Amplification Mechanisms in Liquidity Crises

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Abstract

I describe two amplifications mechanisms that operate during liquidity crises and discuss the scope for central bank policies during crises as well as preventive policies in advance of crises. The first mechanism works through asset prices and balance sheets. A negative shock to the balance sheets of asset-holders causes them to liquidate assets, lowering prices, further deteriorating balance sheets, culminating in a crisis. The second mechanism involves investors' Knightian uncertainty. Unusual shocks to untested financial innovations lead agents to become uncertain about their investments causing them to disengage from markets and increase their demand for liquidity. This behavior leads to a loss of liquidity and a crisis.

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1 Introduction

The credit crisis that began in the Summer of 2007, and continues unabated as of this writing, is a powerful reminder of the importance of financial market liquidity for macroeconomic stability. Central banks have actively intervened to stabilize financial markets and institutions, aiming to forestall any spillovers into the real economy. In many ways, both in the behavior of financial markets and the response of central banks, this recent episode is not unique. It recalls past liquidity crises such as the 1998 hedge fund crisis, the 1987 stock market crash, and the 1970 Penn Central default.

There are broadly two ways to think about liquidity crises. Beginning with Diamond and Dybvig (1983)'s model of bank runs, there is a large literature that models crises as random events; the self-confirming bad equilibrium in a model of multiple equilibria. On the other hand, most crises involve some fundamental shock – often originating in the asset market – that is amplified.[1] For example, in the 1998 event, the Russian default triggered a chain of losses that culminated in the bailout of Long Term Capital Management. In the recent subprime crisis, the direct losses due to household default on subprime mortgages is estimated to be at most $500 bn, yet the effects of the subprime shock have been far reaching (and would have likely been larger if not for central bank intervention). Moreover, in both cases, there was contagion, as the shock led to losses in seemingly unrelated markets: in 1998, the mortgage-backed securities market was affected; while recently, even the market for government guaranteed student loans has been affected.[2]

These observations suggest that there are amplification mechanisms that are at work during liquidity crises. In this paper, I review two important amplification mechanisms. The first is a balance sheet amplification that works through asset prices and balance sheet constraints. Roughly the model I present will illustrate the following story: a negative asset price shock causes balance sheet constraints on asset-holders to tighten, causing assets to be liquidated, lowering asset prices further, and so on. The mechanism here is familiar and is the subject of a substantial literature which I will (partially) review.

I next review an amplification mechanism that works through agents' uncertainty. Many liquidity crises surround financial innovations that were rapidly adopted by market participants. In the subprime crisis, the CDOs and associated credit derivatives that are at the center of the crisis, represent financial assets that

[1] Even in the bank run case, Gorton (1988) shows historically that bank runs were associated with negative economic shocks, casting doubt on the sunspot multiple equilibria model. Papers such as Morris and Shin (1998) and Rochet and Vives (2004) resolve the issue by constructing an amplification mechanism whereby a negative economic shock triggers the bank run.

[2] For further details on the 1998 crisis, see Scholes (2000). See Brunnermeier (2008) for further details on the subprime crisis.

have grown from less than \$1 tn in 2000 to over \$5 tn today. New innovations necessarily mean that market participants have a short time within which they formulate valuation, risk management, and hedging models. A liquidity crisis occurs when the new financial assets behave in unexpected ways. Lacking a historical record to refer to, market participants are faced with risks they don't understand, and treat these risks as Knightian (Knight, 1925). Investors' response in this case is to disengage from risks and seek liquid investments, which can lead to a liquidity crisis dynamic.

Thus, one objective of this paper is to clarify these two amplification mechanisms and discuss their relevance during past crises. The second objective of the paper is to clarify the role for policy during a liquidity crisis. In both models, there is a scope for ex-post policies, such as liquidity provision through a lender of last resort. The same forces that underly the negative-shock amplification mechanism also lead to a beneficial amplification of central bank liquidity provision. There is also scope for ex-ante policies, particularly in the balance sheet amplification mechanism. The private sector generally will not internalize the full costs of a crisis. This opens the door to investigating preventive policies in the form of liquidity requirements and capital requirements on the financial sector. I discuss the scope for these policies in both amplification mechanisms.

This paper is organized as follows. In the next section, I discuss liquidity crises in general terms, addressing the following basic questions: What is liquidity? What happens to liquidity in a crisis? Who are the main actors affected by a lack of liquidity? I then hone in on the two amplification mechanisms mentioned above, describing each in Sections 4 and 5. Section 6 concludes.

2 Liquidity in a Crisis

2.1 Liquidity and Intermediation

Market participants, academics, and policymakers refer to the "lack of liquidity" in a crisis. What do they mean by this? The word liquidity is used to refer to a number of distinct financial phenomena that are each pertinent in a crisis. Here are four ways in which the lack of liquidity becomes most apparent.

- *Market liquidity falls*: An asset is said to have low market liquidity if it is difficult to convert this asset into a liquid medium such as cash or bank reserves. In a crisis, market participants find that their assets lose market liquidity. Potential trading partners are hard to find (Duffie, Garleanu, and Pedersen, 2005). Market-makers are reluctant to accumulate inventories and provide immediacy to investors. It is common in the market micro-structure literature in finance to measure market liquidity in terms of bid-ask spreads or price impact measures (Kyle, 1985, O'Hara, 1995, Chordia, Roll, and Subrahmanyam, 2000) or trading volume and volatility measures (Amihud, 2002, Acharya and Pedersen, 2005). A number of papers measure the resiliency of an asset – how quickly the asset price recovers following a large trade – by measuring the negative serial correlation in asset returns (Campbell, Grossman and Wang, 1993, Pastor and Stambaugh, 2003).[3]

- *Funding liquidity falls*: During crises, borrowers face higher costs in the loan market. A hedge fund that wants to borrow using financial securities as collateral (a "repo agreement"), faces higher collateral requirements (Brunnermeier and Pedersen, 2008). Adrian and Shin (2008a)document that repo market volumes fall during crises. Unsecured lending, backed broadly by the balance sheet of an institution, also becomes more costly to obtain. Volume of issuance in the commercial paper market falls and the spreads of commercial paper yields over Treasury bill yields rise (Gatev and Strahan, 2006). Firms find it difficult to rollover or renew lines of funding from banks. Banks also face higher costs of funding. The interbank market for liquidity does not function smoothly. Many regulators during the recent subprime crisis expressed concern that there was "gridlock" in the interbank market: banks were unwilling to lend to each other and instead hoarded their reserves (McAndrews and Potter, 2002).

- *Balance sheet liquidity falls:* An institution is liquid – or not liquidity constrained – if its balance sheet contains predominantly cash-like or other easily saleable assets, and its liabilities are tilted away from a hard-claim like short-term debt, for which default may lead to bankruptcy, and towards "soft-claims" like equity. In this instance, balance sheet liquidity is high: if needed, the institution can repay all of its short-term debt and forestall bankruptcy. Empirical work in corporate finance often uses cash or leverage to measure balance sheet liquidity (see Kashyap and Stein, 1995, Gatev and Strahan, 2006, Adrian and Shin, 2008ab). A common theme in many accounts of crises is that institutions own long term assets, funded by short term debt, which results in a balance sheet with low liquidity (Diamond and Dybvig, 1983).

- *Flight to liquidity*: Investors scramble for liquidity, exiting illiquid investments and seeking liquid

[3]Pastor and Stambaugh (2003) further distinguish between market liquidity and market liquidity risk – with the latter carrying a risk premium on innovations to market liquidity.

investments. They buy secondary market assets that have high market liquidity, and prefer to hold portfolios in short-term safe claims such as bank deposits that are de-facto liquid. Empirical work finds that the price differences between less and more liquid Treasury bonds, which are otherwise similar, rises during crises (Krishnamurthy, 2001). Price differences between less liquid corporate bonds, mortgage-backed securities, or Agency bonds and more liquid Treasury bonds also rise (Longstaff, 2004, Gabaix, Krishnamurthy, and Vigneron, 2007, Krishnamurthy and Vissing-Jorgensen, 2008).

To organize these different aspects of liquidity shortgage, and provide guidance on which may be more or less relevant in the big picture, it is helpful to center attention on the financial intermediary sector. Beginning with Diamond and Dybvig (1983), there is a long tradition of viewing financial intermediaries as the nexus between the real economy's illiquid assets and households'/investors' need to hold liquid portfolios.

On the asset side, the intermediary sector is a net holder of complex/specialized assets in which households lack knowledge to participate in directly. Investment banks and hedge funds intermediate investment in assets ranging from emerging markets' debt to mortgage-backed securities. Commercial banks own loans to small and medium sized businesses, and offer credit lines to larger businesses.

The intermediary sector also provides immediacy to households, enabling them to own claims that they can easily retrade. Trading desks of investment banks, market makers on stock exchanges, as well as hedge funds all make markets and support the trading of a multitude of assets.

In addition to owning claims that can be traded, households' desire for liquid portfolios is satisfied by their owning de-facto liquid claims in the form of short-term low-risk debt. Commercial banks, investment banks, and hedge funds create short-term debt liquidity through a variety of money-market instruments including bank deposits, repurchase agreements, and commercial paper.

Figure 1 provides a schematic of the functional role of the intermediary sector. For representative references on each of the points raised in the preceding paragraphs, see Grossman and Miller (1988), Gorton and Pennacchi (1990), and He and Krishnamurthy (2008b).

Financial intermediaries play a key role in most liquidity crises. The recent subprime crisis centers on commercial banks and investment banks; the 1998 crisis centered on hedge funds; the 1987 crisis on the market-makers in the stock market; the 1970 Penn Central default on money market mutual funds; and, the Great Depression on commercial banks.

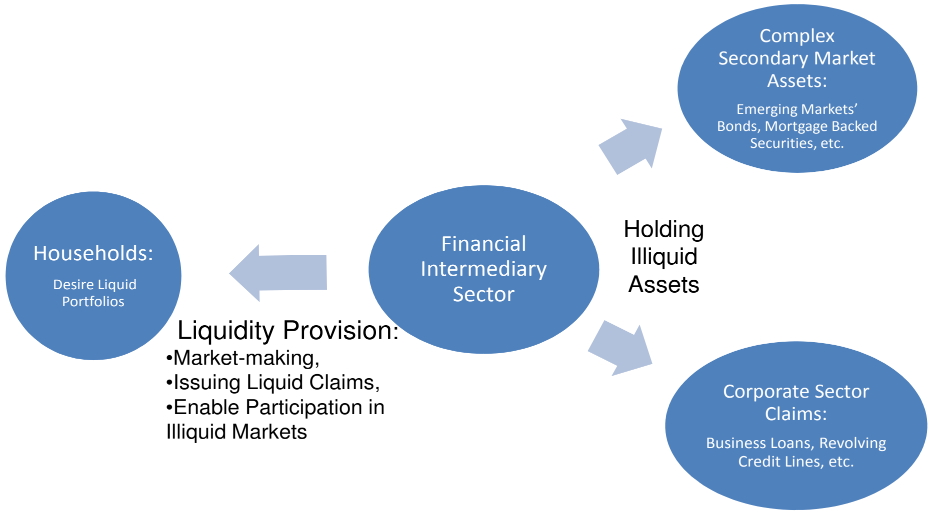

Figure 1: Intermediary Functions

Intermediaries provide liquidity to households (arrow to left) and use funds to purchase both secondary market specialized assets (arrow to top right) and make business loans (arrow to bottom right).

Consider next how a breakdown in financial intermediation – fall in liquidity provision to households and reduced capacity to hold the economy's illiquid assets – can lead to a lack of liquidity in the dimensions I have outlined.

Market liquidity falls because the entities that provide market making and immediacy cut back on these services (Weill, 2007). As hedge funds run into trouble, the markets for the specialized assets they hold shrinks, and these asset markets also lose market liquidity (He and Krishnamurthy, 2008a).

Funding liquidity falls because credit extension, being a core activity of the intermediary sector, is cut back (Holmstrom and Tirole, 1997). Commercial bank lending to firms is curtailed. This is an important channel for central banks because it indicates a manner in which a liquidity crisis spills over to the real sector. Additionally, financing arrangements within the intermediary sector also tighten; e.g., a commercial bank lending to a hedge fund to finance a position on margin raises the margin requirement.

The latter two liquidities are therefore the result of trouble within the intermediary sector. A more direct measure of the state of the intermediary sector is its balance sheet liquidity. If intermediaries have less cash, or lose capital, their balance sheet liquidity suffers. This, in turn, leads them to cut back on their various liquidity provision services. These observations suggest ranking balance sheet liquidity a step above the other liquidities in analyzing crises. A reduction in the balance sheet liquidity of intermediaries leads to a contraction in the supply of liquidity provision services by the intermediaries.

Consider next a rise in liquidity demand/flight to liquidity.[4] During a crisis, households grow uncertain about intermediaries and their investments. Their natural response is to disengage and demand liquid and riskless claims which are less sensitive to their uncertainty. Thus, investors adjust their own investments in the intermediary sector, recontracting the intermediaries' liabilities so that they shift towards the short-term and away from softer claims like long-term debt. These actions in turn reduce the balance sheet liquidity of the intermediary sector, with knock-on effects on other liquidities.

[4] This discussion follows from Caballero and Krishnamurthy (2008) who show how given some exogenous liquidity needs for agents, a rise in Knightian uncertainty increases their liquidity demand. While it is standard in the literature to assume that households have liquidity needs, there is a deeper question of where these liquidity needs come from. Eisfeldt (2007) writes a model of a household whose income process may be different than its desired consumption process. The discrepancy gives rise to a demand for liquid assets. However, Eisfeldt shows that a realistic calibration of the model does very poorly in generating the magnitude of liquidity premia are observed in practice. Faced with a liquid, low return asset, and an illiquid, high return asset, households time their purchases of illiquid assets to take full advantage of the high return asset without incurring any of its illiquidity costs. In short, there is no demand for the low-return liquid asset. The result is echoed in other papers in the literature (e.g., Heaton and Lucas, 1996) and suggests that we need a theory of liquidity demand that does not center on households' consumption needs. Here are some approaches that seem promising. Vayanos (2004) presents a model of delegated fund management in which households withdraw funds from a mutual fund following poor performance. As a result, the mutual fund manager alters his portfolio to favor liquid over illiquid assets, as household withdrawal states appear more likely. In this approach, there are no "true" liquidity needs in the economy. However, the inflexibilities that arise in contractual relationships between households and intermediaries create an endogenous source of liquidity demand. Holmstrom and Tirole (1998) and Eisfeldt and Rampini (2007) study corporate liquidity demand. In their models, firms face external financing constraints that prevent them from undertaking all good investment opportunities. This possibility leads them to insure, ex-ante, against those states in which the constraints are most tightly binding. The insurance can be represented as a demand for liquid assets, which the firm can then use in the needed state.

2.2 Interactions

The lack of liquidity in different dimensions can interact. Such interactions are important because they underly amplification mechanisms. The formal analysis of the following sections explores two amplification mechanisms.

As noted in the introduction, the first interaction I study is between balance sheet liquidity and asset market liquidity. In equilibrium, if the liquidity of assets depends on the condition of the asset holders' balance sheet, then a systematic deterioration in balance sheets can reduce asset liquidity, further deteriorating balance sheets, and so on.

The second interaction I study is between investors' liquidity needs and intermediaries' balance sheet liquidity. I study a model in which a rise in uncertainty causes investors' liquidity demand to rise. Suppose again that the liquidity of an asset depends on the balance sheet liquidity of a financial institution that acts as the market-maker in that asset. A rise in uncertainty makes an investor concerned that the liquidity needs of *other* investors will rise and in satisfying this liquidity need, the balance sheet liquidity of the market-maker will fall. Suppose that every investor is symmetrically in this position. Then, the rise in liquidity needs deteriorates balance sheet liquidity of the market maker, causing a loss of market liquidity in the asset.

In addition to describing these amplification mechanisms, I discuss how central bank policy can alleviate a liquidity crisis. This brings me to another aspect of liquidity that deserves mention.

Macroeconomic or monetary liquidity: The central bank has control over the supply of government-backed liquid assets and increases this supply during crises. Typically, the central bank injects liquidity by expanding the supply of bank reserves, and in the recent subprime crisis the Fed conducted swaps of (less liquid) mortgage-backed securities for (more liquid) Treasury securities. The central bank also directly lends reserves through the discount window in a targeted fashion to commercial, and recently, investment banks. Discount window policy also has an important commitment dimension. The Fed typically commits to making reserves freely available to banks at some predetermined rate. This commitment may be important in cases where banks are unsure whether they will be able to access liquidity from private sources in every contingency.[5]

[5] There is a related notion of government supplied liquidity that appears in the literature, labeled aggregate liquidity by Woodford (1990) and Holmstrom and Tirole (1998). When the central bank acts as a lender of last resort during a crisis, it is ultimately redistributing real resources across private parties. The lender of last resort is valuable because resources, from

In the models I study, the important dimension of monetary policy is in enhancing the financial sector's balance sheet liquidity. The central bank's willingness to absorb less liquid assets on to its balance sheet, providing more liquid assets to the private sector in return, enhances balance sheet liquidity and drives the benefit to policy.

3 Benchmark Model of Liquidity Provision

I begin by introducing a benchmark model that I use to discuss the two amplification mechanisms operating during liquidity crises.

The model has a financial intermediary – commercial bank, market-maker, trading desk, etc. – that sells an asset at date 0 to a class of atomistic investors. While the asset matures at date 2, the investors purchase the asset at date 0 in part because the intermediary stands ready to buy back the asset at date 1 if an investor wishes to liquidate. That is, the intermediary commits to be a liquidity provider for the asset. The intermediary has resources of L — cash, or some other liquid medium — to back up the liquidity commitment. L may be thought of as the balance sheet liquidity of the intermediary.

An investor may receive a shock at date 1 whereby he needs to liquidate the asset. Investors come in two classes, A and B. The shock is independent across the classes, but perfectly correlated within a class. The probability that a shock affects any one agent is ϕ. Thus, there are four liquidity shock possibilities at date 1: $\omega \in \Omega \equiv \{no, A, B, AB\}$. $\omega = no$ is the no shock case and occurs with probability $(1-\phi)^2$; $\omega = A$ is shock to class A, with probability $(1-\phi)\phi$; $\omega = B$ is shock to class B, with probability $(1-\phi)\phi$; $\omega = AB$ is shocks to both classes, with probability ϕ^2.

either the private sector or the central bank, that are injected into the affected parties has a large multiplier. The government is special in executing the transfer only because of credibility: the government can credibly commit to transfer a much larger amount of resources than a private party such as an insurance company. Holmstrom and Tirole consider a model in which the government securitizes this commitment by issuing government bonds that it sells ex-ante to the private sector. The private sector then trades these bonds, along with private securities, to arrange for an optimal state-contingent transfer of resources, in the event of a liquidity crisis. Aggregate liquidity is a measure of how much of these transfers can be arranged. Gorton and Huang (2004) study a related model where both the private and the public sector supplies liquidity against crises. Their analysis sheds light on the moral hazard consequences of supplying public liquidity, and thereby articulates the costs and benefits of public liquidity provision. Caballero and Krishnamurthy (2008) also study a model where both private and public sector supply liquidity. Their analysis demonstrates conditions under which public supply is a complement to private supply of liquidity, hence alleviating the moral hazard concern.

I normalize things as follows. There is a unit measure of each class of agents. Each owns one unit of the asset at date 0, so that there are two shares of the asset outstanding. The asset pays one at date 2.

The intermediary promises to buy back the asset at the price of one, unless it exhausts L, in which case it allocates L equally to all liquidating investors. If l_1 is the measure of investors liquidating, the date 1 asset price is:

$$P_1^{\omega}(L) = \min\left(1, \frac{L}{l_1}\right). \tag{1}$$

I assume that the liquidity of the financial intermediary satisfies:

$$1 < L < 2.$$

Thus, in the cases of no or one shock, the intermediary has sufficient L to redeem shares at a price of one, while in the case of both shocks, the intermediary redeems shares at the price,

$$P_1^{AB}(L) = \frac{L}{2}$$

Assuming investors are risk neutral, the price of the asset at date 0 is,

$$P_0 = 1 - (1 - L/2)\phi^2. \tag{2}$$

We can think of $(1-L/2)\phi^2$ as the liquidity discount on the asset, which is therefore a measure of the asset's market liquidity. Clearly this discount is decreasing in L.

4 Amplification through Balance Sheets and Asset Prices

Most investors in the sophisticated financial markets affected by liquidity crises – i.e. the mortgage and credit markets in the current subprime crisis, or the sovereign debt and derivative markets of the 1998 crisis – are themselves financial intermediaries who are managing the funds of an ultimate household. Examples of these investors are hedge funds or banks, as well as pension funds and mutual funds. Theory then suggests that there may be agency conflicts between the household-investor and the intermediary that can affect the intermediary's investment decisions. As in corporate finance, where a similar manager-shareholder conflict arises, the intermediary's investment will depend on external financing conditions and the condition of its balance sheet.

Hedge funds raise equity from a clientele that is a combination of the managers of the hedge fund and wealthy households/institutional investors. They raise debt, mostly in the form of repo financing, from other investors. Thus a hedge fund manages the funds contributed via both debt and equity contracts of some ultimate household. If hedge funds have lost money, or if they are holding particularly illiquid assets during a period of market illiquidity, investors may be unwilling to invest in either the equity or debt of the hedge fund. In this case, the hedge may have to reoptimize its portfolio, selling some assets and holding more liquid assets.

Mutual funds raise moneys predominantly through equity contributions of households. A well documented regularity is that the investors in mutual funds withdraw their funds following poor performance. Again, there is a relationship between the investment decisions of the mutual fund and its balance sheet (or change thereof).

If the liquidity of the financial market depends on the health of participants' balance sheets, then a feedback mechanism emerges whereby a negative shock in the financial market worsens balance sheets, reducing financial market liquidity, feeding back into balance sheets, and so on.

4.1 Balance Sheet Constraint

I alter the baseline liquidation model to introduce balance sheet concerns. I assume that the investors in the model are themselves intermediaries who are investing the money of some ultimate household. To be concrete, I will label these investment-intermediaries as hedge funds. But as noted above, one can equally think of them as commercial banks or institutional investors. I study a model in which the investment decisions of these hedge funds are a function of the health of their balance sheets.

The hedge funds purchase the asset at date 0. If they need some liquidity at date 1, the liquidity provider with L units of resources commits to buy back the assets. In the subprime case, we can think of the hedge fund as a buyer of mortgage-backed securities (MBS) and the liquidity provider as a commercial or investment bank that makes markets in the MBS. Another labeling that fits the recent crisis is to think of the investor as a small commercial bank or institutional investor that had invested in a structured investment vehicle and the liquidity provider as a large commercial bank that had provided the liquidity backstop for the vehicle.

Note that one can also introduce balance sheet concerns into the liquidity provider of the model — this is another logical path to take — but it turns out to be easier to exposit the amplification mechanism by modifying the investor side.

Figure 2 illustrates the players in the model. At date 0, the hedge fund raises debt from households of

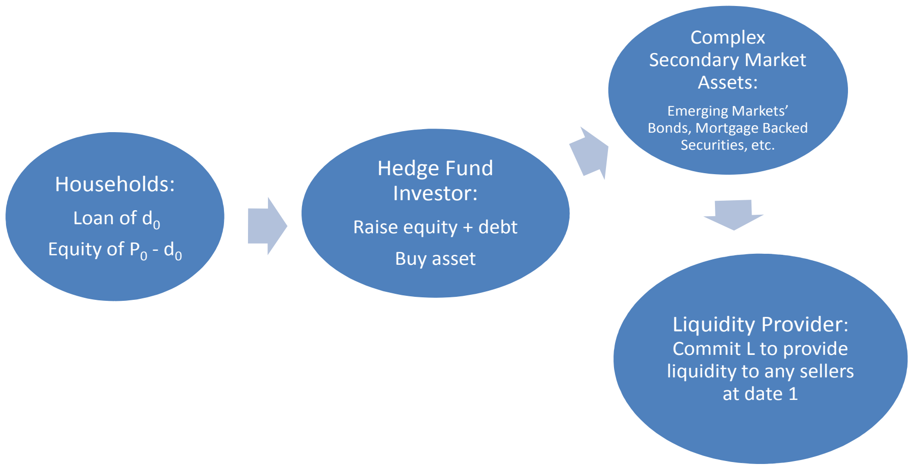

Figure 2: Balance Sheet Model

Households provide funds to hedge funds, who invest the funds in secondary market assets, expecting there to be a liquidity provider that stands ready to buy back the asset if the hedge funds needs to liquidate.

d_0 and equity of $P_0 - d_0$, so that the total funds raised are exactly enough to buy one unit of the asset. The hedge fund is also subject to the exogenous liquidation shocks introduced earlier.

Define the equity capital of the hedge fund at date 1 as,

$$w_1 = P_1 - d_0.$$

This equity capital is a measure of the balance sheet liquidity of the hedge fund. That is, since the hedge fund could sell the asset at date 1 at price P_1 and repay d_0 of debt, the difference $P_1 - d_0$ reflects the liquid resources available to the fund. For now, I fix the debt level exogenously at $d_0 < \frac{L}{2}$, and moreover assume for simplicity that the debt carries no interest. Since the lowest date 1 price is $\frac{L}{2}$ the debt can always be fully repaid.

Suppose that the date 1 holdings of the fund are subject to a margin constraint. In particular, define θ_1

as holdings of the fund at date 1 (after any liquidations). The constraint is that,

$$m\theta_1 \leq w_1. \tag{3}$$

The fund must have equity capital commensurate to the size of its asset market position. We may think of m as a margin requirement per unit of asset holding, so that to hold θ_1 units of the asset, the fund must put up total margin of $m\theta_1$.

Note that since w_1 is decreasing in P_1, the balance sheet constraint tightens as market prices falls. In this sense, the constraint captures an important feature of crises: market conditions and financing conditions worsen at the same time.

The interpretation of m as a margin is close to Brunnermeier and Pedersen (2008) who develop the margin interpretation of this constraint in depth, discussing the rationales in the institutional context of margin setting. An important observation they make is that margins are typically set based on asset price volatility. But, in an equilibrium setting, asset price volatility may be increasing in the tightness of the margin constraint. This describes a feedback loop between volatility and margin constraints.

If we step back from the margins interpretation of constraint (3), it is worth noting that there are other balance sheet constraints that appear in the literature which are close to (3) and importantly preserve the relation between the constraint and market prices.

Kiyotaki and Moore (1997) develop a model in which lenders limit the debtor's investments based on pledged collateral. Suppose that the assets of θ_1 can be pledged as collateral to a lender who forwards up to $\gamma\theta_1 P_1$ against these assets (where $\gamma < 1$). Then, to purchase θ_1 units of asset, the budget constraint for the hedge fund is,

$$\theta_1 P_1 \leq \gamma\theta_1 P_1 + w_1$$

or, rewriting,

$$(1-\gamma)P_1\theta_1 \leq w_1. \tag{4}$$

This constraint, if we define $m = (1-\gamma)P_1$, is identical to the constraint (3). There is a price dependence on m which makes it different. However, in spirit, this constraint also preserves the effect that a reduction in w_1 causes θ_1 to fall.[6]

[6] To see this, substitute $w_1 = P_1 - d_0$ into equation (4), to find that,

$$(1-\gamma)\theta_1 \leq 1 - \frac{d_0}{P_1}.$$

He and Krishnamurthy (2008a) develop a model in which an incentive conflict affects the participation of outside investors in a fund. In their model, a hedge fund manager as part of an optimal contract is required to put up some of his own wealth into the fund. Denote the manager's wealth as w_1. He and Krishnamurthy derive a contract whereby outside investors are willing to coinvest in the fund, at most, a multiple $\mathcal{M}$ times the manager's investment in the fund. The coinvestment constraint implies that the total funds that are under management by the hedge fund must be less than $w_1 + \mathcal{M}w_1$. With these funds, the manager purchases θ_1 units of assets. Then the fund's budget constraint is,

$$P_1\theta_1 \leq w_1(1+\mathcal{M})$$

or,

$$\frac{1}{1+\mathcal{M}}P_1\theta_1 \leq w_1 \tag{5}$$

This constraint is identical to (4), and again preserves the key interaction between w_1 and the asset position θ_1. The fact that there are many contexts in which a constraint similar to (3) arises indicates that the balance sheet mechanism may be pervasive.

Let us analyze the margin constraint in (3) further. Define,

$$l_1 = 1 - \theta_1 \tag{6}$$

as the amount of asset liquidated by the hedge fund at date 1. If a fund receives a liquidity shock then $l_1 = 1$ by definition. The interesting case to study is endogenous liquidation. Thus, let us focus on a fund that does not receive a liquidity shock. For this fund, $l_1 = 0$ if the constraint does not bind, while if the constraint binds, substitute to find:

$$l_1 = 1 + \frac{1}{m}\left(d_0 - P_1^{\omega}\right). \tag{7}$$

Liquidations increase as the price falls and leverage (d_0) rises, capturing the balance sheet effect I have alluded to. The required margin m has two effects on liquidation. First, since d_0 is always less than P_1^{ω}, increasing m increases liquidations. This is also apparent by inspecting the capital constraint (3) where raising m tightens the constraint. Second, from (7) we see that raising m decreases the sensitivity of liquidations to price conditional on the constraint binding. This is also intuitive from (3): if the constraint is binding, a fall in w_1 tightens the constraint less when m is large.

As P_1 falls, the right hand side of this equation falls, tightening the constraint and thereby reducing θ_1.

In state $\omega = no$, total liquidations across both hedge funds are $2l_1$ (which may be zero if $l_1 = 0$). In state $\omega = A$ or $\omega = B$, total liquidations are $l_1 + 1$. State $\omega = AB$ has only the exogenous liquidation of two shares.

The price is set by the liquidity provider as before:

$$P_1^{no} = \min\left(1, \frac{L}{2l_1}\right) , \qquad P_1^A = P_1^B = \min\left(1, \frac{L}{l_1+1}\right) , \qquad P_1^{AB} = \frac{L}{2}$$

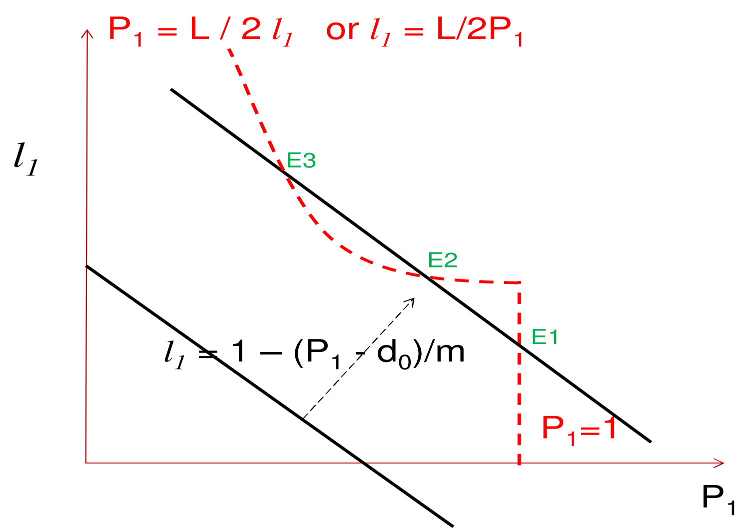

Figure 3: Liquidation Equilibria

Price is on the X-axis and quantity liquidated on the Y-axis. The curved-then-straight line is the liquidity provider's price-setting function. The negatively sloped line is the hedge fund liquidation function, pictured for a low value of d_0 and a high value of d_0.

Figure 3 presents the equilibrium price determination graphically. The price is on the X-axis, while the quantity liquidated is on the Y-axis. If liquidations are low, the liquidity provider has sufficient L to redeem all shares at $P_1 = 1$. This is the vertical segment corresponding to the intermediary's price-setting function. When liquidations exceeds $L/2$ shares, the price falls with liquidations. This is the curved part of the price-setting function.

The hedge fund's liquidation function of (7) is also pictured. This function is linearly decreasing in price, describing the heart of the balance sheet feedback: low prices induce liquidation.[7] Let us take the case of state $\omega = no$. If d_0 is low, we have the no-liquidation equilibrium at $P_1 = 1$. As d_0 rises, the liquidation function shifts to the right. If d_0 rises sufficiently the liquidation function intersects thrice with the price-setting curve. The multiple crossing can also occur in states $\omega = A, B$, where for a given d_0, the exogenous liquidity shock shifts the function up.

The multiple crossing case is one of multiple equilibria and illustrates the balance-sheet feedback loop.[8] Equilibrium $E1$ is a no-liquidation equilibrium in which $P = 1$. Equilibrium $E2$ is a moderate liquidation equilibrium where the price falls below one, and the hedge funds liquidate some of their holdings. Equilibrium $E3$ is the severe liquidation equilibrium: low prices and large liquidations. In both $E2$ and $E3$ if agents expect prices to be low, they liquidate positions and push prices down. Note that of the two liquidation equilibria, $E2$ is an unstable equilibrium (i.e. any perturbation of prices or quantities moves the equilibrium to either $E1$ or $E3$), while $E3$ is the stable liquidation equilibrium. I will focus on equilibria $E1$ and $E3$ in the analysis that follows.

The feedback loop illustrated by the model has been explored in different contexts within the literature. Allen and Gale (2005) present a model in which a bank is funded by demandable deposits and holds assets whose price is determined in a market equilibrium. There is a feedback loop between bank runs, asset liquidation, and lower asset prices and value of bank equity. Diamond and Rajan (2005) present a related analysis in which the contracts underlying relationships are derived from optimal contracting considerations but the liquidation price of the asset is fixed exogenously. Shleifer and Vishny (1997) describe a feedback loop because investors withdraw moneys from an arbitrageur if the arbitrageur loses money. Morris and

[7] Introducing balance sheet concerns into the liquidity provider would tilt the liquidity provider's price-setting function further to the left, flattening the curve. This will create another source of amplification.

[8] If the balance sheet constraint binds, the equilibrium values of l_1 in equilibria $E2$ and $E3$ are the roots of the quadratic equation:

$$l_1^2 - (1 + d_0/m)l_1 + \frac{1}{2m}L = 0$$

The parameters required to arrive at the three-crossing solution depicted in the figure are: $L < 1$, $d_0 < L/2$, and a suitably chosen value of m (slightly lower than $1-d_0$) so that the solutions to the quadratic equation are not complex and in the solution $l_1 < 1$. Note that if $m < 1 - d_0$ the balance sheet constraint does not bind at a value of $P_1 = 1$. This defines the equilibrium $E1$.

Shin (2004) describe a feedback loop in a model where traders have short horizons and trading limits. As mentioned earlier, Brunnermeier and Pedersen (2008) present a model in which the borrowing constraint is a function of market price volatility, which is itself a function of the actions of the constrained agents. Vayanos (2004) presents a related analysis where the investors in a hedge fund withdraw money if volatility rises. Vayanos' model is fully dynamic which helps to understand the behavior of a risk premium on volatility innovations.[9] There is also work in a multiple asset setting in which the feedback loop can be used to explain contagion effects. Kyle and Xiong (2001) is representative of this work. The models in this branch of the literature present the result that a shock in one market tightens balance sheet constraints and causes liquidations and falling asset prices in another market.

The common point in all of this work is that the balance sheet mechanism can help explain how liquidity can deteriorate. Equally, this work demonstrates an amplification mechanism for shocks. If we focus on the liquidation equilibrium $E3$, any shock that perturbs either curve has large effects on the equilibrium price and quantity since both curves are negatively sloped.

4.2 Crisis Policy

The multiple liquidation equilibria is a coordination failure and can motivate central bank intervention. As in Diamond and Dybvig's (1983) bank-run model, central bank policy via a lender of last resort can eliminate the coordination failure. Suppose that the central bank commits to inject $2 - L$ of cash into the liquidity provider if either d_0 is large enough, or there are the exogenous liquidation shocks $\omega = A, B$.[10] This commitment makes the price-setting curve fully vertical at price $P = 1$ and rules out the bad equilibrium.

The policy can be implemented through the discount window with the central bank agreeing to lend funds to the liquidity provider against the collateral of the asset. Another way to implement this policy is for the central bank to act as a "market-maker of last resort" (Buiter and Sibert, 2007). Again if either d_0 is large enough, or if $\omega = A, B$, the central bank commits to purchase the asset directly if $P < 1$. The policy rules out the bad equilibrium. In both cases, the important dimension of the policy is that the central bank commits to absorb the illiquid asset onto its balance sheet, providing a liquid asset such as cash in return.

[9] In these papers, it is because assets are "marked to market" that there is a feedback loop. Plantin, Sapra, and Shin (2008) build on this observation in analyzing the costs and benefits of "mark to market" accounting rules.

[10] Since in state AB there is no coordination failure there is also no special gain to committing to purchase the asset in that state.

The commitment to enhance the balance sheet liquidity of the liquidity provider severs the feedback loop.[11]

4.3 Ex-ante Policy

The balance-sheet model opens the door to studying ex-ante policy. That is, the date 1 multiple equilibria are due to an externality among agents. If we focus on date 0, then the ex-ante debt choice affects the strength of the date 1 externality. As I will show, this logic implies that generally the date 0 debt choice of the hedge fund is inefficient. The result is one of the intellectual underpinnings of prudential capital or liquidity requirements, or more generally, regulation of the balance sheet liquidity of the financial sector.

Since the externality is present with or without the exogenous liquidity shocks, I simplify the presentation by removing these shocks and set $\phi = 0$. I will focus on the case presented in Figure 3, where d_0 is large enough so that equilibrium is either $E1$ or $E3$ (as noted above, I discard $E2$). The selection between these equilibria is determined by the realization of a sunspot. With probability Φ the sunspot realization chooses the $E3$ equilibrium, while with probability $1-\Phi$ the $E1$ equilibrium is chosen.

I carry out the following experiment. Suppose that a hedge fund increases the date 0 debt financing of d_0 by a small amount and reduces the equity contribution by the same small amount so that the date 0 asset position is unchanged. I compute the ex-ante valuation of this change for a single fund and contrast this number with the same computation when the entire hedge fund class increases d_0. As one would conjecture, both of these valuations are negative since increasing d_0 increases date 1 liquidations. The interesting result is that generally the private cost of debt is smaller than the social cost, suggesting that agents may over-leverage in equilibrium. Note that my experiment is silent on the benefit of debt over equity financing. Rather than getting into a model of corporate financing to derive this advantage, I will imagine fixing this benefit and holding it constant across the two computations.

Consider first the single agent computation. A hedge fund's expected value from asset holding, given d_0 and P_1^ω, is:

$$V^P = (1-\Phi) + \Phi\left((1-l_1^{E3}) + l_1^{E3} P_1^{E3}\right)$$

The first term in this expression reflects the expected payoff from not having to liquidate the asset in the $E1$ equilibrium, while the second term is the expected payoff when there is l_1^{E3} liquidation in the $E3$ equilibrium.

[11] Implicitly this policy also requires that the central bank is as knowledgeable as the liquidity provider or hedge fund about how to value the asset.

Differentiating this expression, I find:

$$
\begin{aligned}
\frac{\partial V^P}{\partial d_0} &= -\Phi(1 - P_1^{E3})\frac{\partial l_1^{E3}}{\partial d_0} \\
&= -\Phi(1 - P_1^{E3})\frac{1}{m},
\end{aligned} \tag{8}
$$

where the second line follows from expression (7). The cost of higher leverage is that the asset may be liquidated early at price P_1^{E3} rather than held to maturity to yield a payoff of one.

For the entire hedge fund class, the expected value is,

$$
V^S = (1 - \Phi) + \Phi\left((1 - l_1^{E3}) + \frac{L}{2}\right)
$$

where I have rewritten V^P substituting in from the equilibrium condition that $P_1^{E3} = \frac{L}{2l_1^{E3}}$. Differentiating again,

$$
\begin{aligned}
\frac{\partial V^S}{\partial d_0} &= -\Phi\frac{\partial l_1^{E3}}{\partial d_0} \\
&= -\Phi\left(\frac{1}{m} - \frac{\partial P_1^{E3}}{\partial d_0}\right)
\end{aligned} \tag{9}
$$

Relative to the single agent case, the computation must account for the general equilibrium effect of d_0 on P_1^{E3}. From Figure 3, we see that increasing d_0 shifts the liquidation function to the right and lowers P_1^{E3} (i.e. increases liquidations). Hence,

$$
\left|\frac{\partial V^S}{\partial d_0}\right| > \Phi/m > \Phi(1 - P_1^{E3})/m = \left|\frac{\partial V^P}{\partial d_0}\right|,
$$

where these derivatives are negative.

Intuitively, the social cost of debt is higher than the private cost of debt because a single hedge fund does not take into account that its increasing d_0 at date 0 makes the fund have to liquidate more asset at date 1, pushing prices lower at date 1, which in turn results in greater liquidations by other hedge funds. Since the market price is endogenous at date 1 to the trades of the hedge funds, the trading externality that is present at date 1 translates into a financing externality at date 0.

Thus a central bank policy that restricts date 0 leverage, or imposes a minimum equity capital requirement, can improve welfare. These are common regulations for commercial banks, and some of the recent policy discussion regards extending these regulations to other parts of the financial sector including hedge funds and investment banks.

In more general settings, the externality leads to "underinsurance" against the crisis equilibrium – the leverage externality of the current model is just one instance of underinsurance. Consider for instance a model in which there are many states at date 1 with differing amount of L, and $L < 2$ in only a few of these states. In such a model, the optimal policy will call for agents to have more liquid balance sheets (i.e more w, less d) in only the crisis states. Thus more generally the policy I have outlined concerns the risk management of balance sheet liquidity. The externality leads agents to make ex-ante asset and liability choices that leaves them with a less liquid balance sheet in crisis states.

Caballero and Krishnamurthy (2003) study the underinsurance externality in the context of emerging market's crises and discuss firms' financing choices over domestic and foreign currency debt arguing that firms will undervalue the insurance benefit of denominating debt in domestic currency. Gromb and Vayanos (2002) study a similar externality in a dynamic model of collateral constrained arbitrage and show that the arbitrageurs trade too early in trying to profit from the arbitrage opportunity.[12]

Finally, strictly speaking if I define welfare in the current model as the sum of hedge fund/household and liquidity-provider utility, then since liquidations just leads to transfers, the date 0 debt choice does not affect welfare. It is straightforward to consider setting where quantities (i.e. real investment) adjusts and describe ex-ante policies that are Pareto improving. See Caballero and Krishnamurthy (2004) for a full treatment of ex-ante policies.

4.4 Dynamic Model

It is possible to embed the balance sheet mechanism I have described within a full-fledged dynamic equilibrium model. This is done in papers such as Xiong (2001), Vayanos (2004), and He and Krishnamurthy (2008b). I will briefly sketch the model of He and Krishnamurthy, and present some illustrative results from the calibrated model of that paper.

The economy is set in continuous time. There is a risky asset which pays a flow of dividends D_t, where,

$$\frac{dD_t}{D_t} = gdt + \sigma dZ_t$$

Z_t is a Brownian motion process and represents the risk in the economy.

[12] For related treatments, see Diamond and Rajan (2005) in a banking context, and Holmstrom and Tirole (1998) in a model of government liquidity policy.

The risky asset is held through intermediaries that are each subject to a balance sheet constraint. In particular, the intermediaries are run by owner-managers who own w_t of the equity in the fund. The owner-manager makes decisions on how much money to raise from outside investors, and over the portfolio (risky and riskless asset shares) held by the intermediary. Given w_t, the model imposes a constraint that an intermediary (or the owner-manager of the intermediary) can raise outside funds of at most $\mathcal{M}w_t$, where $\mathcal{M}$ is a constant. Here, w_t may be thought of as the capital of an intermediary and determines the financing conditions for an intermediary.

The lenders/outside-investors in the economy are modeled as households who can directly invest in the riskless bond, but can only gain exposure to the risky asset allocating their wealth to an intermediary. The investment in the intermediary is subject to the capital constraint described above. The portfolio decision within the intermediary is made by the owner-manager rather than the household-investor. Households also receive labor income, calibrated to reflect the typical profile for a U.S. household.

The mechanics of this model are similar to the static model described in the previous section. Past investment decisions of the agents in the economy along with the realizations of dividend shocks determine the w_t of a representative intermediary. At date t, w_t determines the strength of the balance sheet mechanism. The central feature of the model is that there is a cutoff level for w_t so that that the balance sheet mechanism is only operational for values of w_t below this cutoff. Figure 4 graphs the instantaneous risk premium on the risky asset as a function of w_t, illustrating the balance sheet mechanism.

When w_t is lower, the balance sheet constraint is tighter and the risk premium is higher. Intuitively, the model works as follows. As the risky asset is only held through the intermediaries, the amount of economy-wide capital that is devoted to bearing the risk in this asset is the sum of w_t plus the funds that households contribute to the intermediary sector. When w_t is low, households' are willing to contribute at most $\mathcal{M}w_t$. In particular, in the states where w_t is low, the constraint causes households' risk-bearing capital to be sidelined. The total risk-bearing capital is limited to $w_t(1 + \mathcal{M})$. In equilibrium, since this limited capital bears all of the dividend and price risk of the risky asset, the risk premium has to rise to clear the market. The effect is monotonic in w_t.

Because the model is multi-period, it embeds dynamic linkages that are absent in the static model described earlier. In particular, a tighter constraint in the future affects the asset price in the future. But since today's asset price depends in part on the future asset price, today's price is a function of the future

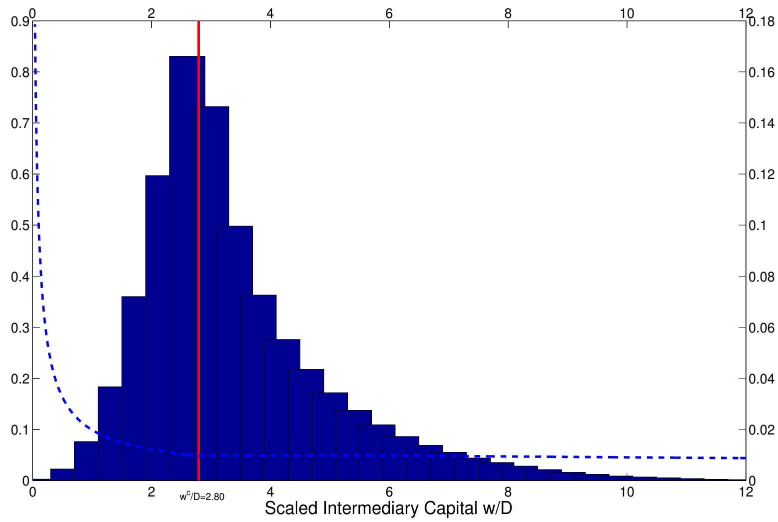

Figure 4: Risk Premium and Steady State Distribution

The dashed line graphs the risk premium (y-axis) as a function of w_t scaled by the dividend, D_t. The red vertical line indicates the region in the state space, to the left of which the constraint binds. Also pictured is the steady state distribution of the state variable in order to indicate the relevant range of variation of the risk premium.

balance sheet constraint. Of course the balance sheet constraint today and in the future are themselves dependent on asset prices, so that the feedback mechanism occurs simultaneously at multiple dates.

This type of dynamic can shed light on how an economy recovers from a crisis. Table 1 presents results from simulating the model begining with an initial "crisis" condition. The initial condition is the state with risk premium of 20% ("Transit from"). Simulating the model from that initial condition, the table reports the first passage time that the state hits the risk premium corresponding to that in the "Transit to" column. The times in the table are reported in years.

There are two forces driving the crisis recovery pattern in Table 1. First, in the midst of the crisis, when the constraints are the tightest, the asset values are low. The intermediary sector earns a high risk premium by holding the distressed assets. This causes w_t to on average rise, with the effect obviously stronger when the risk premium is higher. As a result, the recovery speeds are shorter (third column) when risk premia are higher. Second, as w_t rises, there is reintermediation as households match the rise in w_t by contributing

Table 1: Crisis Recovery

This table presents transition time data from simulating the calibrated model in He and Krishnamurthy (2008b). Fix a state corresponding to an instantaneous risk premium of 20% ("Transit from"). Simulating the model from that initial condition, the table reports the first passage time that the state hits the risk premium corresponding to that in the "Transit to" column. Time is reported in years. The column "Increment time" reports the time between incremental "Transit to" rows.

	Transit from 20	Increment time
Transit to		
15	0.23	0.23
12.5	0.46	0.23
10	1.02	0.56
7.5	2.62	1.6
6	5.91	3.29
5	12.88	7.1

$\mathcal{M}$ times the increase in w_t. That is, the balance sheet constraints loosen as w_t rises, further quickening the recovery.

The magnitudes and patterns of crisis recovery illustrated in Table 1 are similar to crisis recovery patterns witnessed in practice, as discussed in He and Krishnamurthy. For example, in the 1998 hedge fund crisis, the crisis reaches a peak in October of 1998. From this peak, the risk premia in the affected markets take about 9 months to fall halfway back to their pre-crisis levels. This magnitude of half-life is similar to empirical studies of other crisis episodes.

5 Amplification through Uncertainty

In early 2007, banks were well capitalized and flush with cash. Similar statements of health could be made about most of the key pieces of the financial sector. In terms of the model just discussed, L was high and d_0 was low. From the vantage of the balance sheet liquidation mechanism, these benign initial conditions make it hard to understand the extent of the recent crisis.

I argue in this section that an important amplification mechanism in the recent crisis has to do with lack of knowledge and uncertainty. Investors had rapidly adopted a financial innovation – the credit market structures – with which they had a limited history. When AAA subprime tranches suffered losses, investors realized that they had not understood the securitized credit structures they had purchased. Investors were not surprised that high-risk homeowners defaulted on some loans; rather, they were surprised that such defaults had a material effect on the values of the most senior of the tranches backed by pools of subprime mortgages. Moreover, given that a myriad other credit products - not just mortgage – had been structured in much the same way as subprime investments, investors' model-uncertainty was across the entire credit market.

Thus, the small cash-flow shock of subprime defaults resulted in a large shock to investors' uncertainty. Moreover there was contagion across the entire credit market due to investors' model uncertainty. The response of investors to their uncertainty was to disengage. Investors went back to the drawing board to formulate new models. In the meantime, given that they did not have a clear understanding of events, they took decisions to protect themselves against worst-case scenarios on the risks that they did not understand. The result of all of this disengagement was a loss of liquidity, with many attendant effects.

Investors' dramatic disengagement and emphasis on protecting against a worst-case event is hard to capture within a standard model of decision making. In the standard model, agents consider all possible models — for example, models in which AAA tranches are not risky and those in which they are risky — placing probabilities on each of the possible models, and then making decisions that average over the models in a Bayesian fashion. The shock (i.e. AAA defaults) leads agents to adjust their model-probabilities, increasing their weight on the model in which AAA tranches are risky. Of course such a reassessment leads to lower prices on the AAA tranches. But, it does not lead to disengagement nor the worst-case decision rules that have been witnessed recently.[13]

The main difficult with the standard model in capturing these events is that under the Savage axioms for decision-making, model uncertainty and risk regarding cash-flows are treated the same way. Indeed, leaving aside the subprime example that I have given, there is a long tradition in Economics dating back to Knight (1921) that recognizes that risk and uncertainty provoke different behavioral responses. The point is most clearly made in the Ellsberg (1961) paradox. Giving people choices between gambles where some gambles had clearly specified odds, while others did not, Ellsberg found that people consistently avoided the gambles with unknown odds. In fact, Ellsberg found that one could combine the known and unknown gambles in ways that showed that people violated the Savage axioms.

Beginning with Gilboa and Schmeidler (1988), there have been a number of papers aimed at developing

[13]Routledge and Zin (2005) argue similarly that the trading-halts and disengagement we observe during financial crises are an important reason to think that these events are about Knightian uncertainty. Routledge and Zin develop a model in which uncertainty leads to a trading hald and widening of bid-ask spreads.

a theory of decision-making that distinguishes between risk and uncertainty and is consistent with the behavior noted by Ellsberg. Consider a decision problem where a state $\omega \in \Omega$ will be realized tomorrow. The probability distribution over the states is denoted by π. An agent makes a decision $d \in D$ today that results in utility $u(c(\omega, d))$ in state ω. The standard Expected Utility representation of this decision problem is that the agent solves,

$$\max_{d \in D} \; E_\pi[u(c(\omega, d))].$$

Suppose however that agents are uncertain over the probability distribution π. In particular, suppose that this uncertainty is that agents only know that $\pi \in \Pi$. Then, Gilboa and Schmeidler's Maximin Expected Utility representation of Knightian uncertainty aversion is,

$$\max_{d \in D} \; \min_{\pi \in \Pi} \; E_\pi[u(c(\omega, d))].$$

The "min" operator is the key here: agents use a worst-case for the uncertain probabilities π when making their decision.

In the subprime example, one may consider that agents had a model $\pi \in \Pi_0$ in mind at the beginning of 2007. The default events led them to become uncertain, so that they considered a larger class of models Π_1 (e.g., $\Pi_0 \subset \Pi_1$). Although the Gilboa and Schmeidler (1989) theory is not dynamic and therefore does not explain how agents' priors are updated, there has been subsequent work in the decision theory literature that does (see Hanany and Klibanoff, 2007, for discussion of this research).

This section presents extensions of the model to incorporate Knightian uncertainty, in ways guided by the recent credit crisis, and shows that the model can well capture recent events. I then turn to a discussion of some other historical crises in light of the uncertainty model, arguing that the subprime case illustrates a more general pattern present in other financial crises. Finally, I discuss policy in the uncertainty model and compare it to policy in the balance sheet liquidation model.

5.1 Counterparty Risk

I return to the benchmark model described in Section 3 where a class of atomistic investors purchase an asset at date 0, considering that they may receive a liquidity shock at date 1 and that a financial intermediary with balance sheet liquidity of L will be the liquidity provider in such an event.

In the subprime context, we may imagine that the liquidity shock is a loss in the agent's subprime investments that necessitates rebalancing the agent's portfolio. The shock probabilities for each class of investors, A and B, is ϕ. I have thus far treated the shocks as independent, but suppose that the shocks have correlation of ρ, possibly different than zero. Moreover, suppose that agents are *uncertain* over the value of ρ, knowing only that $\rho \in [-1, +1]$.

The problem of an investor at date 0 is to decide how much to pay for the asset, given the agent's probability of liquidation and the price-support provided by the intermediary. Uncertainty enters into this decision because the agent knows that the intermediary has limited balance sheet liquidity, and if shocks to both A and B occur at the same time, the intermediary will not be able to provide full price support.

This way of introducing uncertainty captures "counterparty risk." Will the intermediary be able to deliver on its liquidity when needed, or will other shocks deplete the intermediary's liquidity so that it (partially) defaults? In the current context, the concern may be, how widespread are the subprime losses and where are these losses buried? Note also that even if A can accurately assess his own shock probability to be ϕ, the modeling is that he may be uncertain about the shock distribution for B. This is another way of thinking about counterparty risk – i.e., A's concern is will risks to other agents end up affecting him?

It is obvious that in the simple setup the "worst-case" for the agents is if $\rho = +1$. The maximin decision rule is to purchase the asset assuming that the counterparty risk is the highest. The date 0 price is then

$$P_0 = 1 - (1 - L/2)\phi. \tag{10}$$

Comparing this expression to that in equation (2) where $P_0 = 1 - (1 - L/2)\phi^2$, we see that uncertainty magnifies the importance of the liquidation event from order ϕ^2 to order ϕ.

5.2 Wasted Balance Sheet Liquidity

Consider the following implementation of the price support equilibrium. Suppose that in purchasing the asset at date 0, the investors require that the intermediary set up two divisions, each with $L/2$ units of liquidity. Division A promises liquidity to investor-class A, and likewise for division B. This implementation disentangles the A shock from the B shock, removing counterparty risk. Effectively each agent disengages from the risks that he does not understand and guarantees a known amount of liquidity for himself. Each investor then prices the asset at date 0 requiring only the knowledge of their own shock-probability. Hence, $P_0 = 1 - (1 - L/2)\phi$.

However the over-collateralization comes at a cost. Part of the intermediary's balance sheet liquidity of L is wasted in the equilibrium because there are states at date 1, the ones in which only one of A or B receive the liquidation shock, in which the liquidity of the non-shock division goes unused. Investor's uncertainty causes the effective balance sheet liquidity of the intermediary to fall.

Financial crises are not typically about an aggregate shortage of resources, but rather about the distribution of these resources. Resources that could be valuably deployed stay on the sidelines. For instance, in the recent credit market events, regulators have been concerned that many banks have been hoarding their liquidity, causing the money market to be illiquid. Uncertainty-induced liquidity waste is one way to model these outcomes.

5.3 Individual Exposure to Aggregate Risk

In the recent crisis, banks have been unsure of the extent of their own exposure to subprime and related credit risks. The problem stems from the complexity of these instruments. Different parts of banks have different risk exposures. Taking stock of all of this to provide an overall risk picture for a bank has taken time and proven difficult. The problem has been compounded because in many cases the markets for the relevant assets have become illiquid, making it difficult to measure market values.

On the other hand, it has been easier to estimate the aggregate losses stemming from subprime defaults. That is, beginning with the subprime borrowers, one can estimate default probabilities and recovery rates on default, and provide an upper bound on the subprime losses. Brunnermeier (2008) reports an over-estimate here of $500 bn. The uncertainty over the credit market structures is what has made it difficult to measure individual exposures to the aggregate risk.

Consider the following variation of the model. Suppose that at date 1 there are only two states, shock or no-shock (with probabilities ϕ and $1-\phi$). In the shock state, only one of either agents A or B receive a liquidity shock. Agents are uncertain over which agent will receive the shock. Denote ϕ_A and ϕ_B as the shock probabilities for A and B. Since only one of A or B will receive the shock,

$$\phi_A + \phi_B = \phi$$

However, suppose that A is uncertain over the value of ϕ_A. A only knows that $\phi_A \in [\phi/2 - K, \phi/2 + K]$ ($K \leq \phi/2$). The treatment of B is symmetric.[14]

[14] In this model, one solution to the uncertainty problem is for A and B to cross-insure each other. Both buy insurance against their own shock state and sell insurance against the others' shock state. I am assuming that markets are incomplete so that this trade is not possible. Caballero and Krishnamurthy (2008) develop a related model based on uncertainty over idiosyncratic

It is easy to see how the uncertainty affects the date 0 decision. The worst-case for the agent is if their shock probability is $\phi/2 + K$. Then the agents price the asset so that,

$$P_0 = 1 - (1 - L/2)(\phi/2 + K).$$

Uncertainty reduces the asset price, increasing the importance of the liquidation state. (I have normalized things by assuming that the intermediary has $L/2$ units of liquidity, since liquidity needs have been halved relative to the previous extension.)

5.4 History and Financial Innovation

The preceding extensions illustrate how a rise in Knightian uncertainty can lead to a lack of liquidity. My modeling choices in the extensions are guided by the subprime crisis. In this section, I turn to other historical crisis episodes, interpreting these events in light of the uncertainty model.

The uncertainty model is most suited to environments where market participants have had a limited experience in dealing with a particular asset. These circumstances provide fertile ground for "unusual" events – such as the losses on AAA rated subprime tranches. That is, it is likely that something occurs that is at odds with market participants' models of the world. Knightian uncertainty and market illiquidity follow naturally.

Consider in particular the following narrative of a financial innovation. A successful financial innovation is a product that meets a market demand and is therefore taken up widely. In the subprime case, securitized credit products have come to proliferate the market in the short space of five years. Thus, by its very nature, a successful financial innovation provides market participants with only a short history and there will be outcomes that people do not expect. The subprime case clearly fits this narrative, but consider some other historical episodes.

In 1970, the Penn Central Railroad defaulted on \$82m of prime-rated commercial paper. The commercial paper market at the time was not as mature as it is today. It had developed rapidly through the 1960s to meet growing corporate borrowing needs. However, ratings were not fine tuned and back-up liquidity facilities, which are standard practice today, did not exist. When the default occurred, it spooked money-market

exposures to aggregate shocks where markets are complete, yet the equilibrium is qualitatively similar to the one described here. The main modeling difference is that in Caballero and Krishnamurthy, the model is completely symmetric – there is no distinction between "A" and "B" types.

investors. These investors went back to the drawing board to re-evaluate their credit models and ratings guidelines. The result was disengagement. Investors stopped buying commercial paper completely. Over time, and with the Fed intervening by encouraging banks to buy commercial paper, the market normalized.

Contrast this event with the 1997 Mercury Finance - another commercial paper borrower – default on \$500m of paper. The default was much larger in real terms than Penn Central and was similarly a surprise to the market. In contrast to the Penn Central case, there were no effects on the commercial paper market. The reason is that it quickly became clear that the default was a case of fraudulent accounting in Mercury Finance. The uncertainty element that had been important in 1970 was not present.

Another example to illustrate these points is the stock market crash of October 19, 1987. The new innovation in this episode was portfolio insurance strategies – that is, the synthetic replication of put options. This was a strategy that had become increasingly common among investors in this period. However, in 1987 it was unclear how widespread these strategies were and how financial markets would equilibrate in the presence of portfolio insurance strategies. The speed of the market decline on October 19 took everyone by surprise. Market makers widened their bid-ask spreads and other key market players pulled out of the market completely. The result was a lack of liquidity. Many observers point to the option-replication drive sales into an illiquid market as being an important factor in the market crash. Today, these types of replications strategies are common and well understood by all market participants.

My last example is the hedge fund crisis of the fall of 1998. In this scenario, hedge funds were still a relatively new and opaque financial vehicle. Assets under hedge fund management had grown from around \$10 bn in 1991 to \$80 bn in 1997 (still far less than the trillions under management today). In the fall of 1998, even sophisticated market participants such as Long Term Capital Management were taken by surprise by the unprecedented comovement of Russian government bond spreads, Brazilian spreads, and U.S. Treasury bond spreads. The standard risk management models that hedge funds used were no longer applicable (Scholes, 2000). The result was that financial market participants searched for new models and made decisions based on worst-case scenarios. We now know that hedge funds had similar strategies and had filled up a similar asset space, and that this was the source of the correlations. Indeed risk management strategies post-1998 explicitly account for high-correlation illiquidity events. But at the time neither hedge funds nor their creditor banks understood this point. The result of this uncertainty was illiquidity and crisis.[15]

[15] While in 1998 hedge funds were still a novel financial vehicle, the large reported losses of the Amaranth hedge fund in 2006

The preceding discussion covers three of the major financial crises experienced in the U.S. over the last 50 years.[16] Each of these episodes is associated with a financial innovation, and occurred at a time when market participants had only a limited history within which to understand financial developments, suggesting that Knightian uncertainty is an important factor in many financial crises.

5.5 Policy

As in the liquidation model, the lender of last resort policy is valuable in the uncertainty model. Consider that if the central bank commits to inject one unit of resources into the intermediary (increasing L) in the two-shock state, this commitment can have a large ex-ante effect. In the counterparty risk example I have developed, the increase in P_0 due to this commitment is of order ϕ, while in the baseline model the commitment increases P_0 with order ϕ^2. In the correlation risk example, the increase in P_0 is proportional to $\phi/2+K$, while in the baseline case, the increase is proportional to $\phi/2$. In effect, the central bank delivers resources to the market in the states that agents are most anxious about. Caballero and Krishnamurthy (2008) develop this point more fully, discussing some of the welfare issues that arise when agents' preference do not satisfy the Savage axioms.

While crisis policy is similar across the uncertainty and liquidation model, my conjecture is that there is less scope for ex-ante policy in the model. In the liquidation model, the date 0 policy is to reduce leverage. As I have noted, in a more sophisticated model this policy will be about incentivizing agents to improve risk management. Agents must increase the liquidity of their balance sheets only in the states of the world where a liquidity event will occur. This leads to the following question. Does the central bank know which states are the uncertainty-crisis states? Consider that the central bank is in the same (or worse) position as the private sector in forecasting how a crisis on a new financial innovation will unfold. Of course the central bank can require a blunt policy such as carry more liquidity/reduce leverage/reduce asset positions into *all* states of the world. But such a policy may be prohibitively costly since it distorts private sector actions in non-crisis states, and these states may be the more likely ones. The conjecture needs to be investigated more fully.

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barely caused a ripple in financial markets.

[16] See Calomiris (1994) on the Penn Central default, Melamed (1998) on the 1987 market crash, Scholes (2000) on the events of 1998, and Stewart (2002) or McAndrews and Potter (2002) on 9/11.

5.6 Interactions and Crisis Recovery

I have discussed the two amplification mechanisms – through prices and through uncertainty – separately for pedagogical purposes. But there may be interesting ways in which they interact. What follows are my observations on interactions that may have played a role in recent events. I am investigating these issues more formally in current research.

In the recent subprime crisis, I have argued that the initial market dynamic was driven by uncertainty. As investors grappled with the complexities of credit market instruments, their behavioral response caused asset prices to fall. With banks marking their assets to market, two competing forces emerged. On the one hand, the realization of losses reduced the extent of uncertainty in the marketplace. On the other hand, with banks writing down losses, their balance sheets suffered and the effects highlighted by the balance-sheet/asset-price feedback began to dominate the markets.[17]

Routledge and Zin (2005) develop a micro-structure model of asset trade in which Knightian uncertainty leads to a trading halt and widening of bid-ask spreads. Their model suggests that uncertainty inhibits the process of price discovery, rendering market prices uninformative about fundamental value. This too may be interestingly related to balance sheet effects. Accounting rules require that banks mark their books to market prices. However in an environment of uncertainty, where market prices are suspect, such mark-to-market accounting becomes difficult. This effect has also been apparent in recent events.

These observations can shed light on how a market recovers from an uncertainty-driven crisis. It is clear from the model I have outlined as well as the historical examples I have given that the uncertainty crisis is only resolved over time as investors understand where they went wrong and formulate new models of the world; in short, as the uncertainty is resolved. Part of this process involves information revelation. What mistakes have investors made? Which investors have large exposures to the relevant assets, and how big are their losses? In an environment where the price discovery mechanism is impaired, information revealed from mark-to-market accounting becomes hard to interpret. Further, in an environment where balance sheets are weak, a financial institution will be reluctant to realize losses. These forces tend to perpetuate an uncertain environment, and may reduce recovery times relative to those suggested in Section 4.4.

[17] We can also imagine the interaction going in the other direction. Consider that in 1998, the Russian default triggered a liquidation event in hedge-fund dominated markets. The resulting unexpected correlation among market prices was a surprise to most market participants and triggered an uncertainty dynamic.

6 Conclusion

This paper has described two amplifications mechanisms that operate during liquidity crises and studied the scope for ex-ante and ex-post central bank policies under each mechanism. There are many aspects of the mechanisms I have discussed that require further research. Let me conclude by mentioning two particularly fruitful avenues of research.

First, given the primacy of intermediaries' balance sheet liquidity in crisis dynamics, it is important to understand the microeconomic foundations of balance sheet liquidity. This is a question for corporate finance. The liability side of a financial institution plays an important part in balance sheet liquidity. Short-term debt creates less liquidity, while equity creates more liquidity. How can we explain the financing patterns we observe in practice? Why have some institutions chosen to finance mortgage loans by borrowing in short-term debt market, creating low balance sheet liquidity and increasing the likelihood of a liquidity crisis? Why, as a crisis unfolds, are institutions forced to rely more on short-term debt than equity as a funding source? Moreover, traditional banking oversight concerns the regulation of balance sheet liquidity. As these regulations are extended to the non-bank financial sector, it is important to understand in what dimensions such regulations may be appropriately extended. Without a firm understanding of the corporate financing foundations of balance sheet liquidity, these issues cannot be satisfactorily addressed.

Second, taking a reduced form approach to corporate finance, as I have done in this paper, it is still interesting to explore the quantitative dimensions of these amplification mechanisms. That is, it is important to build calibrated models of these amplification mechanisms in which to explore the quantitative impact of policy. While I have noted some work on this topic in this paper, more still remains to be done.

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