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"Expectation Conformity in Strategic Cognition"
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Abstract

This document contains extensions and additional results mentioned in Section 5 in the main body of the paper but not developed there. All sections, conditions, and results specific to this document have the suffix "S" to avoid confusion with the corresponding parts in the main text. Section S.1 considers games in which cognition is self-directed and takes the familiar form of players receiving additive signals about an exogenous payoff state. Section S.2 considers games of manipulative cognition in which players choose frames to influence other players' recollection of information previously received. Section S.3 considers another class of games of manipulative cognition that generalizes Holmström (1999) signal-jamming model of career concerns. Finally, Section S.4 considers games in which cognition determines the depth of reasoning in the level-k model.

S.1 Noisy information acquisition about exogenous payoff states

In this section, we consider games in which players receive additive signals about an unknown exogenous payoff state, as in most of the literature on information acquisition.

Payoffs are given by

$$u_i(a_i, a_{-i}, \omega) = -(1-\beta)(a_i - g(\omega))^2 - \beta(a_i - \bar{a}_{-i})^2 + \psi(a_{-i}, \omega), \tag{S.1}$$

where $a_i \in A_i = \mathbb{R}$, $a_{-i} \equiv (a_1, ..., a_{i-1}, a_{i+1}, ..., a_n) \in \mathbb{R}^{n-1}$, $\omega \in \mathbb{R}$, and $\bar{a}_{-i} \equiv \Sigma_{j \neq i} a_j/(n-1)$. Because ω is unidimensional, without loss of generality, let $g(\omega) = \omega$. Furthermore, because

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$\psi(a_{-i},\omega)$ plays no role in the computation of the individual best responses, let $\psi(a_{-i},\omega)=0$ for all (a_{-i},ω).

To further simplify the analysis, assume that $n=2$ and that ω is drawn from an improper uniform prior over the entire real line (as it will become clear in a moment, hierarchies of beliefs and expected payoffs are well defined, despite the improperness of the prior). Correlated (exogenous) variation in the players' beliefs about ω is captured by the two players observing a common signal

$$y=\omega+\varepsilon,$$

with ε drawn from a Normal distribution with mean 0 and variance h^{-1}.

The information privately collected by each player, instead, is summarized in the signal

$$x_i=\omega+\eta_i,$$

with η_i drawn from a Normal distribution with mean 0 and variance ρ_i^{-1}, with (η_1,η_2) drawn independently across the two players and independently from (ω,ε).[1] Hence, in this model, cognition determines the precision of a player's private signal about the exogenous payoff state ω.

Now let $s_i\equiv(x_i,y)$, $i=1,2$. Below we show that when, in the stage-2 game, player i expects player j to follow a strategy that, for any $s_j=(x_j,y)$, selects with probability one an action

$$a_j(s_j)=m_jx_j+(1-m_j)y$$

then, given her own cognitive choice ρ_i, player i's best response consists in following a strategy that, for each $s_i=(x_i,y)$, also selects with probability one an action $a_i(s_i)=m_ix_i+(1-m_i)y$ that is a convex linear combination of x_i and y, with

$$m_i=\frac{(1-\beta)\rho_i}{\rho_i+h}+\frac{\beta\rho_i}{\rho_i+h}m_j.$$

Fixing the two players' cognitive postures $\rho=(\rho_1,\rho_2)$, we then show that there exists a unique linear continuation equilibrium σ^ρ for the stage-2 game corresponding to the cognitive profile ρ, and such an equilibrium is such that, for each $s_i=(x_i,y)$, σ_i^ρ selects with probability one the action $a_i^\rho(s_i)=m_i^\rho x_i+(1-m_i^\rho)y$, with

$$m_i^\rho=(1-\beta)\rho_i\frac{\rho_j(1+\beta)+h}{(\rho_i+h)(\rho_j+h)-\beta^2\rho_i\rho_j}. \tag{S.2}$$

We then have the following result:

Proposition S.1 (learning about exogenous states). *Let $\rho=(\rho_1,\rho_2)$ and $\hat\rho=(\hat\rho_1,\hat\rho_2)$ be two arbitrary cognitive profiles. UEC always holds for these profiles, irrespective of whether the stage-2*

[1] As shown in Bergemann and Morris (2013), any joint (Gaussian) distribution between (a_1,a_2,ω) can be generated by the two players observing a perfectly public and two perfectly private Gaussian additive signals.

actions are strategic complements ($\beta > 0$) or strategic substitutes ($\beta < 0$). ID holds if and only if, $\beta(\hat{\rho}_i - \rho_i)(\hat{\rho}_j - \rho_j) \geq 0$. Finally, expectation conformity holds if and only if

$$\beta(\hat{\rho}_j - \rho_j)\,(\hat{\rho}_i - \rho_i)\,(\hat{\rho}_i + h)\,[\rho_i(1+\beta) + h] + \beta^2\,(\hat{\rho}_i - \rho_i)^2\,\hat{\rho}_j\,[\rho_j(1+\beta) + h] \geq 0.$$

The result in the first part of the proposition says that, holding fixed the precision of player j's private signal, the value to player i of increasing the precision of her private signal is higher when player j expects player i to acquire a more precise private signal. That is, the game always satisfies UEC. To understand why, consider first the case where the stage-2 actions are strategic complements ($\beta > 0$). When player j expects player i to acquire a more precise signal, she also expects player i to select an action in the stage-2 game that is more sensitive to player i's private signal. Because $\beta > 0$, player j's stage-2 action is then more sensitive to player j's own private signal (use the formula in (S.2), but with the role of i and j inverted, to verify that m_j^ρ is increasing in ρ_i) which in turn increases player i's incentives to acquire a more precise private signal.

Next, consider the case where the stage-2 actions are strategic substitutes ($\beta < 0$). Again, when player j expects player i to acquire a more precise private signal, she also expects player i's stage-2 action to be more sensitive to player i's private signal. Because $\beta < 0$, player j's best response is then to select a stage-2 action that is less sensitive to player j's own private signal (again, use the formula in (S.2) to verify that, when $\beta < 0$, m_j^ρ is decreasing in ρ_i). Because actions are strategic substitutes, that player j's stage-2 action responds less to her private signal in turn implies that player i's incentives to acquire a more precise private signal are stronger. Hence, irrespective of whether $\beta > 0$ or $\beta < 0$, holding fixed player j's cognition ρ_j, player i's incentives to acquire more precise private information are always higher when player j expects her to acquire more precise private information.

The second part of the proposition says that, holding player j's expectation about player i's cognition fixed, the value to player i of acquiring more precise private information is higher when either (a) player j also acquires more precise private information and actions are strategic complements ($\beta > 0$), or (b) player j acquires less precise private information and actions are strategic substitutes. This is because, irrespective of whether $\beta > 0$ or $\beta < 0$, when player j acquires more precise private information, she then always responds more to it (again, use the formula in (S.2) to verify that m_j^ρ is always increasing in ρ_j, irrespective of the sign of β). Player i's incentives to acquire more precise information are thus higher when either $\beta > 0$ and $\hat{\rho}_j \geq \rho_j$, or when $\beta < 0$ and $\hat{\rho}_j < \rho_j$ (they are lower when $\beta(\hat{\rho}_j - \rho_j) \leq 0$).

Finally, the last part of the proposition says that player i's incentives to acquire more precise information (that is, to choose $\hat{\rho}_i > \rho_i$) are stronger under the profile $\hat{\rho} = (\hat{\rho}_i, \hat{\rho}_j)$ than under the profile $\rho = (\rho_i, \rho_j)$ when, other things equal, $\hat{\rho}_i - \rho_i$ is large relative to $\hat{\rho}_i \rho_i$. For example, when public information is imprecise, that is, when $h \to 0$, EC holds if and only if

$$\beta(\hat{\rho}_j - \rho_j)\hat{\rho}_i\rho_i + \beta^2\,(\hat{\rho}_i - \rho_i)\,\hat{\rho}_j\rho_j \geq 0.$$

The results in Proposition S.1 above hold for *arbitrary* cognitive profiles $\rho = (\rho_i, \rho_j)$ and $\hat{\rho} = (\hat{\rho}_i, \hat{\rho}_j)$, which need not be symmetric across the players. When applied to equilibrium analysis, they shed light on whether equilibrium multiplicity originates in the players benefiting from conforming to other players' expectations about their own cognition (UEC) or in their own expectations about the opponents' cognitive postures (ID). These properties in turn offer a deeper understanding of the sources and nature of the equilibrium multiplicity these games are prone to.

Proof of Proposition S.1. Suppose that, in the stage-2 game, player i expects player j to follow a strategy that, for any $s_j = (x_j, y)$, selects the action

$$a_j(s_j) = m_j x_j + (1 - m_j) y$$

with probability one, for some scalar m_j. Given her cognitive choice ρ_i, for any $s_i = (x_i, y)$, player i's best response then consists in selecting with probability one the action

$$\begin{aligned} a_i &= (1-\beta)\mathbb{E}\left[\omega|\rho_i, s_i\right] + \beta\mathbb{E}\left[a_j(s_j)|\rho_i, s_i\right] \\ &= (1-\beta)\left[\tfrac{\rho_i}{\rho_i+h}x_i + \tfrac{h}{\rho_i+h}y\right] + \beta m_j\left[\tfrac{\rho_i}{\rho_i+h}x_i + \tfrac{h}{\rho_i+y}y\right] + \beta(1-m_j)y. \end{aligned}$$

Player i's optimal action is thus also linear in x_i and y, with coefficients to x_i and y equal to

$$m_i = \frac{(1-\beta)\rho_i}{\rho_i + h} + \frac{\beta\rho_i}{\rho_i + h} m_j$$

and $1 - m_i$, respectively.[2]

Iterating over the two players' best responses to find the fixed point, we then have that, for any cognitive profile $\rho = (\rho_i, \rho_j)$, there exists a unique linear continuation equilibrium for the stage-2 game corresponding to the cognitive profile ρ and is such that, for any $s_i = (x_i, y)$, $i = 1, 2$, σ_i^ρ selects with probability one the action $a_i^\rho(s_i) = m_i^\rho x_i + (1 - m_i^\rho)y$, where[3]

$$m_i^\rho = (1-\beta)\rho_i \frac{\rho_j(1+\beta) + h}{(\rho_i + h)(\rho_j + h) - \beta^2\rho_i\rho_j}.$$

Expectation Conformity.

Next observe that, for any $(\omega, \varepsilon, \eta_1, \eta_2)$, and any (m_1, m_2), when, in the stage-2 game, for any

[2] More generally, one can show that, when player i expects player j to follow a strategy σ_j that, for any $s_j = (x_j, y)$, selects with probability one the action $a_j(s_j) = m_j x_j + k_j y$, for some scalars m_j and k_j, then her best response is to follow a strategy that, for any $s_i = (x_i, y)$, selects with certainty the action $a_i(s_i) = m_i x_i + k_i y$ where the coefficients m_i and k_i are given by

$$m_i = \frac{(1-\beta)\rho_i}{\rho_i + h} + \frac{\beta\rho_i}{\rho_i + h} m_j \qquad \text{and} \qquad k_i = \frac{(1-\beta)h + \beta h m_j}{\rho_i + h} + \beta k_j.$$

One can then use this property to show uniqueness of the linear continuation equilibrium, for any $\rho = (\rho_1, \rho_2)$.

[3] The formulas for the equilibrium sensitivities are obtained after various algebraic simplifications which are omitted for brevity.

$s_i = (x_i, y)$, each player $i = 1, 2$ selects with probability one the action $a_i = m_i x_i + (1 - m_i) y$, then

$$a_i - \omega = m_i \eta_i + (1 - m_i)\varepsilon$$

and

$$a_i - a_j = m_i \eta_i - m_j \eta_j - (m_i - m_j)\varepsilon.$$

This implies that, when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i selects cognition ρ_i', player i's ex-ante expected (gross) payoff (disregarding the externality term ψ in the payoff function in (S.1)) is then equal to

$$V_i(\rho_i'; \rho) = -(1-\beta)\left(m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{\rho_i'} - (1-\beta)\left(1 - m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{h}$$

$$-\beta\left(m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{\rho_i'} - \beta\left(m_j^{\rho}\right)^2 \tfrac{1}{\rho_j} - \beta\left(m_i^{\rho_i';\rho} - m_j^{\rho}\right)^2 \tfrac{1}{h}$$

where

$$m_j^{\rho} = \frac{(1-\beta)\rho_j(\rho_i + h) + \beta\rho_i\rho_j(1-\beta)}{(\rho_i + h)(\rho_j + h) - \beta^2\rho_i\rho_j}$$

is the sensitivity of player j's stage-2 action to her private information x_j when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and where

$$m_i^{\rho_i';\rho} = \frac{(1-\beta)\rho_i'}{\rho_i' + h} + \frac{\beta\rho_i'}{\rho_i' + h} m_j^{\rho}$$

is the sensitivity of player i's stage-2 action to his private information when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i selects cognition ρ_i'.

Simplifying, $V_i(\rho_i'; \rho)$ can be rewritten as

$$V_i(\rho_i'; \rho) = -\left(m_i^{\rho_i';\rho}\right)^2 \tfrac{1}{\rho_i'} - \beta\left(m_j^{\rho}\right)^2 \tfrac{1}{\rho_j}$$
$$-\left[1 - \beta + \left(m_i^{\rho_i';\rho}\right)^2 - 2(1-\beta)m_i^{\rho_i';\rho} + \beta\left(m_j^{\rho}\right)^2 - 2\beta m_i^{\rho_i';\rho} m_j^{\rho}\right]\tfrac{1}{h}.$$

Now to see whether this game satisfies UEC, ID, and EC, take any pair of cognitive levels for player i, $\hat{\rho}_i$ and ρ_i, and let $\rho' = (\rho_i', \rho_j')$ and $\rho'' = (\rho_i'', \rho_j'')$ be two arbitrary cognitive profiles.

Then let

$$D \equiv \left[V_i(\hat{\rho}_i; \rho') - V_i(\rho_i; \rho')\right] - \left[V_i(\hat{\rho}_i; \rho'') - V_i(\rho_i; \rho'')\right].$$

Observe that UEC holds if $D \geq 0$ for $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$, ID holds if $D \geq 0$ for $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$, and EC holds if $D \geq 0$ for $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\rho_i, \rho_j)$.

Using the characterization of the V_i functions above, we have that[4]

[4]Again, the formula for D is obtained after various algebraic simplifications that are omitted for brevity.

$$D=\left[\left(m_i^{\rho_i;\rho'}\right)^2-\left(m_i^{\rho_i;\rho''}\right)^2\right]\left(\tfrac{1}{\rho_i}+\tfrac{1}{h}\right)-\left[\left(m_i^{\hat{\rho}_i;\rho'}\right)^2-\left(m_i^{\hat{\rho}_i;\rho''}\right)^2\right]\left(\tfrac{1}{\hat{\rho}_i}+\tfrac{1}{h}\right)$$

$$+2\left(m_i^{\hat{\rho}_i;\rho'}-m_i^{\rho_i;\rho'}\right)\left[1-\beta+\beta m_j^{\rho'}\right]\tfrac{1}{h}-2\left(m_i^{\hat{\rho}_i;\rho''}-m_i^{\rho_i;\rho''}\right)\left[1-\beta+\beta m_j^{\rho''}\right]\tfrac{1}{h}.$$

Next, use the structure of the best responses derived above to observe that

$$m_i^{\hat{\rho}_i;\rho'}-m_i^{\rho_i;\rho'}=h\frac{(\hat{\rho}_i-\rho_i)\left(1-\beta+\beta m_j^{\rho'}\right)}{(\hat{\rho}_i+h)(\rho_i+h)}$$

and

$$m_i^{\hat{\rho}_i;\rho''}-m_i^{\rho_i;\rho''}=h\frac{(\hat{\rho}_i-\rho_i)\left(1-\beta+\beta m_j^{\rho''}\right)}{(\hat{\rho}_i+h)(\rho_i+h)}.$$

Replacing the above expressions into the formulas for D, and simplifying, we then have that

$$D=\left[\left(m_i^{\rho_i;\rho'}\right)^2-\left(m_i^{\rho_i;\rho''}\right)^2\right]\left(\tfrac{1}{\rho_i}+\tfrac{1}{h}\right)-\left[\left(m_i^{\hat{\rho}_i;\rho'}\right)^2-\left(m_i^{\hat{\rho}_i;\rho''}\right)^2\right]\left(\tfrac{1}{\hat{\rho}_i}+\tfrac{1}{h}\right)$$

$$+\tfrac{2(\hat{\rho}_i-\rho_i)\left[\left(1-\beta+\beta m_j^{\rho'}\right)^2-\left(1-\beta+\beta m_j^{\rho''}\right)^2\right]}{(\hat{\rho}_i+h)(\rho_i+h)}.$$

Using the fact that

$$m_i^{\rho_i;\rho'}-m_i^{\rho_i;\rho''}=\frac{\beta\rho_i}{\rho_i+h}\left(m_j^{\rho'}-m_j^{\rho''}\right)$$

and

$$m_i^{\hat{\rho}_i;\rho'}-m_i^{\hat{\rho}_i;\rho''}=\frac{\beta\hat{\rho}_i}{\hat{\rho}_i+h}\left(m_j^{\rho'}-m_j^{\rho''}\right),$$

after a few simplifications, we then have that

$$D=\tfrac{\beta}{h}\left(m_j^{\rho'}-m_j^{\rho''}\right)\left(m_i^{\rho_i;\rho'}+m_i^{\rho_i;\rho''}-m_i^{\hat{\rho}_i;\rho'}-m_i^{\hat{\rho}_i;\rho''}\right)$$

$$+\tfrac{2(\hat{\rho}_i-\rho_i)\left[\left(1-\beta+\beta m_j^{\rho'}\right)^2-\left(1-\beta+\beta m_j^{\rho''}\right)^2\right]}{(\hat{\rho}_i+h)(\rho_i+h)}.$$

Next, observe that

$$m_i^{\rho_i;\rho'}+m_i^{\rho_i;\rho''}=\frac{2(1-\beta)\rho_i}{\rho_i+h}+\frac{\beta\rho_i}{\rho_i+h}\left(m_j^{\rho'}+m_j^{\rho''}\right)$$

and

$$m_i^{\hat{\rho}_i;\rho'}+m_i^{\hat{\rho}_i;\rho''}=\frac{2(1-\beta)\hat{\rho}_i}{\hat{\rho}_i+h}+\frac{\beta\hat{\rho}_i}{\hat{\rho}_i+h}\left(m_j^{\rho'}+m_j^{\rho''}\right),$$

from which we obtain that

$$m_i^{\rho_i;\rho'} + m_i^{\rho_i;\rho''} - m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''} =$$

$$= \frac{2(1-\beta)\rho_i(\hat{\rho}_i+h)+\beta\rho_i(\hat{\rho}_i+h)\left(m_j^{\rho'}+m_j^{\rho''}\right)-2(1-\beta)\hat{\rho}_i(\rho_i+h)-\beta\hat{\rho}_i(\rho_i+h)\left(m_j^{\rho'}+m_j^{\rho''}\right)}{(\rho_i+h)(\hat{\rho}_i+h)}.$$

The numerator of the last expression is equal to

$$N = 2(1-\beta)\rho_i h + \beta\rho_i h\left(m_j^{\rho'} + m_j^{\rho''}\right) - 2(1-\beta)\hat{\rho}_i h - \beta\hat{\rho}_i h\left(m_j^{\rho'} + m_j^{\rho''}\right).$$

Hence,

$$m_i^{\rho_i;\rho'} + m_i^{\rho_i;\rho''} - m_i^{\hat{\rho}_i;\rho'} - m_i^{\hat{\rho}_i;\rho''} = -h\frac{2(1-\beta)(\hat{\rho}_i-\rho_i) + \beta\left(\hat{\rho}_i-\rho_i\right)\left(m_j^{\rho'} + m_j^{\rho''}\right)}{\left(\rho_i+h\right)\left(\hat{\rho}_i+h\right)}.$$

Replacing the last expression into the formula for D, we have that

$$D = \frac{\hat{\rho}_i - \rho_i}{\left(\rho_i+h\right)\left(\hat{\rho}_i+h\right)}\beta\left(m_j^{\rho'} - m_j^{\rho''}\right)\left[2(1-\beta) + \beta\left(m_j^{\rho'} + m_j^{\rho''}\right)\right]$$

from which we obtain that

$$D \overset{\text{sgn}}{=} \beta(\hat{\rho}_i - \rho_i)\left(m_j^{\rho'} - m_j^{\rho''}\right).$$

We are now ready to establish the properties in the proposition.

To see that this game satisfies UEC, take $\rho' = (\hat{\rho}_i, \rho_j)$ and $\rho'' = (\rho_i, \rho_j)$. Using the fact that, for any $\rho = (\rho_i, \rho_j)$,

$$m_j^{\rho} = \frac{(1-\beta)\rho_j\left(\rho_i+h\right) + \beta\rho_i\rho_j(1-\beta)}{\left(\rho_i+h\right)\left(\rho_j+h\right) - \beta^2\rho_i\rho_j},$$

we have that

$$\frac{\partial m_j^{\rho}}{\partial\rho_i} \overset{\text{sgn}}{=} \beta.$$

We thus conclude that, independently of the sign of β, $D \geq 0$, which implies that UEC holds.

Next, to see whether this game satisfies ID, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\hat{\rho}_i, \rho_j)$. Using the fact that, for any $\rho = (\rho_i, \rho_j)$, and any $\beta \in (-1, +1)$, m_j^{ρ} is non-decreasing in ρ_j, we then have that

$$D \overset{\text{sgn}}{=} \beta(\hat{\rho}_i - \rho_i)(\hat{\rho}_j - \rho_j).$$

We thus have that, for any pair of information structures $\hat{\rho} = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho = (\rho_i, \rho_j)$, this game satisfies ID if and only if $\beta(\hat{\rho}_i - \rho_i)(\hat{\rho}_j - \rho_j) \geq 0$.

Finally, to see whether this game satisfies EC, take $\rho' = (\hat{\rho}_i, \hat{\rho}_j)$ and $\rho'' = (\rho_i, \rho_j)$. We then have that

$$\frac{m_j^{\rho'} - m_j^{\rho''}}{1-\beta} = \frac{\hat{\rho}_j\left(\hat{\rho}_i+h\right) + \beta\hat{\rho}_i\hat{\rho}_j}{\left(\hat{\rho}_i+h\right)\left(\hat{\rho}_j+h\right) - \beta^2\hat{\rho}_i\hat{\rho}_j} - \frac{\rho_j\left(\rho_i+h\right) + \beta\rho_i\rho_j}{\left(\rho_i+h\right)\left(\rho_j+h\right) - \beta^2\rho_i\rho_j}$$

from which we obtain that

$$\frac{m_j^{\rho'}-m_j^{\rho''}}{1-\beta}=\frac{[\hat{\rho}_j(\hat{\rho}_i+h)+\beta\hat{\rho}_i\hat{\rho}_j]\left[(\rho_i+h)(\rho_j+h)-\beta^2\rho_i\rho_j\right]-[\rho_j(\rho_i+h)+\beta\rho_i\rho_j]\left[(\hat{\rho}_i+h)(\hat{\rho}_j+h)-\beta^2\hat{\rho}_i\hat{\rho}_j\right]}{[(\hat{\rho}_i+h)(\hat{\rho}_j+h)-\beta^2\hat{\rho}_i\hat{\rho}_j][(\rho_i+h)(\rho_j+h)-\beta^2\rho_i\rho_j]}.$$

The denominator is always positive, so focus on the numerator. This is equal to

$$n=[\hat{\rho}_j(\hat{\rho}_i+h)+\beta\hat{\rho}_i\hat{\rho}_j]\left[(\rho_i+h)(\rho_j+h)-\beta^2\rho_i\rho_j\right]-[\rho_j(\rho_i+h)+\beta\rho_i\rho_j]\left[(\hat{\rho}_i+h)(\hat{\rho}_j+h)-\beta^2\hat{\rho}_i\hat{\rho}_j\right]$$

$$=[\hat{\rho}_j(\hat{\rho}_i+h)+\beta\hat{\rho}_i\hat{\rho}_j][(\rho_i+h)(\rho_j+h)]-\beta^2\rho_i\rho_j\hat{\rho}_j h$$

$$-[\rho_j(\rho_i+h)+\beta\rho_i\rho_j][(\hat{\rho}_i+h)(\hat{\rho}_j+h)]+\beta^2\hat{\rho}_i\hat{\rho}_j\rho_j h.$$

Simplifying, we have that

$$\tfrac{n}{h}=(\hat{\rho}_j-\rho_j)[\hat{\rho}_i\rho_i(1+\beta)+(\hat{\rho}_i+\rho_i)h]+(\hat{\rho}_i-\rho_i)\hat{\rho}_j\rho_j(\beta+\beta^2)$$

$$+\hat{\rho}_i\hat{\rho}_j\beta h+h^2\hat{\rho}_j-\rho_i\rho_j\beta h-h^2\rho_j.$$

Hence, EC holds with respect to $\hat{\rho}=(\hat{\rho}_i,\hat{\rho}_j)$ and $\rho=(\rho_i,\rho_j)$ if and only if

$$\beta(\hat{\rho}_j-\rho_j)(\hat{\rho}_i-\rho_i)[\hat{\rho}_i\rho_i(1+\beta)+(\hat{\rho}_i+\rho_i)h]+\beta^2(\hat{\rho}_i-\rho_i)^2\hat{\rho}_j\rho_j(1+\beta)$$

$$+\beta h[\beta(\hat{\rho}_i\hat{\rho}_j-\rho_i\rho_j)+h(\hat{\rho}_j-\rho_j)](\hat{\rho}_i-\rho_i)\geq 0$$

which, after some simplifications, can be rewritten as

$$\beta(\hat{\rho}_j-\rho_j)(\hat{\rho}_i-\rho_i)(\hat{\rho}_i+h)[\rho_i(1+\beta)+h]+\beta^2(\hat{\rho}_i-\rho_i)^2\hat{\rho}_j[\rho_j(1+\beta)+h]\geq 0,$$

as claimed in the proposition. Q.E.D.

S.2 Framing and defensive memory management

In many environments of interest, cognition has a *manipulative* dimension: a player's cognition impacts her opponents' understanding of the game. In this section, we consider situations in which players choose "frames," or other manipulative devices, to influence other players' recollection of information.

A player (the persuader) tries to induce another player (the receiver) to act favorably to her, by manipulating the receiver's recollection of information relevant for a decision. The manipulation is done by means of "frames," that is, through the design of a contextual purchasing experience—see Salant and Siegel (2018) for various examples along these lines.

We capture such situations as follows. Player 2 (the receiver) has a payoff equal to

$$u_2(a_1,a_2,\omega)=-(a_2-\omega)^2$$

where $a_2 \in \mathbb{R}$ is player 2's action and where $\omega \in \mathbb{R}$ is the underlying state of Nature. Player 1 (the persuader), instead, has a payoff

$$u_1(a_1, a_2, \omega) = a_2$$

that is invariant in ω and in her own action, and increasing in player 2's action.[5] Hence, player 2 wants to "do the right thing" (i.e., align her action with the underlying state ω), whereas player 1 wants player 2 to take as high an action as possible (e.g., to increase her purchases of player 1's product, irrespectively of whether or not this is good for player 2). This structure has received considerable attention in the recent persuasion and information design literature. Contrary to what typically assumed in this literature, though, here player 1 cannot commit to her choice of a frame (i.e., to her manipulative information structure).

Player 2, the receiver, is originally endowed with a primary (exogenous) signal $s_2^P = \omega + \varepsilon$ but recalls such a signal only imperfectly. Such a primary signal may represent the information a buyer received about a seller's product from exogenous sources, or past experiences. In such a context, a "frame" by player 1 is a device influencing player 2's ability to recollect her primary signal. Importantly, such a frame may operate asymmetrically across states, facilitating the recollection of information favorable to player 1 relative to the less favorable one. The choice of a frame may also depend on the information that player 1 herself has about the state. However, because this channel is not essential to the results, we do not consider it here. Instead, we allow the receiver, player 2, to exert effort to increase her recollection of the primary information, thus reducing the effect of player 1's frame on her decision. We interpret such efforts broadly as "defensive memory management." Allowing for such efforts also permits us to investigate whether ID holds in this context.

Let $\rho_1, \rho_2 \in \mathbb{R}_+$ and denote by $r(s_2^P; \rho)$ the probability that player 2 recalls her primary signal when the latter takes value s_2^P and the two players engage in cognition $\rho = (\rho_1, \rho_2)$. Let $s_2^R \in \mathbb{R} \cup \{\emptyset\}$ denote player 2's recalled signal, with $s_2^R = \emptyset$ in case player 2 does not recall, and $s_2^R = s_2^P$ in case she does recall. Without loss of generality, then let $s_2^P = \omega$, with ω drawn from $\mathbb{R}$ according to some cdf F.

Interpret ρ_1 as player 1's choice of a frame and assume that ρ_1 increases uniformly the probability that player 2 recalls any positive signal and leaves it unaltered the probability that player 2 recalls any negative signal. Such a stark structure is not essential to the results. What matters is that the likelihood that the receiver recollects information that is more favorable to the persuader relative to the less favorable one is non-decreasing in ρ_1. Formally, there exist non-negative and non-decreasing functions r^+ and r^- such that, when the state is ω and the two players' cognitive choices are given by $\rho = (\rho_1, \rho_2)$, the probability that player 2 recollects the state is equal to

$$r(\omega; \rho) = \begin{cases} r^-(\rho_2) & \text{if} \quad \omega < 0 \\ r^+(\rho_1, \rho_2) & \text{if} \quad \omega \geq 0. \end{cases}$$

[5]The above payoff specification is thus essentially the same as the one in the previous section, with $\beta = 0$, $\psi(a_{-i}, \omega) = a_{-i}$, $g(\omega) = \omega$, and $|A_1| = 1$.

Given the cognitive profile $\rho = (\rho_1, \rho_2)$, then let

$$\mathbb{E}\left[\tilde{\omega}|s_2^R;\rho\right] = \begin{cases} \omega & \text{if} \quad s_2^R = \omega \\ \bar{\omega}(\rho) & \text{if} \quad s_2^R = \emptyset \end{cases}$$

denote player 2's posterior expectation of the state (equivalently, of her optimal action) given the recalled information. Let $\omega^- = \mathbb{E}\left[\tilde{\omega}|\tilde{\omega} < 0\right]$ and $\omega^+ = \mathbb{E}\left[\tilde{\omega}|\tilde{\omega} \geq 0\right]$, where both expectations are under the prior distribution F. We then have that, in the absence of any recollection of her primitive information, player 2's expected value of ω is equal to

$$\bar{\omega}(\rho) = \frac{(1 - r^-(\rho_2))F(0)\omega^- + (1 - r^+(\rho_1, \rho_2))(1 - F(0))\omega^+}{(1 - r^-(\rho_2))F(0) + (1 - r^+(\rho_1, \rho_2))(1 - F(0))}.$$

Note that $\bar{\omega}(\rho)$ is weakly decreasing in ρ_1, that is, in the beliefs player 2 has about player 1's use of manipulative frames. It may be either increasing or decreasing in player 2's own cognition, ρ_2. In particular, $\bar{\omega}(\rho)$ is decreasing in ρ_2 if $\frac{dr^-(\rho_2)}{d\rho_2} = \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}$ and $r^+(\rho_1, \rho_2) \geq r^-(\rho_2)$, that is, if more cognition by player 2 has an equal effect on her ability to recollect positive and negative information, and if the likelihood that she recollects positive information is no smaller than the likelihood that she recollects negative information. On the other hand, $\bar{\omega}(\rho)$ is increasing in ρ_2, when $\frac{dr^-(\rho_2)}{d\rho_2} > \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}$ and $r^+(\rho_1, \rho_2) \cong r^-(\rho_2)$.

Given the quadratic loss function, for any cognitive profile ρ, and any recalled memory s_2^R, player 2's optimal action is equal to

$$a_2^\rho(s_2^R) = \mathbb{E}\left[\tilde{\omega}|s_2^R;\rho\right]$$

implying that, for any cognitive profile $\rho = (\rho_1, \rho_2)$ and any actual choice of frame ρ_1', player 1's ex-ante expected gross payoff when the two players are expected to engage in cognition $\rho = (\rho_1, \rho_2)$ and, instead, player 1 chooses ρ_1' is equal to

$$V_1(\rho_1';\rho) = F(0)\left[(1 - r^-(\rho_2))\bar{\omega}(\rho) + r^-(\rho_2)\omega^-\right] + (1 - F(0))\left[(1 - r^+(\rho_1', \rho_2))\bar{\omega}(\rho) + r^+(\rho_1', \rho_2)\omega^+\right].$$

Similarly, given any cognitive profile $\rho = (\rho_1, \rho_2)$ and any actual choice ρ_2' of memory management, player 2's ex-ante expected gross payoff when the two players are expected to engage in cognition $\rho = (\rho_1, \rho_2)$ and, instead, player 2 invests ρ_2' in memory management is equal to

$$V_2(\rho_2';\rho) = -F(0)(1 - r^-(\rho_2'))\mathbb{E}\left[\left(\bar{\omega}(\rho_1, \rho_2') - \tilde{\omega}\right)^2|\tilde{\omega} < 0\right]$$

$$-(1 - F(0))(1 - r^+(\rho_1, \rho_2'))\mathbb{E}\left[\left(\bar{\omega}(\rho_1, \rho_2') - \tilde{\omega}\right)^2|\tilde{\omega} \geq 0\right].$$

The next result illustrates how cognitive expectations shape the players' incentives to engage into manipulative framing and defensive memory management in such environments.

Proposition S.2 (**framing and memory management**). *Consider any pair of cognitive profiles* $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ *and* $\rho = (\rho_1, \rho_2)$*. UEC always holds for such profiles (weakly for the receiver, player 2,* *and strongly for the persuader, player* 1*). ID holds for player* 1 *(the persuader) if and only if*

$$[r^+(\hat{\rho}_1, \hat{\rho}_2) - r^+(\rho_1, \hat{\rho}_2)]\,[\omega^+ - \bar{\omega}(\hat{\rho}_1, \hat{\rho}_2)] \geq [r^+(\hat{\rho}_1, \rho_2) - r^+(\rho_1, \rho_2)]\,[\omega^+ - \bar{\omega}(\hat{\rho}_1, \rho_2)] \qquad \text{(S.3)}$$

which is the case for example when (a) $\hat{\rho}_1 \geq \rho_1$*,* $\hat{\rho}_2 \geq \rho_2$*, (b)* r^+ *is weakly supermodular, and (c)* $\bar{\omega}$ *is weakly decreasing in* ρ_2*.*

The persuader's incentives to engage into manipulative framing are stronger when she is expected to invest more into manipulative framing. This is because, the more the receiver expects the persuader to engage in manipulative framing, the more she interprets the lack of recollection of her primitive information as a signal of the state being unfavorable to the persuader. But then the stronger the incentives for the persuader to engage into manipulative framing to reduce the risk that the receiver does not recall.

Next, consider the receiver, player 2. Her optimal action depends only on her beliefs about player 1's manipulation and not on her belief about player 1's expectation of her own defensive cognition. As a result, UEC also holds for player 2 but in the trivial sense of player 2's incentives being invariant in player 1's expectations about player 2's cognition.

The second part of the proposition identifies a condition under which player 1's incentives to engage in manipulative framing are stronger when she expects player 2 to invest more in defensive memory management. The condition holds, for example, when the more player 2 invests in recollecting her primitive information, the larger the marginal effect of player 1's manipulation on player 2's recollection of positive information and, in the absence of any recollection, the lower player 2's optimal action. Increasing differences for player 1 also holds when r^+ is submodular (that is, when the more player 2 invests in defensive memory management, the smaller the marginal effect of player 1's manipulation on player 2's recollection of positive information) provided that, in the absence of any recollection, player 2's optimal action is smaller when player 2 invests more in memory management than when she invests less.

Whether increasing differences holds for player 2 (the receiver) is more convoluted and depends on a complicated condition which we do not discuss here.

Proof of Proposition S.2. First, consider UEC. Given any pair of cognitive profiles $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$, the value for player 2 (the receiver) of going from cognition ρ_2 to cognition $\hat{\rho}_2$ when player 1's cognition is ρ_1 is invariant in the level of cognition that player 1 expects player 2 to select. Hence, UEC trivially holds for player 2, albeit never strictly. Next, consider player 1 (the persuader). Using the characterization of the ex-ante expected gross payoffs in the paragraphs preceding the proposition, we have that

$$\Gamma_1^{UEC}\big(\rho,\hat{\rho}\big) \equiv \Big[V_1\big(\hat{\rho}_1;(\hat{\rho}_1,\rho_2)\big) - V_1\big(\rho_1;(\hat{\rho}_1,\rho_2)\big)\Big] - \Big[V_1\big(\hat{\rho}_1;(\rho_1,\rho_2)\big) - V_1\big(\rho_1;(\rho_1,\rho_2)\big)\Big]$$

$$= F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\rho_1,\rho_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$+F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\rho_1,\rho_2) + r^+(\rho_1,\rho_2)\omega^+\right].$$

After simplifying, we have that

$$\Gamma_1^{UEC}\big(\rho,\hat{\rho}\big) = [1-F(0)]\left[r^+(\hat{\rho}_1,\rho_2) - r^+(\rho_1,\rho_2)\right]\left[\bar{\omega}(\rho_1,\rho_2) - \bar{\omega}(\hat{\rho}_1,\rho_2)\right] \geq 0$$

where the inequality follows from the fact that

$$\bar{\omega}(\rho) = \frac{(1-r^-(\rho_2))F(0)\omega^- + (1-r^+(\rho_1,\rho_2))(1-F(0))\omega^+}{(1-r^-(\rho_2))F(0) + (1-r^+(\rho_1,\rho_2))(1-F(0))}$$

is decreasing in ρ_1. Hence, UEC holds strictly for player 1, for any pair of cognitive profiles $\hat{\rho} = (\hat{\rho}_1,\hat{\rho}_2)$ and $\rho = (\rho_1,\rho_2)$ such that $\hat{\rho}_1 \neq \rho_1$ (irrespective of the sign of $\hat{\rho}_1 - \rho_1$).

Next, consider ID. Note that

$$\Gamma_1^{ID}\big(\rho,\hat{\rho}\big) \equiv \Big[V_1\big(\hat{\rho}_1;\hat{\rho}_1\,,\hat{\rho}_2\big) - V_1\big(\rho_1;\hat{\rho}_1\,,\hat{\rho}_2\big)\Big] - \Big[V_1\big(\hat{\rho}_1;\hat{\rho}_1\,,\rho_2\big) - V_1\big(\rho_1;\hat{\rho}_1\,,\rho_2\big)\Big]$$

$$= F(0)\left[(1-r^-(\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2) + r^-(\hat{\rho}_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\hat{\rho}_1,\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2) + r^+(\hat{\rho}_1,\hat{\rho}_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2) + r^-(\hat{\rho}_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\rho_1,\hat{\rho}_2))\bar{\omega}(\hat{\rho}_1,\hat{\rho}_2) + r^+(\rho_1,\hat{\rho}_2)\omega^+\right]$$

$$-F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] - (1-F(0))\left[(1-r^+(\hat{\rho}_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\hat{\rho}_1,\rho_2)\omega^+\right]$$

$$+F(0)\left[(1-r^-(\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^-(\rho_2)\omega^-\right] + (1-F(0))\left[(1-r^+(\rho_1,\rho_2))\bar{\omega}(\hat{\rho}_1,\rho_2) + r^+(\rho_1,\rho_2)\omega^+\right].$$

After simplifying, we have that

$$\tfrac{\Gamma_1^{ID}\left(\rho,\hat{\rho}\right)}{(1-F(0))} = \left[r^+(\hat{\rho}_1,\hat{\rho}_2) - r^+(\rho_1,\hat{\rho}_2)\right]\left[\omega^+ - \bar{\omega}(\hat{\rho}_1,\hat{\rho}_2)\right]$$

$$-\left[r^+(\hat{\rho}_1,\rho_2) - r^+(\rho_1,\rho_2)\right]\left[\omega^+ - \bar{\omega}(\hat{\rho}_1,\rho_2)\right].$$

Hence, ID holds for player 1 with respect to the cognitive profiles $\hat{\rho} = (\hat{\rho}_1,\hat{\rho}_2)$ and $\rho = (\rho_1,\rho_2)$ if and only if Condition (S.3) in the proposition holds. It is easy to see that Condition (S.3) is satisfied when properties (a)-(c) in the proposition hold.

Finally, note that

$$\frac{\partial \bar{\omega}(\rho)}{\partial \rho_2} \overset{sgn}{=} \left[-\frac{dr^-(\rho_2)}{d\rho_2}F(0)\omega^- - \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}(1-F(0))\omega^+\right]\left[(1-r^-(\rho_2))F(0)+(1-r^+(\rho_1,\rho_2))(1-F(0))\right]$$

$$-\left[(1-r^-(\rho_2))F(0)\omega^- + (1-r^+(\rho_1,\rho_2))(1-F(0))\omega^+\right]\left[-\frac{dr^-(\rho_2)}{d\rho_2}F(0) - \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}(1-F(0))\right].$$

After some algebra, we have that

$$\frac{\partial \bar{\omega}(\rho)}{\partial \rho_2} \overset{sgn}{=} \frac{dr^-(\rho_2)}{d\rho_2} - \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2} - r^+(\rho_1,\rho_2)\frac{dr^-(\rho_2)}{d\rho_2} + \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}r^-(\rho_2)\ .$$

Therefore, $\bar{\omega}(\rho)$ is decreasing in ρ_2 for example when $\frac{dr^-(\rho_2)}{d\rho_2} = \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}$ and $r^+(\rho_1,\rho_2) \geq r^-(\rho_2)$, whereas $\bar{\omega}(\rho)$ is increasing in ρ_2 when $\frac{dr^-(\rho_2)}{d\rho_2} > \frac{\partial r^+(\rho_1,\rho_2)}{\partial \rho_2}$ and $r^+(\rho_1,\rho_2) \cong r^-(\rho_2)$, as claimed above. Q.E.D.

S.3 Generalized Career Concerns

The manipulative frames considered in the previous section are instances of "signal jamming," akin to those studied in the industrial organization literature. For example, signal jamming occurs when a firm secretly cuts its price so as to reduce its rivals' profits and induce them to believe that demand is low (or that costs are high) and exit the market. Cognitive traps are common in such games. In this section, we discuss another class of signal jamming considered in the literature, inspired by Holmström (1999)'s celebrated career concerns model.

A worker exerts effort to convince a competitive labor market that her talent is high. The worker's performance depends on her talent, which is unknown to the worker, her effort, and noise. When talent and effort are complements, such signal jamming often generates expectation conformity and expectation traps (see Dewatripont et al. 1999).[6] To see this, consider a generalized version of the career-concerns model in which both the worker and the competitive labor market can invest to influence the information the labor market receives about the worker's talent. Let player 1 be the worker and player 2 the competitive labor market, and assume that payoffs are as in the previous section. Denote player 1's effort by ρ_1 and player 2's effort by ρ_2. Given $\rho = (\rho_1, \rho_2)$, player 2 receives a signal

$$s_2 = A(\rho) + M(\rho)\omega + R(\rho)\varepsilon_2$$

about player 1's talent ω, where A is an "additive" term akin to the one in Holmström (1999)'s original model, M is a "multiplicative" term akin to the one in Dewatripont et al. (1999), and R is a term capturing player 2's ability to "recall," as in the framing model in the previous section. Each of these functions is non-negative, with A and M non-decreasing, and R non-increasing. Holmström

[6] See also Horner and Lambert (2019) for a more recent analysis of these games.

(1999)'s original model corresponds to $A(\rho) = \rho_1$ and $M(\rho) = R(\rho) = 1$, all $\rho = (\rho_1, \rho_2)$ (only the worker invests and effort has an additive effect on performance), whereas the multiplicative model of Dewatripont et al. (1999) corresponds to $M(\rho) = \rho_1$, $A(\rho) = 0$, and $R(\rho) = 1$, all $\rho = (\rho_1, \rho_2)$ (again, only the worker invests, but the impact of effort on performance now depends on the state ω). In either model, recall is exogenous.

Suppose that ω is normally distributed with mean $\omega_0 > 0$ and variance $1/h_\omega$, and that ε_2 is normally distributed with mean 0 and variance $1/h_\varepsilon$. Fixing $\rho = (\rho_1, \rho_2)$, for any s_2, player 2's optimal action is then given by

$$a_2^\rho(s_2) = \mathbb{E}[\tilde{\omega}|s_2; \rho] = \left[\frac{M^2(\rho)h_\varepsilon}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\right]\left[\frac{s_2 - A(\rho)}{M(\rho)}\right] + \left[\frac{R^2(\rho)h_\omega}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\right]\omega_0.$$

Given $\rho = (\rho_1, \rho_2)$, for any actual choice ρ_1' by player 1, player 1's ex-ante expected payoff (gross of the cognitive cost but net of all terms that do not depend on her actual choice ρ_1') is equal to

$$V_1(\rho_1'; \rho) = \frac{M(\rho)h_\varepsilon}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega}\left[M(\rho_1', \rho_2)\omega_0 + A(\rho_1', \rho_2)\right].$$

Likewise, given $\rho = (\rho_1, \rho_2)$, for any actual choice ρ_2' by player 2, player 2's ex-ante expected payoff (gross of the cognitive cost but net of all terms that do not depend on her actual choice ρ_2') is equal to

$$V_2(\rho_2'; \rho) = -\frac{M^2(\rho_1, \rho_2')R^2(\rho_1, \rho_2')h_\varepsilon + R^4(\rho_1, \rho_2')h_\omega}{\left(M^2(\rho_1, \rho_2')h_\varepsilon + R^2(\rho_1, \rho_2')h_\omega\right)^2}.$$

It is then easy to see that, when only player 1 invests, UEC (and hence EC) never obtains in Holmström (1999) additive model, where the optimal level of ρ_1' is implicitly given by

$$C_1'(\rho_1') = \frac{h_\varepsilon}{h_\varepsilon + h_\omega}$$

and is independent of the level ρ_1 expected by player 2. Instead, UEC (and hence EC) can easily obtain in the multiplicative model of Dewatripont et al. (1999) where the optimal level of ρ_1' solves

$$C_1'(\rho_1') = \frac{\rho_1 h_\varepsilon \omega_0}{\rho_1^2 h_\varepsilon + h_\omega}$$

and is increasing in ρ_1 for $\rho_1 \leq \sqrt{h_\omega/h_\varepsilon}$. Consistently with the results in Proposition 1 in the main body, the equilibrium is thus unique in Holmström (1999), whereas multiple equilibria are possible in the multiplicative model of Dewatripont et al. (1999). Furthermore, when this is the case, the worker is better off in the low-effort equilibrium. This multiplicative version of this game is thus prone to expectation traps. Whether or not the above conclusions are robust to the possibility that player 2 (the labor market) also invests to influence the quality of the information received and/or to the possibility of endogenous recall depends on the specific assumptions one makes about the A,

M, and R functions, as indicated in the proposition below. Let

$$L(\rho) \equiv M(\rho)\omega_0 + A(\rho),$$

$$G(\rho) \equiv \frac{M(\rho)h_\varepsilon}{M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega},$$

and

$$Z(\rho) \equiv \frac{M^2(\rho)R^2(\rho)h_\varepsilon + R^4(\rho)h_\omega}{\left(M^2(\rho)h_\varepsilon + R^2(\rho)h_\omega\right)^2}.$$

Proposition S.3 (generalized-career-concerns). *Take any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ with $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$. UEC trivially holds as equality for player 2. It holds for player 1 if the function G is non-decreasing in ρ_1. ID holds for player 1 if the function G is increasing in ρ_2 and the function L is supermodular. ID holds for player 2 if the function Z is submodular.*

Proof of Proposition S.3. First, consider UEC. Using the expressions for $V_1(\rho_1'; \rho)$ and $V_2(\rho_2'; \rho)$ derived above, we have that

$$\Gamma_1^{UEC}(\rho, \hat{\rho}) = \tfrac{M(\hat{\rho}_1,\rho_2)h_\varepsilon}{M^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^2(\hat{\rho}_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

$$-\tfrac{M(\rho_1,\rho_2)h_\varepsilon}{M^2(\rho_1,\rho_2)h_\varepsilon + R^2(\rho_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

and $\Gamma_2^{UEC}(\rho, \hat{\rho}) = 0$. Hence, for any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ with $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$, we have that $\Gamma_1^{UEC}(\rho, \hat{\rho}) = 0$ if the function L is constant in ρ_1. When, instead, L is strictly increasing in ρ_1, then $\Gamma_1^{UEC}(\rho, \hat{\rho}) \geq 0$ if the function G is non-decreasing in ρ_1 (with the inequality strict when G is strictly increasing in ρ_1). When, instead, G is strictly decreasing in ρ_1, then $\Gamma_1^{UEC}(\rho, \hat{\rho}) < 0$.

Next, consider ID. Using again the formulas for the functions $V_1(\rho_1'; \rho)$ and $V_2(\rho_2'; \rho)$, we have that

$$\Gamma_1^{ID}(\rho, \hat{\rho}) = \tfrac{M(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon}{M^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon + R^2(\hat{\rho}_1,\hat{\rho}_2)h_\omega}\left[M(\hat{\rho}_1, \hat{\rho}_2)\omega_0 + A(\hat{\rho}_1, \hat{\rho}_2) - M(\rho_1, \hat{\rho}_2)\omega_0 - A(\rho_1, \hat{\rho}_2)\right]$$

$$-\tfrac{M(\hat{\rho}_1,\rho_2)h_\varepsilon}{M^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^2(\hat{\rho}_1,\rho_2)h_\omega}\left[M(\hat{\rho}_1, \rho_2)\omega_0 + A(\hat{\rho}_1, \rho_2) - M(\rho_1, \rho_2)\omega_0 - A(\rho_1, \rho_2)\right]$$

and

$$\Gamma_2^{ID}(\rho, \hat{\rho}) = -\tfrac{M^2(\hat{\rho}_1,\hat{\rho}_2)R^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon + R^4(\hat{\rho}_1,\hat{\rho}_2)h_\omega}{\left(M^2(\hat{\rho}_1,\hat{\rho}_2)h_\varepsilon + R^2(\hat{\rho}_1,\hat{\rho}_2)h_\omega\right)^2} + \tfrac{M^2(\hat{\rho}_1,\rho_2)R^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^4(\hat{\rho}_1,\rho_2)h_\omega}{\left(M^2(\hat{\rho}_1,\rho_2)h_\varepsilon + R^2(\hat{\rho}_1,\rho_2)h_\omega\right)^2}$$

$$+\tfrac{M^2(\rho_1,\hat{\rho}_2)R^2(\rho_1,\hat{\rho}_2)h_\varepsilon + R^4(\rho_1,\hat{\rho}_2)h_\omega}{\left(M^2(\rho_1,\hat{\rho}_2)h_\varepsilon + R^2(\rho_1,\hat{\rho}_2)h_\omega\right)^2} - \tfrac{M^2(\rho_1,\rho_2)R^2(\rho_1,\rho_2)h_\varepsilon + R^4(\rho_1,\rho_2)h_\omega}{\left(M^2(\rho_1,\rho_2)h_\varepsilon + R^2(\rho_1,\rho_2)h_\omega\right)^2}.$$

Hence, for any pair of profiles $\rho = (\rho_1, \rho_2)$ and $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ with $\hat{\rho}_i \geq \rho_i$, $i = 1, 2$, we have that $\Gamma_1^{ID}(\rho, \hat{\rho}) \geq 0$ if the function G is increasing in ρ_2 and the function L is supermodular (strictly, if G is strictly increasing and/or if L is strictly supermodular). Similarly, for the same profiles, we have that $\Gamma_2^{ID}(\rho, \hat{\rho}) \geq 0$ if the function Z is submodular (with the inequality strict, if Z is strictly submodular). Q.E.D.

S.4 Endogenous depth of reasoning

The analysis in the main body as well as in the preceding sections of this Online Supplement assumes that the players are fully rational and that cognition takes the form of learning about payoffs and/or about other players' beliefs. In this section, we consider an alternative situation in which payoffs are common knowledge, but where players are boundedly rational and cognition determines the players' ability to compute iterated best responses. The analysis builds on the celebrated level-k model, in which k is a player's depth of reasoning, that is, the maximal number of steps of iterated best responses performed by the player. Contrary to the earlier literature (see, e.g., Crawford, Costa-Gomes, and Iriberri (2013) for a detailed overview), a player's depth of reasoning is *endogenous*. Alaoui and Penta (2016, 2017, 2018) are the first to endogenize the depth of reasoning in the level-k model. The main difference relative to their analysis is that we allow the value of expanding cognition to depend on a player's beliefs about (a) her opponents' cognition and (b) her opponents' expectations of her own cognitive level.

S.4.1 The environment

Consider the following two-player game in which payoffs are common knowledge. For each $i = 1, 2$, and each $k \in \mathbb{N}$, there is a mixed action $\alpha_i^k \in \Delta(A_i)$ such that α_i^k is a best response for player i to player j playing according to α_j^{k-1}, with α_i^0 specified exogenously, but reflecting a natural "anchor" that depends on the stage-2 game under consideration (up to this point, the formalism is the same as in the original model, where k is exogenous). Each player's cognitive level $\rho_i \in \mathbb{N}$ determines the player's endogenous depth of reasoning, that is, the number of steps of iterated best responses performed by the player. A player with depth of reasoning ρ_i who expects his opponent to have performed $\rho_j \geq \rho_i - 1$ steps of iterated best responses plays $\alpha_i^{\rho_i}$ in the stage-2 game. A player with depth of reasoning ρ_i who, instead, expects his opponent to have performed $\rho_j < \rho_i - 1$ steps of iterated best responses plays $\alpha_i^{\rho_j+1}$ in the stage-2 game. Formally, for any cognitive profile $\rho = (\rho_i, \rho_j)$, and any ρ_i', player i's period-2 mixed action when the two players are expected to engage in cognition ρ and, instead, player i chooses cognition ρ_i' is given by

$$\sigma_i^{\rho_i';\rho} = \begin{cases} \alpha_i^{\rho_i'} & \text{if} \quad \rho_i' \leq \min\{\rho_i + 1, \rho_j\} + 1 \\ \alpha_i^{\min\{\rho_i+1,\rho_j\}+1} & \text{if} \quad \rho_i' > \min\{\rho_i + 1, \rho_j\} + 1. \end{cases}$$

The idea is that player i plays the action corresponding to his cognitive capacity, unless, given the player's beliefs over the two players' cognitive capacities, player i believes his cognitive capacity exceeds the level that is necessary to perfectly predict the opponent's mixed action. This modeling of the stage-2 behavior is the same as in Alaoui and Penta (2016, 2017, 2018). As anticipated above, the key point of departure is in how players choose ρ_i. In Alaoui and Penta (2016, 2017, 2018), the choice of ρ_i is determined by a cost-benefit analysis in which both the costs and the benefits do not depend on a player's beliefs about her opponents' cognitive sophistication and about their expectations of player i's own cognitive capacity. Here, instead, we allow for such dependence and investigate its implications for the selection of the cognitive levels.

Consistently with the notation in the paper, denote by

$$V_i(\rho_i'; \rho) = U_i(\sigma_i^{\rho_i';\rho}, \sigma_j^{\rho}; \rho_i', \rho_j)$$

the ex-ante expected gross value of choosing cognition ρ_i' when the two players are expected to engage in cognition $\rho = (\rho_i, \rho_j)$ and, instead, player i chooses cognition ρ_i'. Contrary to the case in which cognition takes the form of information acquisition, note that, in this model, V_i need not coincide with player i's value function. This naturally reflects the limited cognitive ability of the players (recall that this model is meant to be a description of the strategic reasoning of boundedly rational agents). Also note that we dropped ω from the player's payoff function because, as explained above, in this game, payoffs are common knowledge (equivalently, $|\Omega| = 1$).

In the spirit of Alaoui and Penta (2016, 2017, 2018), also assume that, when it comes to choosing their depth of reasoning, the players correctly perceive how V_i depends on the players' cognition, even if they are not able to determine their correct best responses. Importantly, in this cognitive game, a player understands that, by increasing her cognition, she may end up with a lower payoff. This may happen despite the players' cognitive choices being covert. As explained in the main text, the reason is that a player who increases her depth of reasoning but not to the point of being able to correctly identify the opponent's true mixed action may find herself trapped into a cognitive loop that induces her to select a stage-2 mixed action that is farther away from her true best response than the one identified by computing a smaller number of iterated best responses.[7]

S.4.2 Discussion

As mentioned above, the players correctly understand how their gross payoffs V_i depend on their own cognition, their opponent's cognition, and their opponent's expectation about their own cognition. This may feel at odds with the assumption that the players are boundedly rational and cannot iterate their best responses to identify the rationalizable actions. It may also look strange that players be able to predict how their stage-2 actions depend on their actual cognition, on their opponent's cognition, and on their opponent's expectations about their own cognition, without however being able to compute the precise best responses. These concerns are normal in models of bounded rationality. The model, however, should not be interpreted literally. It is meant to capture *forces* that shape the choice of the depth of reasoning. It seems plausible that such a choice depends on a player's expectations of her opponent's sophistication, as well as on her beliefs about her opponent's expectation of her own sophistication. That a player's perceived (gross) payoff $V_i(\rho_i'; \rho)$ from choosing cognition ρ_i' when the two players are expected to choose cognition ρ_i and ρ_j correctly reflects the dependence of the stage-2 actual actions on the players' cognitive

[7] Clearly, on path, a player whose depth of knowledge exceeds her opponents' never experiences a lower payoff. That higher cognition may backfire in the level-k model applies only to off-path cognitive choices.

levels (and hence coincides with the player's true payoff) is not essential. What matters is that a player correctly anticipates the forces that shape her stage-2 action, how they depend on the two players' actual and perceived depth of reasoning, and how such forces in turn affect her payoff. In particular, what matters is that a player understands that (a) more cognition need not translate into higher payoffs when insufficient to identify the opponent's action (it can even backfire by bringing a player's action more farther apart from her true best response), (b) going significantly deeper into the understanding of the game than the opponent need not bring any advantage relative to going slightly deeper, and (c) once at her cognitive capacity, a player is unable to respond to variations in her opponent's behavior due to a deeper understanding of the game. These features seem plausible and extend beyond the specific formalization above.[8]

S.4.3 Arad and Rubinstein "11–20" game

For concreteness, we illustrate the role of expectation conformity in a specific game that has received considerable attention in the level-k literature. The stage-2 game described below was first introduced in Arad and Rubinstein (2012), and then simplified by Alaoui and Penta (2016). The players simultaneously announce an integer between 11 and 20. The players receive a number of tokens equal to the integer they announce. However, if a player announces an integer equal to the one announced by her opponent minus one, she receives extra x tokens, where $x \geq 20$. If the two players announce the same integer, they receive 10 tokens in addition to the integer they announce. Each token corresponds to one payoff unit. Letting $A_i = \{11, 12, ..., 20\}$, $i = 1, 2$, we thus have that the ex-post payoffs are equal to

$$u_i(a_i, a_j) = \begin{cases} a_i + x & \text{if} \quad a_i = a_j - 1 \\ a_i + 10 & \text{if} \quad a_i = a_j \\ a_i & \text{otherwise.} \end{cases}$$

This game, which is intended for experimental work, captures, in a stark and simplified manner, some of the forces that arise in certain strategic situations where players benefit from matching, or undercutting by little, the rivals' actions. For example, the two players could be firms selling imperfectly substitutable goods to different segments of the market. If firm i's price exceeds the rival's, firm i sells only to those consumers who do not value the rival's product (the "loyalists"). Firm i is a monopolist on this segment of the market and its monopolistic price on this segment is $a_i = 20$. Reducing the price below $a_i = 20$ without attracting consumers who see the two goods as substitutes comes with a loss of profits. When, instead, firm i matches its rival's price, in addition to selling to its loyalists, it also sells to 1/2 of those consumers who see the two products as substitutes. If it undercuts its rival, it sells to all consumers who see the two products as substitutes. However, if it undercuts the rival by a lot, the extra profits from conquering the contestable buyers are less than the losses from the loyalists. The Arad and Rubinstein (2012) game is meant to be a (highly simplified) version of the strategic situation that firms face in such circumstances.

[8] Some of these properties are in common with the sparsity model in the main body.

Following Arad and Rubinstein (2012) and Alaoui and Penta (2016), let the "anchor" α_i^0 be the degenerate mixed action that selects the largest integer 20 with probability one. This action is often interpreted as the most natural one in the absence of strategic reasoning. The version originally introduced in Arad and Rubinstein (2012) does not feature the bonus of 10 tokens in case the players announce the same integer. The simplification proposed by Alaoui and Penta (2016) has two advantages. It implies that, if the game was played by fully rational players, the unique rationalizable action would have both players select $a_i = 11$ with certainty. It also implies that, for all i, and all $k \geq 9$, α_i^k is the degenerate mixed action that selects the integer 11 with certainty; that is, iterated best responses converge to the unique rationalizable action after 9 iterations. Because of this property, we simplify the analysis by assuming that $\rho_i \in \{0, 1, ..., 9\}$, $i = 1, 2$. We then have the following result (the proof follows directly from the arguments after the proposition):

Proposition S.4. *(a) Consider any pair of cognitive profiles $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$ such that $\hat{\rho}_1 > \rho_1$, $\rho_2 = \rho_1 + 1$, and $\hat{\rho}_2 = \hat{\rho}_1 + 1$. Then $\Gamma_1^{UEC}(\rho, \hat{\rho}) = \Gamma_2^{UEC}(\rho, \hat{\rho}) = 0$, whereas $\Gamma_1^{ID}(\rho, \hat{\rho}) < 0 < \Gamma_2^{ID}(\rho, \hat{\rho})$. (b) Next, let $\hat{\rho} = (\hat{\rho}_1, \hat{\rho}_2)$ and $\rho = (\rho_1, \rho_2)$ be such that $\hat{\rho}_2 = \hat{\rho}_1 > \rho_2 = \rho_1$. Then, for $i = 1, 2$, $\Gamma_i^{UEC}(\rho, \hat{\rho}) = 0$ whereas $\Gamma_i^{ID}(\rho, \hat{\rho}) < 0$.*

Hence, this game features a negative form of expectation conformity, at least with respect to the cognitive profiles under consideration. Consider first case (a). The idea behind this specific pair of cognitive profiles is the following. Suppose the two players are known to have different cognitive costs, with player 2 being the "leader" (that is, the player with the lowest cognitive cost). Further assume that both players' cognitive costs are strictly increasing in their cognition. Then in any equilibrium in which the follower's cognition is equal to ρ_1, the leader's cognition is equal to $\rho_2 = \rho_1 + 1$. Similarly, in any equilibrium in which the follower's cognition is equal to $\hat{\rho}_1 > \rho_1$, the leader's cognition is equal to $\hat{\rho}_2 = \hat{\rho}_1 + 1$.[9] We are interested in whether *multiple asymmetric equilibria* are possible in such a situation, driven by expectation conformity. The answer is no. To see why this is the case, consider first the situation faced by the follower (player 1). Fixing player 2's cognitive level to $\rho_2 = \rho_1 + 1$, the value to player 1 of expanding her cognition from ρ_1 to $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ is (weakly) smaller when player 2 expects player 1 to choose cognition $\hat{\rho}_1$ than when she expects her to choose cognition $\rho_1 < \hat{\rho}_1$. To see this, note that the gross value to player 1 of expanding cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ is equal to

$$\begin{aligned} V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2)) - V_1(\rho_1; (\hat{\rho}_1, \rho_2)) &= [20 - \rho_2 + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x-1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)] - (20 - \rho_1) \\ &= 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x-1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1) - (\rho_2 - \rho_1). \end{aligned}$$

This is because player 2 announces $a_2 = 20 - \rho_2$ when she chooses cognition $\rho_2 = \rho_1 + 1$ and expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$, whereas player 1, when she chooses cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and expects player 2 to choose cognition $\rho_2 = \rho_1 + 1$, she announces $a_1 = 20 - \rho_2$

[9] It is also easy to verify that there exists no equilibrium in which the follower's cognition is strictly higher than the leader's.

if $\hat{\rho}_1 = \rho_1 + 1 = \rho_2$ and $a_1 = 20 - \rho_2 - 1$ if $\hat{\rho}_1 > \rho_1 + 1 = \rho_2$. When, instead, player 1 chooses cognition ρ_1 and expects player 2 to choose cognition $\rho_2 = \rho_1 + 1$, she then announces $a_1 = 20 - \rho_1$.

Likewise, when player 2 expects player 1 to choose cognition $\rho_1 = \rho_2 - 1$, she announces $a_2 = 20 - \rho_1 - 1 = 20 - \rho_2$. It follows that the gross value to player 1 of increasing her cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects her to choose cognition ρ_1 is the same as when player 2 expects her to choose cognition $\hat{\rho}_1$, implying that

$$\Gamma_1^{UEC}(\rho, \hat{\rho}) \equiv [V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2)) - V_1(\rho_1; (\hat{\rho}_1, \rho_2))] - [V_1(\hat{\rho}_1; (\rho_1, \rho_2)) - V_1(\rho_1; (\rho_1, \rho_2))] = 0.$$

Next, consider player 2 (the leader) and fix player 1's cognition to be equal to ρ_1. The gross value to player 2 of expanding her cognition from $\rho_2 = \rho_1 + 1$ to $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is the same no matter whether player 1 expects her to choose cognition ρ_2 or $\hat{\rho}_2$. This is because, in either case, player 1's ability to predict player 2's action is bounded by player 1's own cognitive capacity. In fact, player 1 announces $a_1 = 20 - \rho_1$, that is the action identified by ρ_1 steps of iterated best responses, irrespective of how far ahead she thinks player 2 is in the understanding of the game. This last property, which is the same as is Alaoui and Penta (2016), is similar in spirit to the one discussed in the context of sparsity in games. Hence, for player 2 as well, $\Gamma_2^{UEC}(\rho, \hat{\rho}) = 0$.

Next, consider ID, focusing again on the cognitive profiles of part (a) in the proposition. For player 1, the gross value of expanding her cognition from ρ_1 to $\hat{\rho}_1$ when player 2 expects her to choose $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and chooses cognition $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is equal to

$$V_1(\hat{\rho}_1; (\hat{\rho}_1, \hat{\rho}_2)) - V_1(\rho_1; (\hat{\rho}_1, \hat{\rho}_2)) = (20 - \hat{\rho}_1) - (20 - \rho_1) = -(\hat{\rho}_1 - \rho_1).$$

This is because, in this case, player 2 announces $a_2 = 20 - \hat{\rho}_1 - 1$, whereas player 1 announces $a_1 = 20 - \hat{\rho}_1$ when choosing cognition $\hat{\rho}_1$ and $a_1 = 20 - \rho_1$ when choosing cognition ρ_1. The increase in cognition thus induces player 1 to lower her announcement, without, however, matching player 2's announcement, or undercutting it by one.

When, instead, player 2 expects player 1 to choose cognition $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$ and chooses cognition ρ_2, she then announces $a_2 = 20 - \rho_2$, in which case the value to player 1 of expanding her cognition from ρ_1 to $\hat{\rho}_1$ is equal to

$$V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2)) - V_1(\rho_1; (\hat{\rho}_1, \rho_2)) = [20 - \rho_2 + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)] - (20 - \rho_1)\,.$$

Hence,

$$\Gamma_1^{ID}(\rho, \hat{\rho}) \equiv [V_1(\hat{\rho}_1; (\hat{\rho}_1, \hat{\rho}_2)) - V_1(\rho_1; (\hat{\rho}_1, \hat{\rho}_2))] - [V_1(\hat{\rho}_1; (\hat{\rho}_1, \rho_2)) - V_1(\rho_1; (\hat{\rho}_1, \rho_2))]$$

$$= -(\hat{\rho}_1 - \rho_1) - [10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1) - (\rho_2 - \rho_1)]$$

$$= -(\hat{\rho}_1 - \rho_2) - [10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1) + (x - 1)\mathbb{I}(\hat{\rho}_1 > \rho_1 + 1)] < 0.$$

The reason why this game features a negative form of increasing differences for the player with the highest cognitive cost (equivalently, with the lowest expected cognitive level) is that player 2, when expecting player 1's cognition to be equal to $\hat{\rho}_1 \geq \rho_1 + 1 = \rho_2$, announces $a_2 = 20 - \hat{\rho}_1 - 1$ when choosing the high cognitive level $\hat{\rho}_2 = \hat{\rho}_1 + 1$, whereas she announces $a_2 = 20 - \rho_1 + 1$ when choosing the low cognitive level $\rho_2 = \rho_1 + 1$. Thus player 1 (the one who is expected to be behind in the exploration of the game) suffers from an increase in cognition by her opponent.

Next consider player 2 (the one who is expected to be ahead in the exploration of the game). When player 1 expects her to choose cognition $\hat{\rho}_2$, the gross value of increasing her cognition from $\rho_2 = \rho_1 + 1$ to $\hat{\rho}_2 = \hat{\rho}_1 + 1$ is equal to

$$V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2)) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2)) = 20 - \hat{\rho}_2 + x - [(20 - \rho_2) + 10\mathbb{I}(\hat{\rho}_1 = \rho_1 + 1)] > 0$$

when player 1's cognition is equal to the high level $\hat{\rho}_1$ and is equal to

$$V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2)) - V_1(\rho_2; (\rho_1, \hat{\rho}_2)) = 0$$

when player 1's cognition is equal to the low level ρ_1 (In this latter case, player 2 expects her capacity to be large enough to perfectly predict player 1's announcement, no matter whether she chooses $\rho_2 = \rho_1 + 1$ or $\hat{\rho}_2 = \hat{\rho}_1 + 1 > \rho_2$). It follows that, for the leader, this game features positive increasing differences:

$$\Gamma_2^{ID}(\rho, \hat{\rho}) \equiv [V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2)) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2))] - [V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2)) - V_1(\rho_2; (\rho_1, \hat{\rho}_2))] > 0.$$

Combining the results for unilateral expectation conformity with those for increasing differences, we conclude that, in this game, $\Gamma_1^{EC}\big(\rho, \hat{\rho}\big) < 0 < \Gamma_2^{EC}\big(\rho, \hat{\rho}\big)$. Arguments similar to those establishing Proposition 1 in the main body then imply that, when the cognitive costs are strictly increasing, there cannot exist multiple asymmetric equilibria.

Next, consider the cognitive profiles in part (b) of the proposition. What motivates considering such profiles is their relation to the possibility of *multiple symmetric equilibria*. The result in the proposition implies that such a multiplicity is not possible.

First, consider UEC and, without loss of generality, focus on player 2. When player 1 chooses cognition ρ_1, in the stage-2 game, she then announces $a_1 = 20 - \rho_1$, no matter whether she expects player 2 to choose $\rho_2 = \rho_1$ or $\hat{\rho}_2 > \rho_2 = \rho_1$. This is because, in either case, player 1 is at her cognitive capacity. As a consequence, the value to player 2 of increasing her cognition from ρ_2 to $\hat{\rho}_2 > \rho_2$ is positive but invariant to player 1's expectations about player 2's cognition. From the definition of $\Gamma_2^{UEC}\big(\rho, \hat{\rho}\big)$, we then have that

$$\Gamma_2^{UEC}\big(\rho, \hat{\rho}\big) \equiv [V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2)) - V_2(\rho_2; (\rho_1, \hat{\rho}_2))] - [V_2(\hat{\rho}_2; (\rho_1, \rho_2)) - V_2(\rho_2; (\rho_1, \rho_2))]$$

$$= (20 - \rho_1 - 1 + x) - (20 - \rho_1) - [(20 - \rho_1 - 1 + x) - (20 - \rho_1)] = 0.$$

Because the two players face the same situation under the cognitive profiles under consideration, the same conclusion applies to player 1, that is, $\Gamma_1^{UEC}(\rho, \hat{\rho}) = 0$.

Next, consider ID. When player 1 expects player 2 to choose a higher cognitive level $\hat{\rho}_2$, she then announces $a_1 = 20 - \rho_1$ when choosing the low cognitive level $\rho_1 = \rho_2$ and $a_1 = 20 - \hat{\rho}_1$ when choosing the high cognitive level $\hat{\rho}_1 = \hat{\rho}_2$ (in both cases, player 1 is constrained by her cognitive capacity). The value to player 2 of increasing her cognition from ρ_2 to $\hat{\rho}_2$ is then equal to

$$V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2)) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2)) = 20 - \hat{\rho}_2 + 10 - (20 - \rho_2)$$

when player 1 chooses the high cognitive level $\hat{\rho}_1 = \hat{\rho}_2$, whereas it is equal to

$$V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2)) - V_2(\rho_2; (\rho_1, \hat{\rho}_2)) = 20 - \rho_2 - 1 + x - (20 - \rho_2 + 10)$$

when player 1 chooses the low cognitive level $\rho_1 = \rho_2$. Because $x > 20$, we then have that

$$\Gamma_2^{ID}(\rho, \hat{\rho}) \equiv [V_2(\hat{\rho}_2; (\hat{\rho}_1, \hat{\rho}_2)) - V_2(\rho_2; (\hat{\rho}_1, \hat{\rho}_2))] - [V_2(\hat{\rho}_2; (\rho_1, \hat{\rho}_2)) - V_2(\rho_2; (\rho_1, \hat{\rho}_2))] < 0.$$

The same conclusion applies to player 1. This game thus features a form of negative ID with respect to the profiles under consideration: increasing cognition is more valuable when the opponent chooses a lower cognition. Again, arguments similar to those in Proposition 1 in the main body then imply that this game, despite being played by boundedly rational players, cannot feature multiple symmetric (pure-strategy) equilibria. The reason is that, in this game, the benefit of increasing cognition (starting from a symmetric situation) is lower when the opponent is also expected to increase her cognition, for the opponent call a lower number after expanding her cognition. Hence, no matter the cognitive costs, this game features a unique symmetric equilibrium.

References

Alaoui, L. and A,. Penta (2016), "Endogenous Depth of Reasoning," *Review of Economic Studies*, 83, 1297–1333.

Alaoui, L. and A,. Penta (2022), "Cost-Benefit Analysis in Reasoning," *Journal of Political Economy*, 130(4) 881-925.

Alaoui, L., K. Janezic, and A,. Penta (2020), "Reasoning about Others' Reasoning," *Journal of Economic Theory*, 189.

Arad, A. and A. Rubinstein (2012), "The 11-20 Money Request Game: A Level-k Reasoning Study," *American Economic Review*, 102 (7): 3561-73.

Crawford, V., M. Costa-Gomes, N. Iriberri (2013) "Structural Models of Non-equilibrium Strategic Thinking: Theory, Evidence, and Applications," *Journal of Economic Literature*, 51 (1): 5-62.

Dewatripont, M., Jewitt, I., and J. Tirole (1999) "The Economics of Career Concerns. I: Comparison of Information Structures," *Review of Economic Studies*, 66 (1): 183–198.

Hellwig, C., and L. Veldkamp (2009) "Knowing What Others Know: Coordination Motives in Information Acquisition," *Review of Economic Studies*, 76(1): 223–251.

Holmström B. (1999) "Managerial Incentive Problems: A Dynamic Perspective," *Review of Economic Studies*, 66(1): 169–182.

Hörner, J. and N. Lambert, (2019), "Motivational Ratings," mimeo Yale and Stanford Universities.

Salant, Y., and R. Siegel (2018), "Contracts with Framing," *American Economic Journal: Microeconomics*, 10 (3): 315-46.