k - \alpha_i$ (resp., $\Delta_\omega = \omega_i - k$) represents citizen i's incentive to support the challenger over the incumbent in the event that the incumbent survives (resp., is ousted). Solving for p_i yields:

$$p_i = \frac{\Delta_\omega}{\Delta_\omega - \Delta_\alpha} = \frac{\omega_i - k}{\alpha_i + \omega_i}.$$

This functional form is reminiscent of the argument of the integral in equation (5).[11]

[11] To spell things out further: since θ is drawn from a uniform distribution, the ex ante probability of

Finally, it is remarkable that, when $k = 0$, the right hand side in equation (5) looks like the objective function in a multi-battlefield Tullock contest. Such contests have been studied in the literature under the name of "lottery Colonel Blotto" games, but the players' objective function has hitherto been assumed, not derived from a global game structure.[12] It is remarkable, and unnoticed so far to my knowledge, that when politicians compete to win a coordination game by redistributing resources, they are engaged in a "lottery Colonel Blotto" game.

Takeaways from this section For a given constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, all citizens coordinate on supporting the incumbent if θ, which they observe almost perfectly, satisfies the inequality (5). Else, all citizens coordinate on supporting the challenger.

4 Politicians' equilibrium promises

This section shows that the incumbent must treat all voiceful citizens equally ("incumbent must equalize" principle), but the challenger may not ("challenger need not equalize" principle). I start with an intuitive derivation of these two principles.

Intuitive derivation of the "incumbent must equalize" and "challenger need not equalize" principles The functional form of the vulnerability index affords an intuitive derivation of the "incumbent must equalize" and "challenger need not equalize" principles. Suppose, for the purpose of developing intuition only, that $(\eta, \lambda) = (1, 0)$ so that Lemma 1 can be applied directly. Then, the incumbent seeks to *minimize* the right hand side in (5) subject to the budget constraint (1). The right hand side in (5) may be written as:

$$\int_0^1 v(\alpha_i, \omega_i; k) \; di, \tag{6}$$

event (5) is a linear affine function of the right hand side of (5). Maximizing or minimizing this function is the same as maximizing or minimizing its argument, which is similar to p_i. This intuition is not yet complete because p_i is citizen-specific, but in a global game we expect coordination to be approximately perfect when $\sigma \to 0$, meaning that all the p_i's must converge to the same number for almost all θ's. Sakovics and Steiner (2012) deliver the last part of the intuition, showing that this common number is the average of all p_i's. The integral in equation (5) is exactly this average, where the indicator $\mathbf{1}[\omega_i \geq k]$ ensures that the average is taken only over those citizens who don't have a dominant strategy to vote for the incumbent.

[12] See Friedman (1958), Snyder (1989), and Kovenock and Rojo Arjona (2019).

where I denote:

$$v(\alpha, \omega; k) = \left(\frac{\omega - k}{\omega + \alpha}\right) \cdot \mathbf{1}[\omega \geq k]. \tag{7}$$

Since v is a convex function of α, if the challenger treats all citizens symmetrically then (6) is minimized by promising every citizen an equal share of the budget. This is the root of the "incumbent must equalize principle."

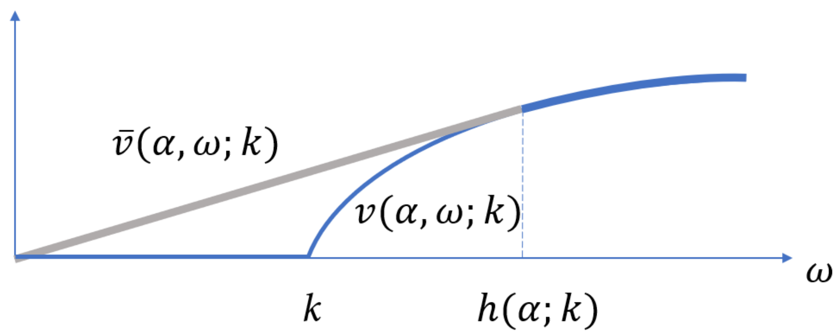

Figure 1: Why the challenger may redistribute inequitably. The incumbent promises α. If the challenger's budget is below $h(\alpha; k)$, the challenger benefits from making inequitable promises: in fact, the incumbent's vulnerability is maximized by promising $h(\alpha; k)$ with some probability, and 0 with complementary probability. If the challenger's budget is greater than $h(\alpha; k)$, the challenger's best response is to treat every voiceful citizen equally.

The challenger's problem is somewhat more complex: he seeks to *maximize* (6), but v is *not* a globally concave function of ω: refer to Figure 1, which plots v as a function of ω. Therefore the challenger may benefit from not treating all citizens equally. To understand why, suppose the incumbent promises the same α to all citizens, and form the *concave envelope* $\overline{v}(\alpha, \omega)$ as a function of ω. Figure 1 plots v and $\overline{v}$: the concave envelope $\overline{v}$ is never smaller than v, and it is strictly greater for small values of ω. If the challenger's available resources b_2 are less than $h(\alpha; k)$, equal treatment only gets the challenger $v(\alpha, b_2; k)$; however, the challenger can attain the full $\overline{v}(\alpha, b_2; k)$ through the following inequitable strategy: each citizen is promised $h(\alpha; k)$ with some probability, and zero with complementary probability, with the probability being chosen such that the budget constraint is met. The fact that the challenger may deviate from equal treatment is due to the disadvantage embodied in k: if the challenger treats everyone equally, he risks spreading his resources too thin. Thus it is optimal for the challenger to use an inequitable strategy: the "challenger need not equalize."

Finally, the challenger must pick the lucky citizens randomly to prevent the incumbent from "picking off" the most receptive citizens. This randomness produces the ex ante symmetric treatment of citizens which was a maintained assumption in the derivation of the "incumbent must equalize" principles.

Formal analysis I now formalize the intuition developed above and remove the restriction that $(\eta, \lambda) = (1, 0)$. In addition, I provide comparative static results on η, κ, and λ.

A politician's strategy is defined very generally as a set of probability distributions from which the voters' promises are drawn. The probability distributions are allowed to depend on the citizens' identities;[13] however, following Myerson (1993) I will restrict attention to equilibria in symmetric strategies, i.e., in strategies such that the promises to all voiceful voters are drawn from the same distribution (note: this is not a restriction on the strategy space).[14] The formal definition follows.

Definition 1 (symmetric strategy) *A strategy is called symmetric if promises to citizens are drawn from a probability distribution that conditions only on whether a citizen has voice.*

Symmetric strategies require that promises to all voiceful citizens be drawn from a single probability distribution. This does not imply that they will all receive the same promise: indeed, if the probability distribution is non-degenerate, different voiceful citizens will receive different promises. Although I do not prove this here, it is likely that all equilibria are in symmetric strategies because non-symmetric strategies are vulnerable: if a politicians' strategy is not symmetric, it allows the opponent to identify and pick off pools of citizens who might be particularly responsive to his/her promises.

Proposition 1 (equilibrium promises) *Assume $\lambda < 1$. For candidate $j = 1, 2$ denote*

$$\overline{B}_j = \frac{(1-\lambda)}{\eta} B_j,$$

and

$$h(\alpha; k) = k + \sqrt{k\alpha + k^2}. \tag{8}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

[13] For example: citizen i may be promised 2 and citizen i' may be promised 4 with probability 1/2, and 6 with probability 1/2.

[14] In other words, I will have to show that deviating to non-symmetric, i.e., personalized strategies is not profitable in equilibrium.

1. ***(politicians don't waste resources)*** *Voiceless citizens are promised zero by both politicians.*

2. ***(incumbent must equalize)*** *The incumbent promises all voiceful citizens the egalitarian distribution, i.e.,* $\alpha_i^* = B_1/\eta$ *for all* $i \in [0, \eta]$.

3. ***(conditions under which challenger equalizes)*** *If* $\overline{B}_2 \geq h\left(\overline{B}_1; k\right)$ *the challenger promises all voiceful citizens the egalitarian distribution, i.e.,* $\omega_i^* = B_2/\eta$ *for all* $i \in [0, \eta]$.

4. ***(conditions under which challenger does not equalize)*** *If* $\overline{B}_2 < h\left(\overline{B}_1; k\right)$ *the challenger promises the voiceful citizens an inequitable distribution: some of them, chosen at random, are offered* $h\left(\overline{B}_1; k\right) / (1 - \lambda)$*, the rest are offered zero.*

Proof. See the Appendix. ■

Part 1 is obvious: voiceless citizens get zero because no rational politician would waste resources on citizens with no political power. The rest of the proposition, intuitively, says that the incumbent distributes her budget equally among all voiceful citizens, but the challenger only does so if he "can afford it," meaning that the disadvantage is below some threshold (part 3). The challenger's disadvantage is expressed in terms of the function h, which is increasing in the incumbent's budget and in k. If the challenger's disadvantage exceeds the threshold (part 4), the challenger is better off making disparities: he must give zero to a subset to the voiceful citizens in order to give enough to the rest, as explained intuitively earlier in this section.

The rescaled budgets $\overline{B}_1$ and $\overline{B}_2$ represent "incentive budgets:" they can be interpreted as the amount of *incentives* (as opposed to resources) that the politicians have available to distribute, relative to the size of k. The incentive budgets are more generous when η and λ are small. This is intuitive: when η is small, more is left over to distribute to the voiceful citizens after the voiceless citizens are expropriated; and when λ is small, the incentives available to the politicians are, in effect, more powerful relative to k. The incentive budgets $\overline{B}_1$ and $\overline{B}_2$ determine whether the challenger will promise the egalitarian distribution because they appear in the inequality in part 4 of the proposition. Inspecting this inequality immediately yields the following comparative statics.

Corollary 1 (factors that lead to inequitable promises by the challenger)

1. *If* $k = 0$ *the challenger promises all voiceful citizens the egalitarian distribution.*

2. *If $k > 0$ the challenger is more likely to treat the voiceful citizens inequitably if:*
   (a) *k is larger*
   (b) *his budget B_2 is smaller relative to the incumbent's budget B_1.*
3. *If $k > 0$, the challenger treats the voiceful inequitably for λ sufficiently close to one.*
4. *The challenger treats the voiceful equally for η sufficiency close to zero.*

All these comparative static reflect a simple principle: k has a special role in creating unequal promises by the challenger. Indeed, when $k = 0$ the challenger does not create arbitrary inequality. But when k is large enough, the challenger will allocate incentives unequally among identical citizens because the marginal potency of the challenger's incentives is non-monotonic. Indeed, a citizen will not respond to small incentives due to the dominant strategy property discussed at page 12. But once incentives are large enough, their marginal potency is positive and decreasing. This non-monotonicity creates an incentive to treat citizens differently when the challenger has a small budget. Ultimately, the special role that k has in generating inequality is due to the fact that k can create dominant strategies in a global game, and this is possible because the penalty administered by k in *unconditional on the outcome of the political contest*: refer to the discussion at page 11.

Remark 1 (limit behavior when $k \to 0$ and $\lambda \to 1$) *Proposition 1 requires $\lambda < 1$. What happens to the politicians' behavior as $\lambda \to 1$? Proposition 1 says that the incumbent's strategy is unaffected: equal treatment for the voiceful prevails. The challenger's limit strategy depends on k: if $k = 0$, the challenger treats the voiceful equally for any $\lambda < 1$. If $k > 0$, as λ approaches 1 the challenger's limit strategy is not well defined: the incumbent concentrates all his resources on a vanishing mass of citizens and gives zero to the rest. If, simultaneously, $k \to 0$ and $\lambda \to 1$, the limit may or may not be well defined, depending on the relative speed of convergence.*

My interpretation of Remark 1 is that taking the limit $(k, \lambda) \to (0, 1)$ is not an insightful exercise. Indeed, the case $(k, \lambda) = (0, 1)$ will be analyzed separately in Section 6 and shown to have quite different properties from the limit $(k, \lambda) \to (0, 1)$.

Takeaways from this section The incumbent must treat voiceful citizens equally ("incumbent must equalize" principle), but the challenger may not ("challenger need not equalize" principle). Voiceless citizens are promised zero by both politicians.

5 Uncoordinated citizens

In this section I introduce a pool of citizens who, while voiceful, do not observe the signal z_i about the realization of θ. I call these citizens "uncoordinated" because they lack the information that "coordinated" (i.e., regular) citizens use. Unlike coordinated citizens, whose behavior does not depend on their prior over θ (refer to the discussion at page 13), the behavior of uncoordinated citizens is sensitive to the prior. The reason is simple: uncoordinated citizens must choose knowing only the promises that politicians made, so they must rely on their prior over θ to guess who will be the winner; whereas coordinated citizens do not need to rely on the prior because, when they act, they have an extremely precise signal about θ which supersedes the prior. This section shows that, if the prior says that the incumbent is sufficiently likely ex ante to hold on to power, uncoordinated citizens are so inclined to "go with their prior" that they become too expensive for the challenger to woo. As a result, they receive nothing from the challenger – and from the incumbent as well.

The model is as follows. Fix some $\mu \in (0, 1]$. Suppose citizens with index $i \in [0, \mu]$ are coordinated, meaning that each observes z_i before choosing their action, and those with index $i \in (\mu, 1]$ are uncoordinated, meaning that they do not observe z_i. Politicians simultaneously choose how to split their budgets B_1 and B_2 between the coordinated and uncoordinated citizen pools, and how to distribute this budget within each pool. After observing the entire profile of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$ and, in the case of the coordinated citizens, the signal z_i, all citizens simultaneously choose a_i. More detail about the model is provided in Appendix A.3.

Proposition 2 *For any given constellation of parameters $\eta, \mu < 1, k > 0, \lambda < 1, B_1, B_2, \overline{\theta}$ there exists a threshold $\underline{\theta}^T < 0$ such that for all values of $\underline{\theta} < \underline{\theta}^T$, in every symmetric equilibrium both the challenger and the incumbent promise no resources to the uncoordinated. In these equilibria all the resources go to the coordinated and they are distributed according to Proposition 1 with the parameter η being replaced by $min[\eta, \mu]$.*

Proof. See the Appendix. ■

In the above proposition, the condition that $\underline{\theta}$ be small enough (i.e., sufficiently negative) means that the prior probability of a realization θ that could catalyze regime change is sufficiently small. This condition is realistic: for example, in the illustrative example at page 4, the rural population, while living in the countryside, had every reason to fear rising up *at any given time* (that is, based only on the prior probability that θ is large enough) because, in the absence of *specific information* that everyone else was also likely to rise up (that is, precise information about the realization θ), rising up at any given time would, most likely, mean rising up alone and getting hit with k. This last statement says that the rural population perceived a small prior probability that the realization θ would be large enough. Note that this argument relies on $k > 0$, which is in fact a required assumption in Proposition 2.

The political economy literature has not, to my knowledge, formally made the point that citizens who are able to coordinate in order to affect regime change are, thereby, able to extract more resources from the political system. However, this point has been made descriptively in many different contexts.[15] Proposition 2 contributes to the literature by establishing this result within a formal model.

Takeaways from this section If the incumbent is sufficiently likely ex ante to hold on to power, uncoordinated citizens become too expensive for the challenger to woo. As a result, they receive nothing from the challenger. This, in turn, implies that the incumbent will not need to spend any resources on these citizens either.

6 Contrast with anonymous voting

The goal of this section is to illustrate how and why the mechanics of the present model are different from the mechanics of voting models. I will show that, when $k = 0$ and $\lambda = 1$, the model can be interpreted as a voting model. This section analyzes what happens in our model when $k = 0$ and $\lambda = 1$ and compares it to the case where, as in the previous sections, $\lambda < 1$.[16]

For the action a_i to be interpreted as anonymous voting it must be that rewards and punishment cannot be conditioned on a_i; hence, it must be that $k = 0$ and $\lambda = 1$. In this case, the payoff matrix (3) reduces to the following matrix.

[15]For example, focusing on post-colonial Africa, Bates (2014) argues that politicians distorted their policy agendas in order to favor city dwellers at the expense of rural dwellers because the former are coordinated while the latter are not.

[16]A maintained assumption in the previous sections is that $\lambda < 1$, see page 9.

Anonymous voting	Incumbent replaced	Incumbent survives
$a_i = 1$ (vote for challenger)	ω_i	α_i
$a_i = 0$ (vote for incumbent)	ω_i	α_i

(9)

Unlike payoff matrix (3), this matrix admits no individual retaliation: citizen i receives whatever the winning candidate has promised her, irrespective of who she voted for. Also, since $k = 0$ here, voters are not especially wary of voting for the challenger.

Matrix (9) presents a familiar issue in voting models: because an individual's vote has a negligible impact on the outcome, technically, voters are indifferent as to who they vote for or even whether to vote at all. Conventionally, this issue is resolved by stipulating that citizens behave *as if* their vote matters; so they vote, and they vote for the candidate that promises them the most. Following this convention I, too, stipulate that when faced with the payoff structure in matrix (9), citizen i votes for challenger if (and only if) $\omega_i > \alpha_i$.

With this stipulation about voter behavior, minimizing the probability that event (4) happens becomes the same as solving:

$$\min_{\{\alpha_i\}} \int_0^\eta \mathbf{1}\left[\omega_i > \alpha_i\right] di. \tag{10}$$

Expression (10) should be familiar: the indicator function $\mathbf{1}[\cdot]$ captures the challenger's vote share which, in a voting model, the incumbent indeed seeks to minimize. Conversely, for the challenger, maximizing the probability that event (4) happens is the same as maximizing (10). The appearance of expression (10) confirms that, when $k = 0$ and $\lambda = 1$, the previous sections' model is strategically equivalent to (i.e., gives politicians the same incentives as) a voting model.[17]

Expression (10) clarifies why in many anonymous voting models – including Myerson (1993), and Groseclose and Snyder (1996) – politicians seek to create different favored and disfavored groups among otherwise identical citizens. This is because, unlike the function v defined in (7), the vote share (10) is *not* a convex function of $\{\alpha_i\}$: rather, it is a step function. Therefore, unlike problem (6), minimizing the challenger's vote share (10) is not achieved by treating all citizens equally but, rather, by promising $\omega_i + \varepsilon$ to citizens with relatively low ω_i, and nothing to the rest. The logic, essentially, is that politicians do

[17] A caveat: this is strategic equivalence, not outcome equivalence because, when $k = 0$ and $\lambda = 1$, the equilibrium probability of winning is not necessarily 50% for each candidate. For that, one must further assume that $\underline{\theta} + \overline{\theta} = 1/2$.

not want to waste resources courting groups that are very well-treated by their opponent. This logic is very different from the one that drives resource allocation in the previous sections' model.[18]

A second major difference between the previous sections' model and voting models has to do with the citizens' coordination (or lack thereof). In the previous sections' model, coordinated citizens vote based on z_i and, in the limit, they are able to coordinate perfectly; as a result, politicians receive either 100% or 0% support among the coordinated. In voting models, in contrast, citizens need not concern themselves with what other citizens do or which politician is more likely to win: it is a dominant strategy to vote for the politician who offers them the most, and each politician receives 50% support in equilibrium.[19]

The next table summarizes some key differences between the mechanics of voting models and those of the present model.

Key differences between voting models and this model

	Anonymous voting	**This model**
Citizen allocates her support based on	individual promise	collective perception of regime stability (*)
Politicians receive	50% of the votes	either 0 or 100% of the coordinated citizens' support
Politicians create different favored groups among similarly situated citizens	depends on ancillary assumptions (**)	tendency toward equal treatment of similarly situated citizens (§)
Citizen's ability to coordinate (†)	does not determine how much they are promised	determines how much they are promised

Notes: (*) Refer to the discussion at page 14. (**) With the assumptions made in Myerson (1993) or Groseclose and Snyder (1996), politicians create different favored groups. With the assumptions made in Lindbeck and Weibull (1987), they do not. (§) Similarly situated means "similarly voiceful and coordinated." If k is small, both politicians treat voiceful and coordinated citizens the same; else, only the incumbent does. Non-voiceful citizens are promised zero by both politicians, as are uncoordinated citizens under the assumptions of Proposition 2. (†) Technically, this means the ability to observe z_i.

[18] Lindbeck and Weibull (1987) make assumptions designed to smother this logic, and reach the conclusion that politicians treat identical citizens equally. However, this happens for very different reasons than the ones presented in the previous sections' model.

[19] Correspondingly, matrix (9) makes it clear that information about z_i or θ is not helpful to the voter. Moreover, expression (10) in equilibrium must equal 0.5 whenever $B_1 = B_2$.

Takeaways from this section This sections shows that, when $k = 0$ and $\lambda = 1$, the model can be interpreted as a voting model. In this voting model, which happens to coincide with Myerson (1993), equilibrium resource allocation is qualitatively different that in the case $\lambda < 1$ (readers who are curious about the limit $\lambda \to 1$ may refer to Remark 1 at page 20). Other notable voting models of redistributive politics exist (e.g., Lindbeck and Weibull 1987); but regardless of which voting model one picks, their mechanics differ qualitatively from the present paper.

7 Provision of an egalitarian public good

In this section I enlarge the policy space by adding a policy which I call an *egalitarian public good*. I interpret the egalitarian public good as the use of available tax revenue to pay for a policy with broad-based benefits, for example: agricultural machines that improve the productivity of collective farms (G); or universal education/health care (G); or national defense (G). This section characterizes public good provision, then compares and contrasts it with public good provision under anonymous voting.

The model is as follows. I start by setting $B_1 = B_2 = B$, which means that both politicians have access to the same amount of resources. I assume that either politician can either invest all of B to produce a public good that gives exactly $G > 0$ to each citizen; or, alternatively, the politician can redistribute B as s/he was free to do in the previous section. Just like the benefits of redistribution are decreased by λ for citizens who failed to back the winning politician so, too, are the benefits from the public good.[20] Finally, in this section I assume that $\underline{\theta} < \underline{\theta}^T$ so that, as stated in Proposition 2, neither politician benefits from promising anything to the uncoordinated.

Equilibrium public good provision Providing the public good is socially efficient whenever $G > B$. However, politicians only care about pleasing citizens who are simultaneously voiceful and coordinated, that is, citizens with index $i \in [0, min[\eta, \mu]]$. Therefore, if $G < B/min[\eta, \mu]$, no politician will promise the public good because promising redistribution and targeting it entirely to the pool of voiceful and coordinated citizen delivers more total value to them. So, whenever $G \in (B, B/min[\eta, \mu])$, the public good will not be provided even though it is socially efficient. The root cause of this inefficiency is that,

[20]This implies that the public good must be *excludable*. This is not a strong assumption: in authoritarian regimes, citizens can be excluded from the enjoyment of most public goods through coercion, incarceration, or worse. More on this at page 27 below.

if a politician promises the public good, s/he loses the ability to target resources to the voiceful and coordinated citizens. The next proposition describes the equilibrium.

Proposition 3 ***(provision of an egalitarian public good)*** *Assume $\lambda < 1$ and $\underline{\theta} < \underline{\theta}^T$. Suppose $B_1 = B_2 = B$, and denote:*

$$\begin{aligned} \overline{B} &= \frac{(1-\lambda)}{min[\eta,\mu]} \cdot B \\ \overline{G} &= (1-\lambda)\, G \\ M &= \max\left[\overline{B}, \overline{G}\right]. \end{aligned}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

1. ***(politicians don't waste targetable resources)*** *Both politicians, if they promise redistribution, promise nothing to citizens who are voiceless or uncoordinated.*

2. ***(incumbent equalizes)*** *The incumbent promises the public good if, and only if, $G \geq B/min[\eta,\mu]$. If the incumbent promises redistribution, she treats the voiceful and coordinated citizens equally.*

3. ***(conditions under which the challenger equalizes)*** *If*

$$v\,(M, M; k) \geq \overline{v}\left(M, \overline{B}; k\right), \tag{11}$$

the challenger uses the same strategy as the incumbent. Else, the challenger promises unequal redistribution among the voiceful and coordinated citizens.

Proof. See the Appendix. ■

Parts 1 and 2 say that the incumbent will promise whatever is best for the voiceful and coordinated citizens, be it redistribution (featuring expropriation of the voiceless/uncoordinated) or the public good. Therefore, the incumbent operates as the faithful agent of the voiceful and coordinated citizens. In contrast, the challenger does not necessarily promise what's best for the voiceful and coordinated citizens: part 3 says that, if condition (11) fails, the challenger will promise inequitable redistribution among the voiceful and coordinated.

As mentioned before Proposition 3, if the equilibrium is socially inefficient, it is because of public good under-provision. The next result speaks to the factors that lead to public good (under-)provision in equilibrium.

Corollary 2 (Factors that lead to public good underprovision)

1. *For any given triple* (B, η, μ), *parameters* $G > B/min[\eta, \mu]$ *and* $k > 0$ *exist such that the challenger does not promise the public good even though the voiceful and coordinated citizens would prefer that it be provided.*

2. *For both challenger and incumbent, the set of values* (B, G) *such that the public good is promised grows with* $min[\eta, \mu]$.

3. *The probability that the incumbent promises the public good is independent of* λ. *The challenger promises inequitable redistribution for all* $\lambda > (G-k)/G$.

4. *Given any pair* $\left(M, \overline{B}\right)$, *for any* k *that is small enough the challenger promises the policy preferred by the voiceful and coordinated citizens.*

Proof. See the Appendix. ■

Part 1 establishes that an underprovision problem can exist for any pair (B, η), even if $\eta = \mu = 1$. This inefficiency arises because, when k is large, the challenger wants to target large benefits to a small fraction of the citizens, and using redistribution gives him this flexibility. Part 2 says that, as η or μ increase, the public good is more likely to be provided (which, as discussed above, is socially desirable). This is because as η or μ increases there are fewer voiceless and uncoordinated citizens on which, from a strategic perspective, the benefits of the public good are "wasted." Part 3 is subtle. As λ grows, i.e., as citizens become more protected from political retribution, the incumbent's promises don't change (which may still fall short of the socially efficient provision level); in contrast, for large enough λ the challenger will under-provide the public good. This effect arises because the challenger's "incentive budget" shrinks relative to k, which increases the challenger's disadvantage relative to the incumbent, leading the challenger to create inequality among the voiceful. Part 4 says that, if k is small, even the challenger operates as a faithful agent of the voiceful and coordinated citizens, meaning that the public good will be provided if and only if it is optimal for them.

Contrast with public good provision under anonymous voting The provision of an egalitarian public good under anonymous voting, i.e., in the special case of this model where $(k, \lambda) = (0, 1)$, has been studied in Lizzeri and Persico (2001). The forces that shape public good provision under anonymous voting are quite different from the ones highlighted in Corollary 2. Next, I discuss two key differences.

First, the present analysis has relied on the implicit assumption that the public good is excludable: refer to footnote 20. If the public good is non-excludable, meaning that its benefits cannot be scaled back selectively for citizens who supported the wrong politician, Proposition 3 does not apply: a non-excludable public good will not be provided by either politician because it does not give citizens any incentive to support either politician. In contrast, under anonymous voting, Lizzeri and Persico (2001) show that even an excludable public good will sometimes be provided in equilibrium. The reason for the difference is that, in the present paper, the incentives that citizens receive from politicians are much more powerful than the incentives that voters receive under anonymous voting (on this point, compare payoff matrices(3) and (9)). In a world of powerful incentives, the weak incentives provided by non-excludable public goods are not worth the politicians' investment in the public good.

Second, in the present model the considerations that lead to underprovision of an egalitarian public good are of two types: the public good "wastes" benefits on citizens with index $i > min[\eta, \mu]$ who are either voiceless or uncoordinated; and, from the challenger's perspective only, the public good does not give the flexibility to focus sufficient resources on a few citizens, which might be necessary when k is large (refer to Corollary 2). To confirm this insight, observe that absent these considerations, i.e., if $\eta = \mu = 1$ and $k = 0$, Proposition 3 says that public good provision is efficient. The story is different under anonymous voting: Lizzeri and Persico (2001) show that an egalitarian public good is underprovided in equilibrium *even if* $\eta = \mu = 1$ and $k = 0$.[21] The difference arises because, in the present model, the politicians' incentives to create favored and disfavored groups are purely a function of k, η, and μ; but, in anonymous voting models, these incentives are independent of these parameters, and more elemental: refer to the discussion at page 23. These elemental incentives to treat citizens unequally in anonymous voting militate against egalitarian public good provision. In sum, the factors that drive the unequal treatment of citizens, and hence the strategic penalty for politicians of promising an egalitarian public good, are fundamentally different in anonymous voting models than in the present model.[22]

[21]In anonymous voting $k = 0$ by definition, and all citizens can vote in Lizzeri and Persico (2001) so $\eta = 1$. As for μ, this parameter doesn't play any role in anonymous voting so, technically, we can set it to 1.

[22]A small caveat: in Lindbeck and Weibull's (1987) anonymous voting model, the incentives to treat citizens differently are less elemental: they depend on the citizens differential sensitivity to the politicians' promises. Nevertheless, my main point stands: the factors that drive equal or unequal treatment are different in anonymous voting models.

Takeaways from this section When providing a public good comes at the cost of losing the ability to target resources, the public good will be underprovided from a social perspective. In equilibrium, the incumbent will promise whatever is best for the voiceful citizens, be it redistribution or the public good. The challenger has an extra bias toward redistribution. The factors that drive the unequal treatment of citizens, and hence the strategic penalty for politicians of promising an egalitarian public good, are fundamentally different in anonymous voting models than in the present model.

8 Transition to democracy: some lessons from the model

Transition from non-democracy to democracy is desirable not only for economic reasons but, also, for the important freedoms (of speech, of thought, etc.) that democracy affords. This section asks: if politicians could set the rules of the political game, would they ever commit to transitioning to democracy? Within this model the answer is, unfortunately, negative: even if all citizens prefer democracy, and despite the competition between politicians, in this model, politicians will not commit to transition to democracy.

Within the model presented in this paper, transition to democracy means transitioning to a game where: both politicians have an equal chance of winning, every citizen has voice, supporting the challenger entails no special risk, no retaliation by the winner of the political contest is possible, and information about θ does not matter to the citizens' decision process. Technically, this means transitioning to the parameter constellation studied in Section 6, where $(\eta, k, \lambda) = (1, 0, 1)$ and, consequently, θ does not matter; in addition, $\underline{\theta} + \overline{\theta}$ must equal 1/2 so that both politicians have a fair chance of winning (refer to footnote 17).

Before getting started, a caveat: a formal model of transition is not presented here – that would require too much additional modeling, and space is limited. So, here, I merely sketch out some of the forces that the present model brings out. It must be said clearly that this section is not rigorous: it is an attempt, based on the previous rigorous sections, to glimpse into the forces that shape the evolution of the rules of the political game.

Let's, then, look at the incentives for an office-motivated incumbent to commit to democracy. If the status quo is as in Proposition 2, meaning that the incumbent has a higher-than-50% chance of winning the contest, then she would not want to move to democracy because her probability of winning would plummet mechanically to 50% by definition of democracy. This is realistic: fair competition is not something that incumbent autocrats long for.

As for the challenger, suppose he commits to implementing democracy immediately after winning the political contest. Democracy forbids retaliation against citizens who failed to support him. In this case, the payoff matrix changes from (3) to:

	Incumbent replaced	**Incumbent survives**
$a_i = 1$ (support challenger)	$\omega_i - k$	$\lambda\alpha_i - k$
$a_i = 0$ (support incumbent)	ω_i	α_i

(12)

Because λ has been set to 1 in the first column only, it is a dominant strategy for the citizen to support the incumbent *no matter how appealing ω_i might be.* Therefore, even if switching to democracy produces great present or future benefits for citizens (i.e., large ω_i's), no citizen will reward the challenger for committing to such a switch. Instead, citizens will defect *en masse* to the incumbent (while, perhaps, secretly hoping that the challenger prevails).

The reason why citizens would not support a pro-democracy challenger is fear: democracy means committing to $\lambda = 1$, and that unilateral commitment by the challenger removes any fear of supporting the incumbent. Meanwhile, citizens still fear supporting the challenger, so their choice is clear (if dismal). Therefore, an office-motivated challenger will not commit to implement democracy immediately after winning the political contest. The same logic extends to more complicated commitment schemes where the incumbent commits to switching to democracy one period after gaining power, two periods after, etc.

Takeaways from this section The existing literature (Acemoglu and Robinson 2000, Lizzeri and Persico 2004) suggests that politicians in non-fully democratic systems commit to extending democracy in order to stay in power. Not so here. The reason is that committing to democracy implies forgoing political retaliation, and this tool is too valuable strategically for any politician to give up unilaterally. In this paper, therefore, committing to democracy actually *reduces* a politician's chances of keeping (or obtaining) power. The lesson, then, is that if democracy breaks out, it must do so through a citizen-led process, not through a politician-led one.

9 Conclusions

Beyond votes, politicians also compete for public endorsements and other forms of political support which are *observable* and for which citizens *may be individually punished or rewarded*. This paper provides a model of resource allocation by two office-motivated politicians who compete for the citizens' observable support. The promises made by the politicians set up a coordination game among citizens. This is the first model, to my knowledge, of competitive redistribution that is functional to winning a coordination game among citizens.

In this model, citizens don't behave like voters, and politicians make promises that are qualitatively different from the promises made by candidates in an election. Whereas each voter in an election would favor the candidate who promises her the most, citizens in this model support the politician who they expect will win the political contest. And, whereas candidates in an election would make promises to create favored and disfavored groups, or court especially responsive groups, the forces that drive promises in this model are quite different. The incentives for politicians to provide public goods are different, as well, from electoral incentives, and therefore so are the sources of inefficient public good provision. Moreover, citizens who are better able to coordinate with others receive, by virtue of this fact alone, more resources in equilibrium. This last point is intuitive but, to my knowledge, it had not previously been made within a formal model.

The politics in this model is non-democratic because citizens are exposed to retaliation and because politicians are not replaced through free and fair elections. Still, the model can help highlight some forces, and frictions, in the transition to democracy. The analysis suggests that here, unlike in the literature on the extension of the franchise (Acemoglu and Robinson 2000, Lizzeri and Persico 2004), a transition to democracy will not come about through a process that is intentionally guided by the politicians but, rather, it must come about through a citizen-led process.

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A Online Appendix: Proofs and ancillary results

A.1 Proofs for Section 3

Proof of Lemma 1.

Proof. Citizen i's payoff is:

	Regime change	Status quo
Citizen i supports challenger	$\omega_i - k$	$-k$
Citizen i supports incumbent	0	α_i

Subtracting α_i from the right-hand side column does not alter the citizen's incentives, so we get:

	Regime change $(a \geq 1-\theta)$	Status quo $a < 1-\theta$
Citizen i supports challenger	$\omega_i - k$	$-\alpha_i - k$
Citizen i supports incumbent	0	0

For notational convenience we set

$$b_i = (\omega_i + \alpha_i), c_i = (\alpha_i + k),$$

so that we get:

	Regime change $(a \geq 1-\theta)$	Status quo $(a < 1-\theta)$
Citizen i supports challenger	$b_i - c_i$	$-c_i$
Citizen i supports incumbent	0	0

Now partition citizens into equally treated groups, so that all members of a group g receive the same b_g, c_g. In this setting, Sakovics and Steiner (2012, Proposition 1) show that, in equilibrium, group g supports the challenger if and only if $z_i \geq z_g^*$, as $\sigma \to 0$, all thresholds converge to a common limit $\theta^* = \sum_g m_g \frac{c_g}{b_g}$, so that incumbent survives if and only if:

$$\theta < \sum_g m_g \frac{c_g}{b_g}. \tag{13}$$

This formula, however, requires $b_g > c_g$ (this is a maintained assumption in Sakovics and Steiner 2012). If this condition is violated for some group g' then that group supports the incumbent for sure (dominant strategy). Lemma 1 claims that when $b_g \leq c_g$ is permitted, the equilibrium condition for incumbent survival is:

$$
\begin{aligned}
\theta - 1 &< \sum_g m_g \left(\frac{c_g}{b_g} - 1 \right) \mathbf{1} \left[b_g \geq c_g \right] \\
&= \sum_g m_g \cdot \left(\frac{\alpha_g + k}{\omega_g + \alpha_g} - 1 \right) \cdot \mathbf{1} \left[\omega_g \geq k \right].
\end{aligned}
$$

To derive this condition observe that if $b_{g'} \leq c_{g'}$ for some group g' then that group does not revolt for sure. In that case, we can eliminate group g' from the game, and there is a new game with new weights

$$
\widetilde{m}_g = \frac{m_g}{\sum_{g \neq g'} m_g}.
$$

Let's express the condition on behavior for incumbent survival (same as in the old game) using the new-game notation. The condition on behavior using the old notation is:

$$
\begin{aligned}
\sum_{g \neq g'} m_g a_g &\leq 1 - \theta \\
\sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{1 - \theta}{\sum_{g \neq g'} m_g} \\
\sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{\sum_{g \neq g'} m_g - \sum_{g \neq g'} m_g + 1 - \theta}{\sum_{g \neq g'} m_g} \\
\sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{\sum_{g \neq g'} m_g - 1 + \theta}{\sum_{g \neq g'} m_g} \\
\sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{-m_{g'} + \theta}{\sum_{g \neq g'} m_g} \\
\sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \widetilde{\theta}.
\end{aligned}
$$

The condition on behavior for incumbent survival in the new game involves the transformed random variable $\widetilde{\theta}$. Plug into the Sakovics-Steiner condition (13) to get the equilibrium condition (on primitives, not on behavior) for incumbent survival in the new game:

$$\begin{aligned}
\frac{-m_{g'}+\theta}{\sum_{g\neq g'} m_g} &< \sum_{g\neq g'} \widetilde{m}_g \frac{c_g}{b_g} \\
-m_{g'}+\theta &< \sum_{g\neq g'} m_g \frac{c_g}{b_g} \\
\theta &< m_{g'}\cdot 1+\sum_{g\neq g'} m_g \frac{c_g}{b_g}
\end{aligned}$$

So, letting g' index any group such that $b_g \leq c_g$, the equilibrium condition for survival (now back in old game notation) is:

$$\begin{aligned}
\theta &< \sum_g m_g \cdot \mathbf{1}\left[b_g < c_g\right] + \sum_g m_g \cdot \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= \sum_g m_g \cdot \left\{1-\mathbf{1}\left[b_g \geq c_g\right] + \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \sum_g m_g \cdot \left\{1+\left(\frac{c_g}{b_g}-1\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \left(\sum_g m_g\right) + \sum_g m_g \left(\frac{c_g}{b_g}-1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1+\sum_g m_g \left(\frac{c_g}{b_g}-1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1+\sum_g m_g \cdot \left(\frac{\alpha_g+k}{\omega_g+\alpha_g}-1\right) \cdot \mathbf{1}\left[\omega_g \geq k\right].
\end{aligned}$$

Note that the condition reduces to the Sakovics-Steiner condition (13) when $b_g > c_g$. Rearranging the above inequality we get the following expression for the equilibrium condition for survival:

$$\begin{aligned}
1-\theta &> \sum_g m_g \left(1-\frac{\alpha_g+k}{\omega_g+\alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right] \\
&= \underbrace{\sum_g m_g \left(\frac{\omega_g-k}{\omega_g+\alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right]}_{\text{incumbent vulnerability index}}.
\end{aligned}$$

In my setting there is a continuum of targetable units, so the integral sign must replace the summation sign. With this replacement Lemma 1 is proved. ■

A.2 Proofs for Section 4

Lemma 2 ***(characterizing*** $\overline{v}$***)*** *The concave envelope* $\overline{v}$ *of the function* v *defined in (7) has the following form:*

$$\overline{v}(\alpha,\omega;k)=\begin{cases}\frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}\cdot\omega & \text{for } \omega<h(\alpha;k)\\ v(\alpha,\omega;k) & \text{for } \omega\geq h(\alpha;k)\end{cases},$$

where

$$h(\alpha;k)=k+\sqrt{k\alpha+k^2}. \tag{14}$$

Proof. Compute the derivative at any point ω:

$$\frac{dv}{d\omega}(\alpha,\omega;k)=\frac{\alpha+k}{(\omega+\alpha)^2}\cdot\mathbf{1}[\omega\geq k]. \tag{15}$$

Now compute the slope r_α of the ray going through any $v(\alpha,\omega;k)$ with $\omega>k$:

$$r_\alpha=\frac{v(\alpha,\omega;k)}{\omega}=\frac{1}{\omega}\left(1-\frac{\alpha+k}{\omega+\alpha}\right). \tag{16}$$

At the tangency point $\omega=h(\alpha;k)$ it must be $v'=r_\alpha$. Use this condition to solve for $h(\alpha;k)$:

$$\begin{aligned}\frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(1-\frac{\alpha+k}{h(\alpha;k)+\alpha}\right)\\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(\frac{h(\alpha;k)-k}{h(\alpha;k)+\alpha}\right)\\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)} &= \frac{1}{h(\alpha;k)}(h(\alpha;k)-k)\\ (\alpha+k)h(\alpha;k) &= (h(\alpha;k)-k)(h(\alpha;k)+\alpha)\end{aligned}$$

Solving for $h(\alpha;k)$ yields two solutions: $k\pm\sqrt{k\alpha+k^2}$, but we are looking for the one exceeding k, so the relevant solution is:

$$h(\alpha;k)=k+\sqrt{k\alpha+k^2}.$$

The slope r_α is:

$$\begin{aligned} r_\alpha &= v'\left(h\left(\alpha;k\right);\alpha\right) \\ &= \frac{\alpha+k}{\left(h\left(\alpha;k\right)+\alpha\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k\alpha+k^2}\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k}\sqrt{\alpha+k}\right)^2} \\ &= \frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}. \end{aligned}$$

■

Proof of Proposition 1.

Proof. First, suppose $\eta < 1$. No rational politician would make any positive promises to powerless citizens. This proves part 1.

Now I state the politicians' problems when (η, λ) does not necessarily equal $(1, 0)$. Refer back to matrix (3), and subtract $\lambda\alpha_i$ from the right-hand column and $\lambda\omega_i$ from the left-hand one. This does not alter the citizen's incentives, and results in:

	Incumbent replaced	**Incumbent survives**	
$a_i = 1$ (support challenger)	$(1-\lambda)\,\omega_i - k$	$-k$	(17)
$a_i = 0$ (support incumbent)	0	$(1-\lambda)\,\alpha_i$	

This game is strategically equivalent to the case $\lambda = 0$ that was analyzed in Lemma 1 except that here: the politician's control variables are $x_i = (1-\lambda)\,\alpha_i$ and $y_i = (1-\lambda)\,\omega_i$; and, also, the mass of voiceful citizens is $\eta < 1$. We now show that the latter difference is strategically irrelevant for voters.

Consider a game where the set of players is $[0, \eta]$, payoffs are given by (17), and the condition for regime change is (4). Then one can define a strategically equivalent "replica game" where the set of players is $[0, 1]$, payoffs are still given by (17), and the condition for regime change is now:

$$\int_0^1 a_i \; di \geq \frac{1-\theta}{\eta}.$$

Lemma 1 required only two assumptions on the distribution of $1-\theta$ in order to yield the right-hand side in (5). First, the random variable $1-\theta$ must be uniformly distributed; second, the interval $\left[\left(1-\overline{\theta}\right),\left(1-\underline{\theta}\right)\right]$ must be a superset of $[0,1]$. Since these assumptions have been made already, it follows that for any $\eta \in (0,1]$, the random variable $\xi = (1-\theta)/\eta$ is uniformly distributed and, furthermore, ξ's support is a superset of $[0,1]$ being equal to the interval $\left[\left(1-\overline{\theta}\right)/\eta,\left(1-\underline{\theta}\right)/\eta\right]$. Therefore, after replacing ξ for $(1-\theta)$, the replica game satisfies all the conditions required by Lemma 1. It follows that the value of η does not affect the citizens' equilibrium behavior in the replica game, and therefore the politicians' objective function is described by Lemma 1 except that $\{\boldsymbol{\alpha},\boldsymbol{\omega}\}$ are replaced by $\{\mathbf{x},\mathbf{y}\}$.

Therefore in the original (not the replica) game the incumbent seeks to minimize:

$$\int_0^\eta \frac{y_i-k}{y_i+x_i}\cdot \mathbf{1}\left[y_i\geq k\right]\ di. \tag{18}$$

With the change of variables, and taking account of the fact that voiceless citizens must receive zero, the incumbent's budget constraint (1) rewrites as:

$$\int_0^\eta \frac{x_i}{(1-\lambda)}\ di \leq \frac{B_1}{\eta}.$$

Multiplying through by $(1-\lambda)$ yields:

$$\int_0^\eta x_i\ di \leq \overline{B}_1.$$

The challenger's problem is dealt with symmetrically.

Incumbent's best response: In either case 3 or 4, the challenger's strategy may be described as follows. The challenger sets $y_i = y^*$ with probability p independent of i, and $y_i = 0$ with probability $(1-p)$. Using expression (7) for v we may write the incumbent's problem as:

$$\min_{\mathbf{x}} \int_0^\eta p \cdot v\left(x_i, y^*; k\right)\ di. \tag{19}$$
$$\text{s.t.} \int_0^\eta x_i\ di \leq \overline{B}_1.$$

The function v is symmetric and strictly convex in $\mathbf{x}$ because $y^* = h\left(\overline{B}_1; k, \lambda\right) > k$ (refer to expression 7), so the solution to problem (19) is $x_i^* = \overline{B}_1$ for all $i \in H$, or $\alpha_i^* = B_1/\eta$.

Challenger's best response. The challenger maximizes incumbent vulnerability, i.e., expression (18), given $x_i^* = \overline{B}_1$ for all i. Using expression (7) for v we may write the challenger's problem as:

$$\max_{\mathbf{y}} \int_0^\eta v\left(\overline{B}_1, y_i; k\right)\ di \tag{20}$$
$$\text{s.t.} \int_0^\eta y_i\ di \leq \overline{B}_2.$$

Let $\overline{v}\left(\alpha, \omega\right)$ denote the concave envelope of $v\left(\alpha, \omega\right)$ (refer to Figure 1). The following problem

$$\max_{\mathbf{y}} \int_0^\eta \overline{v}\left(\overline{B}_1, y_i; k\right)\ di \tag{21}$$
$$\text{s.t.} \int_0^\eta y_i\ di \leq \overline{B}_2.$$

is a relaxed version of problem (20) because $\overline{v}\left(\overline{B}_1, y; k\right) \geq v\left(\overline{B}_1, y; k\right)$. Because the objective function in problem (21) is symmetric and concave in $\mathbf{y}$, the problem's solution is $y_i = \overline{B}_2$ for all i. Therefore the value of the relaxed problem must be $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right)$.

In case 3, $\overline{B}_2 \geq h\left(\overline{B}_1; k\right)$ implies $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) = v\left(\overline{B}_1, \overline{B}_2; k\right)$ (refer to Lemma 2). Therefore the value of the relaxed problem (21) is achievable in the original problem (20) by setting $y_i^* = \overline{B}_2$ for all $i \in [0, \eta]$. This implies that $y_i^* \equiv \overline{B}_2$, or $\omega_i^* = B_2/\eta$ for all $i \in [0, \eta]$, is the solution to the original problem (20).

In case 4, $\overline{B}_2 < h\left(\overline{B}_1;k\right)$ implies $\overline{v}\left(\overline{B}_1,\overline{B}_2;k\right) > v\left(\overline{B}_1,\overline{B}_2;k\right)$, and so the value of the relaxed problem (21) is *not* achievable in the original problem (20) by setting $y_i^* = \overline{B}_2$ for all i. By construction of the concave envelope we have:

$$\overline{v}\left(\overline{B}_1,\overline{B}_2;k\right) = v\left(\overline{B}_1,h\left(\overline{B}_1;k\right)\right)\cdot\frac{\overline{B}_2}{h\left(\overline{B}_1;k\right)}, \tag{22}$$

where $h(\alpha;k)$ is as in expression (8) in light of Lemma 2. Expression (22) shows that the value of the relaxed problem is achievable in the original problem (20) by promising $y_i^* = h\left(\overline{B}_1;k\right)$, or $\omega_i^* = h\left(\overline{B}_1;k\right)/(1-\lambda)$ to a mass $\overline{B}_2/h\left(\overline{B}_1;k\right)$ of the voiceful citizens, and $y_i^* = 0$ to the rest.

The proof of uniqueness is deferred to Lemma 3 below. ■

A.3 Proofs for Section 5

For expositional simplicity, set $\eta = 1$, meaning that all citizens are voiceful (else, the proof applies simply by applying all the reasoning to the interval $[0,\eta]$). Denote by $\left(\alpha^C,\omega^C\right)$ and $\left(\alpha^U,\omega^U\right)$ the total resources devoted by the two politicians to the coordinated and uncoordinated citizens' pool, such that the budge constraints hold for either politician: $\alpha^C + \alpha^U = B_1$ and $\omega^C + \omega^U = B_2$. We allow for α^C and ω^C to be drawn randomly from distributions G_1 and G_2 with support $[0,B_1]$ and $[0,B_2]$, respectively. For any given realization of α^C and ω^C, we denote by $\left\{\alpha_i^C,\omega_i^C\right\}_{i\in[0,\mu]}$ and $\left\{\alpha_i^U,\omega_i^U\right\}_{i\in(\mu,1]}$ the promises made to individual coordinated (resp., uncoordinated) citizens. The promises to individual citizens meet the following budget constraints:

$$\begin{aligned}
\int_0^\mu \alpha_i^C\,di &= \alpha^C\\
\int_0^\mu \omega_i^C\,di &= \omega^C\\
\int_\mu^1 \alpha_i^U\,di &= \alpha^U\\
\int_\mu^1 \omega_i^U\,di &= \omega^U.
\end{aligned}$$

Proof of Proposition 2

Step 1: challenger's return from offering ω^U to uncoordinated We show that, regardless of the incumbent's budget α^U and of the behavior of the coordinated citizens, the challenger's marginal return from increasing the budget ω^U by one dollar is bounded above by a small number independent of ω^U. The return is measured in terms of how many uncoordinated citizens' support the challenger, i.e., in terms of how much the mass $\int_\mu^1 a_i di$ increases with ω^U.

Suppose an uncoordinated citizen i receives promises $\left(\alpha_i^U, \omega_i^U\right)$ and places prior probability p on the incumbent remaining in power. In choosing whom to support, this citizen compares his expected payoff when supporting the incumbent,

$$p\alpha_i^U + (1-p)\,\lambda\omega_i^U,$$

with his payoff from supporting the challenger

$$p\left(\lambda\alpha_i^U - k\right) + (1-p)\,\omega_i^U.$$

Citizen i supports the challenger iff the second term exceeds the first, i.e., if

$$\begin{aligned} p\left(\lambda\alpha_i^U - k\right) + (1-p)\,\omega_i^U &> p\alpha_i^U + (1-p)\,\lambda\omega_i^U \\ (1-p)\,(1-\lambda)\,\omega_i^U &> p\,(1-\lambda)\,\alpha_i^U + pk \\ \omega_i^U &> \frac{p}{(1-p)}\left[\alpha_i^U + \frac{k}{(1-\lambda)}\right]. \end{aligned}$$

The right hand side can be made arbitrarily large by making p close to 1 provided $k > 0$, irrespective of the value of α_i^U. Therefore, for p large enough, no budget $\omega^U \leq B_2$ exists that suffices to treat give every uncoordinated citizen more than the RHS. The optimal strategy for the challenger in this case is to offer zero to some randomly chosen citizens; and to the rest, offer something more than the RHS, hence never less than $\frac{p}{(1-p)}\left[\frac{k}{(1-\lambda)}\right]$. When ε is added to ω^U, the optimal use of the extra resources is to increase the number of well-treated citizens, rather than to improve the lot of those who were well treated already. The newly-well treated citizens must receive at least $\frac{pk}{(1-p)(1-\lambda)}$, so at most

$$\varepsilon\frac{(1-p)\,(1-\lambda)}{pk}$$

new citizens can be persuaded to support the challenger. Therefore, the marginal return to giving resources to uncoordinated citizens is bounded above by

$$\delta = \frac{(1-p)(1-\lambda)}{pk}.$$

Now, since the incumbent survives whenever $\theta < 0$ regardless of how promises are allocated, the probability p that the incumbent remains in power is no less than (recall that $\underline{\theta}$ is a negative number):

$$\Pr[\theta < 0] = -\frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}}.$$

This lower bound for p can be made arbitrarily close to 1 by making $\underline{\theta}$ a sufficiently large negative number. Therefore, for any $\delta > 0$ there is a threshold $\underline{\theta}^T < 0$ such that for all values of $\underline{\theta} < \underline{\theta}^T$ the challenger's marginal returns to investing ω^U in the uncoordinated are bounded above by δ irrespective of (α^C, ω^C) and (α^U, ω^U).

Step 2: incumbent's best-response resource distribution to the coordinated First, an important preliminary remark. The behavior of the coordinated does not depend on how many uncoordinated citizens' support the challenger, i.e., on their forecast about the value of $\int_\mu^1 a_i di$. This is because, given any profile of promises $\{\alpha_i^C, \omega_i^C\}$ and $\{\alpha_i^U, \omega_i^U\}$, the quantity $\int_\mu^1 a_i di$ is a number, not a random variable, by the law of large numbers so, for the purpose of determining whether the incumbent survives, the coordinated citizens view $\int_\mu^1 a_i di$ as a horizontal shift in the support of the uniform distribution from which θ is drawn.[23] However, Lemma 1 shows that the coordinated citizens' behavior is independent of the support of θ. So, the coordinated citizens' forecast of $\int_\mu^1 a_i di$ does not affect their behavior, which is still summarized by the vulnerability index in Lemma 1.

With this being said, suppose the challenger draws a random budget ω^C from the distribution G_2 and then offers i.i.d. promises ω_i^C to citizens drawn i.i.d. from the c.d.f. $F_2^{\omega^C}$ with mean ω^C. We seek to show that, in this scenario, the incumbent's best response given any budget α^C is to distribute this budget equally among all coordinated players. To see this, observe that if the incumbent distributes whatever total resources she has according to $\{\alpha_i^C\}$, the vulnerability index reads:

$$\int_0^{B_2} \int_0^{\mu} \int_0^{\infty} v\left((1-\lambda)\alpha_i^C, (1-\lambda)\omega; k\right)\ dF_2^{\omega^C}(\omega)\ di\ dG_2\left(\omega^C\right),$$

[23] For example, if all the uncoordinated support the challenger so that $\int_\mu^1 a_i di = (1-\mu)$, the coordinated citizens process this as a distribution of θ whose support is more favorable to the challenger.

where v is as specified in expression (6), and the term $(1-\lambda)$ obtains from the proof of Proposition 1. The symmetric function

$$Q_1\left(\alpha_i^C\right)=\int_0^{B_2}\int_0^{\infty} v\left((1-\lambda)\,\alpha_i^C,(1-\lambda)\,\omega;k\right)\ dF_2^{\omega^C}(\omega)\ dG_2\left(\omega^C\right)$$

must be strictly convex in α_i^C. Indeed, Q_1 is the weighted average of zero functions (when $(1-\lambda)\,\omega<k$) and strictly convex functions (when $(1-\lambda)\,\omega>k$); note that in any challengers' best response $(1-\lambda)\,\omega$ must sometimes exceed k. Therefore, regardless of the challenger's strategy encoded in $G_2\left(\omega^C\right)$ and $F_2^{\omega^C}(\omega)$, the incumbent's best response in any equilibrium is to distribute equally among the coordinated whatever total resources she devotes to them. Formally, in any equilibrium, $\alpha_i^C=\frac{\alpha^C}{\mu}$ for all $i\in[0,\mu]$.

Step 3: challenger's return from offering ω^C to the coordinated Since in any equilibrium the incumbent distributes her resources α^C equally among the μ coordinated players, the challenger's problem for any ω^C is:

$$\begin{aligned}&\max_{\left\{\omega_i^C\right\}}\int_0^{B_1}\int_0^{\mu} v\left((1-\lambda)\,\frac{\alpha^C}{\mu},(1-\lambda)\,\omega_i^C;k\right)\ di\ dG_1\left(\alpha^C\right)\\&\text{s.t.}\ \int_0^{\mu}\omega_i^C\ di=\omega^C.\end{aligned}\tag{23}$$

The symmetric function

$$Q_2\left(\omega_i^C;G_1\right)=\int_0^{B_1} v\left((1-\lambda)\,\frac{\alpha^C}{\mu},(1-\lambda)\,\omega_i^C;k\right)\ dG_1\left(\alpha^C\right)$$

is a weighted sum of functions indexed by α^C which are zero for $(1-\lambda)\,\omega_i^C<k$, positive and strictly increasing and strictly concave in ω_i^C for $(1-\lambda)\,\omega_i^C>k$, and horizontally asymptoting to 1 as $\omega_i\to\infty$. Therefore, $Q_2\left(\omega_i^C\right)$ is also zero for $\omega_i^C<k/(1-\lambda)$, positive and strictly increasing and strictly concave in ω_i^C for $\omega_i^C>k/(1-\lambda)$, with $\lim_{\omega\to\infty}Q_2(\omega)=1$. So the challenger's best response is based on the concave envelope $\overline{Q}_2$, just as in the case with non-stochastic budgets, and the value of problem (23) is

$$\mu\cdot\overline{Q}_2\left(\omega^C;G_1\right).$$

Consequently, the challenger's marginal return to increasing the total resources ω^C devoted to the coordinated is $\mu\cdot\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C}$.

Denote $h(G_1;k,\lambda) > k/(1-\lambda)$ the point where the concave envelope $\overline{Q}_2\left(\omega_i^C;G_1\right)$ changes from flat to curved. We have

$$\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C} = \begin{cases} \left.\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C}\right|_{\omega^C=h(G_1;k,\lambda)} & \text{for } \omega^C \leq h(G_1;k,\lambda) \\ \frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C} & \text{for } \omega^C > h(G_1;k,\lambda) \end{cases}.$$

The second line in the RHS is decreasing in ω^C due to the concavity of $Q_2\left(\omega^C\right)$. Therefore, we have:

$$\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C} \geq \min\left[\left.\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C}\right|_{\omega^C=h(G_1;k,\lambda)}, \left.\frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C}\right|_{\omega^C=B_2}\right]. \tag{24}$$

The first argument of the minimum operator is bounded away from zero uniformly for all G_1. To see this, observe that, for any G_1, the function Q_2 is bounded by the functions

$$v\left((1-\lambda)\frac{B_1}{\mu},(1-\lambda)\omega^C;k\right) \leq Q_2\left(\omega^C;G_1\right) \leq v\left(0,(1-\lambda)\omega^C;k\right).$$

For both bounding functions, the point where their concave envelope changes from flat to curved exceeds k. Therefore, the point $h(G_1;k,\lambda)$ exceeds $k/(1-\lambda)$ uniformly for all G_1. Thus, $\overline{Q}_2(h(G_1;k,\lambda);G_1)$ is greater than zero uniformly for all G_1. Therefore, the slope of the flat part of the concave envelope $\overline{Q}_2\left(\omega^C;G_1\right)$ is bounded below away from zero uniformly for all G_1.

The second argument of the minimum operator (24) is either equal to the first argument if $B_2 \leq h(G_1;k,\lambda)$ or, if $B_2 > h(G_1;k,\lambda)$, it is uniformly bounded away from zero for all G_1. To see this, observe that for any $\omega^C > h(G_1;k,\lambda)$ we have

$$\begin{aligned} \frac{d\overline{Q}_2\left(\omega^C;G_1\right)}{d\omega^C} &= \frac{dQ_2\left(\omega^C;G_1\right)}{d\omega^C} \\ &= \int_0^{B_1} \frac{d}{d\omega^C} v\left((1-\lambda)\frac{\alpha^C}{\mu},(1-\lambda)\omega^C;k\right)\, dG_1\left(\alpha^C\right). \end{aligned}$$

When evaluated at $B_2 > h(G_1; k)$, this expression reads

$$\begin{aligned}
&\int_0^{B_1} \frac{d}{dB_2} \frac{(1-\lambda) B_2 - k}{(1-\lambda) B_2 + (1-\lambda) \frac{\alpha^C}{\mu}} \, dG_1\left(\alpha^C\right) \\
= \; & (1-\lambda) \int_0^{B_1} \frac{(1-\lambda) \frac{\alpha^C}{\mu} + k}{\left[(1-\lambda) B_2 + (1-\lambda) \frac{\alpha^C}{\mu}\right]^2} \, dG_1\left(\alpha^C\right) \\
\geq \; & \int_0^{B_1} \frac{\min_{\alpha^C} \left[\frac{\alpha^C}{\mu} + \frac{k}{(1-\lambda)}\right]}{\max_{\alpha^C} \left[B_2 + \frac{\alpha^C}{\mu}\right]^2} \, dG_1\left(\alpha^C\right) \\
= \; & \int_0^{B_1} \frac{k}{(1-\lambda) \left[B_2 + \frac{B_1}{\mu}\right]^2} \, dG_1\left(\alpha^C\right) \\
= \; & \frac{k}{(1-\lambda) \left[B_2 + \frac{B_1}{\mu}\right]^2}.
\end{aligned}$$

Hence, if $B_2 > h(G_1; k, \lambda)$, for any G_1 and any ω^C, the challenger's marginal return to increasing the total resources ω^C devoted to the coordinated is $\mu \frac{d\overline{Q}_2(\omega^C; G_1)}{d\omega^C} \geq \mu k / \left[(1-\lambda)\left(B_2 + \frac{B_1}{\mu}\right)^2\right]$.

In sum, both arguments of the minimum operator (24) are greater than zero uniformly for all G_1 and they are independent of $\underline{\theta}$. Hence, the challenger's marginal return to increasing the total resources α^C devoted to the coordinated is uniformly bounded away from zero for all G_1 by a number independent of $\underline{\theta}$.

Step 4: condition for incumbent survival From condition (4), the incumbent is replaced if and only if

$$\int_0^{\mu} a_i di + \int_{\mu}^{1} a_i di \geq 1 - \theta.$$

The first (resp., second) integral on the LHS represents the mass of coordinated (resp., uncoordinated) citizens who support the challenger. The value of these terms depends on $\left\{\alpha_i^C, \omega_i^C\right\}_{i \in [0,\mu]}$ and $\left\{\alpha_i^U, \omega_i^U\right\}_{i \in [\mu,1]}$, the promises made by the two politicians to the coordinated and uncoordinated citizens. For any given configuration of promises $\left\{\alpha_i^C, \omega_i^C\right\}_{i \in [0,\mu]}$ to coordinated agents and of uncoordinated agent actions $\{a_i\}_{i \in [\mu,1]}$, using condition (5), the limit condition for incumbent survival is:

$$\int_0^{\mu} \frac{(1-\lambda)\, \omega_i^C - k}{(1-\lambda)\, \omega_i^C + (1-\lambda)\, \alpha_i^C} \cdot \mathbf{1}\left[(1-\lambda)\, \omega_i^C \geq k\right] \, di + \int_{\mu}^{1} a_i di \geq 1 - \theta.$$

We have shown in Step 2 above that, in equilibrium, $\alpha_i^C = \frac{\alpha^C}{\mu}$ for all $i \in [0, \mu]$. For any random strategy $G_1\left(\alpha^C\right)$ employed by the incumbent and any given $\left\{\omega_i^C\right\}_{i\in[0,\mu]}$, the limit condition for incumbent survival is:

$$\int_0^{B_1}\int_0^{\mu} \frac{(1-\lambda)\,\omega_i^C - k}{(1-\lambda)\,\omega_i^C + (1-\lambda)\,\frac{\alpha^C}{\mu}} \cdot \mathbf{1}\left[(1-\lambda)\,\omega_i^C \geq k\right]\; di\, dG_1\left(\alpha^C\right) + \int_{\mu}^{1} a_i di \geq 1 - \theta. \quad (25)$$

The challenger chooses $\left\{\omega_i^C\right\}_{i\in[0,\mu]}$ to maximize the LHS of this expression within a budget of ω^C, which is problem (23). The value of this problem, as we have shown in Step 3 above, is $\mu \cdot \overline{Q}_2\left(\omega^C; G_1\right)$. Hence condition (25) rewrites as

$$\mu \cdot \overline{Q}_2\left(\omega^C; G_1\right) + \int_{\mu}^{1} a_i di \geq 1 - \theta. \quad (26)$$

The challenger chooses her budget allocation $\left(\omega^C, \omega^U\right)$ to maximize this expression given the incumbent's equilibrium behavior.

Step 5: challenger's incentives to invest in either group Let's first look at the challenger's incentive to invest resources ω^C into increasing the first term in the LHS of (26). Step 3 has shown that, regardless of the distribution $G_1\left(\alpha^C\right)$ used by the incumbent to select a budget and of the budget ω^C invested by the challenger, if the incumbent follows equilibrium behavior by distributing α^C equitably among the μ coordinated agent, the marginal return $\mu d\overline{Q}_2\left(\omega^C; G_1\right)/d\omega^C$ is bounded below by a number greater than zero.

Next, look at the challenger's incentive to invest resources ω^U into increasing the second term in the LHS of (26). Step 1 has shown that for any distribution of resources α_i^U that the incumbent may provide to the uncoordinated, if p is large enough the marginal return to increasing ω^U is bounded above by

$$\delta = \frac{(1-p)\,(1-\lambda)}{pk},$$

where p can be made arbitrarily close to 1 by making $\underline{\theta}$ a sufficiently large negative number. Therefore, regardless of the incumbent's resource distribution α^U and on the budget ω^U invested by the challenger, for any small $\delta > 0$ there exists a negative enough threshold $\underline{\theta}^T < 0$ such that for all values of $\underline{\theta} < \underline{\theta}^T$ the challenger's marginal returns to investing ω^U in the uncoordinated are bounded above by δ.

In sum, take any incumbent's budget allocation $\left(\alpha^C, \alpha^U\right)$ where α^C is drawn from any feasible distribution G_1 and α^U is drawn from any arbitrary distribution, and any challenger budget allocation $\left(\omega^C, \omega^U\right)$; if the incumbent follows equilibrium behavior by distributing α^C equitably among the μ coordinated agent, then the challenger's return to the best use of budget ω^C is bounded below by a number greater than zero, whereas the return to the best use of budget ω^U is bounded above by δ which can be made arbitrarily close to zero by choosing $\underline{\theta}$ large enough. Therefore, for any given constellation of parameters $\mu, k, \lambda, B_1, B_2$, there exists a threshold $\underline{\theta}^T < 0$ such that for all values of $\underline{\theta} < \underline{\theta}^T$ the challenger's optimal allocation ω^U is zero in every equilibrium.

Step 6: incumbent's incentive to invest in the uncoordinated citizens Since in every equilibrium the challenger's optimal allocation ω^U is zero, all uncoordinated citizens support the incumbent even if the incumbent offers them nothing. Therefore, the incumbent's best response in every equilibrium is to offer the uncoordinated citizens nothing.

A.4 Proofs for Section 7

Proof of Proposition 3

Proof. For uniqueness, see Lemma 3.

Part 1. Obvious.

Part 2. Observe that the incumbent does not take advantage of the targetability of redistribution (see Proposition 1), therefore the incumbent will promise the voiceful-optimal policy.

Part 3. Part 2 guarantees that the incumbent's strategy is to promise M, so by the same logic as in the proof of Proposition 1 the challenger's is to promise M to everyone if $v(M, M; k) \geq \overline{v}\left(M, \overline{B}; k\right)$, else he will redistribute the budget to the voiceful citizens, and unequally among them. ■

Proof of Corollary 2

Proof. Part 1. Fix $(B, \eta, \mu = 1)$ (if $\mu < 1$, simply replace the symbol η with $\min[\eta, \mu]$ in this proof). Let us look for parameter constellations such the challenger promises inequitable redistribution even though $\overline{G} > \overline{B}$. First, let us set k large enough that $\overline{B} < h\left(\overline{B}, k\right)$. We then have $\overline{B} < h\left(\overline{G}; k\right) = h(M; k)$ for any $\overline{G} > \overline{B}$. Finally, refer to Figure 1: in the region $\omega < h(\alpha, k)$ any two values of ω sufficiently close to each other violate $v \geq \overline{v}$. As any choice of $\overline{G} > \overline{B}$ sufficiently close to $\overline{B}$ lies within the region $\omega < h(M,k)$, this choice of $\overline{G}$ produces a violation of condition (11). This means that the challenger's best response is to redistribute B unequally.

Part 2. The incumbent promises the public good iff $G > B/\eta$, and the pairs (G, B) that satisfy this inequality grows as η increases. The challenger promises the public good if, simultaneously, $G \geq B/\eta$ (else redistribution strategically dominates the public good) and condition (11) holds. When $G \geq B/\eta$ holds, condition (11) is violated if and only if:

$$v\left(\overline{G}, \overline{G}; k\right) < \overline{v}\left(\overline{G}, \overline{B}; k\right) = \frac{v\left(\overline{G}, h\left(\overline{G}; k\right)\right)}{h\left(\overline{G}; k\right)} \overline{B}. \tag{27}$$

This inequality depends on η only through $\overline{B} = (1-\lambda) B/\eta$. As η increases the set of pairs (G, B) that satisfy condition (27) , i.e., that violate condition (11), shrinks.

Part 3. The incumbent promises the public good iff $G > B/\eta$, which is independent of λ. Let us now turn to the challenger. The statement is vacuous for $k = 0$, so let's focus on the case $k > 0$. The challenger promises the public good if, simultaneously, $G > B/\eta$ (which is independent of λ) and if condition (27) fails. The right hand side of (27) is strictly positive for every $\lambda < 1$. The left hand side equals zero whenever $\mathbf{1}\left[\overline{G} \geq k\right] = 0$. Therefore, condition (27) holds whenever $k > \overline{G} = (1-\lambda) G$. This condition rewrites as $\lambda > (G-k)/G$.

Part 4. Given any $\alpha > 0, \omega > 0$, picking k sufficiently small ensures that $\overline{v}(\alpha, \omega; k) = v(\alpha, \omega; k)$. In particular, given $\left(M, \overline{B}\right)$, picking k sufficiently small ensures that $\overline{v}\left(M, \overline{B}; k\right) = v\left(M, \overline{B}; k\right) \leq v(M, M; k)$, where the inequality follows because v is non-decreasing in ω. Therefore condition (11) holds for any pair $\left(M, \overline{B}\right)$ when k is small. The desired result then follows from Proposition 3 part 3. ■

Lemma 3 *There is a unique equilibrium in symmetric strategies in Propositions 1 and 3.*

Proof. The proof deals with the case $(\eta, \lambda) = (1, 0)$. The case $(\eta, \lambda) \neq (1, 0)$ is a straightforward extension.

Take any equilibrium in which the challenger uses the symmetric strategy where promises are drawn from the distribution F_2. Expression (6) reads:

$$\begin{aligned} & \int_0^\eta v(\alpha_i, \omega_i; k)\ di \\ = & \int_0^\eta \int_0^\infty v(\alpha_i, \omega; k)\ dF_2(\omega)\ di \\ = & \int_0^\eta Q(\alpha_i; k)\ di, \end{aligned} \tag{28}$$

where the function

$$Q(\alpha_i; k) = \int_0^\infty v(\alpha_i, \omega; k)\ dF_2(\omega)$$

is convex in α_i, and indeed strictly so because rationality requires F_2 placing positive probability on some $\omega > k$. Therefore, the problem of minimizing (28) subject to the incumbent's budget constraint (1) yields $\alpha_i = \alpha$ for all i. Hence, in any equilibrium where the challenger uses a symmetric strategy F_2, the incumbent uses the symmetric strategy which is either to promise the public good, or to promise $\frac{B}{\eta}$ (redistribution) to the voiceful citizens only, depending on which strategy gives the most welfare to the voiceful citizens $\max\left[\frac{B}{\eta}, G\right]$. Now, the challenger's best response to this strategy is unique and symmetric, as shown in the proof of Propositions 1 and 3. Therefore, the equilibrium in Proposition 1 is the unique equilibrium in symmetric strategies. ■