

Speedtrading: Index Options Trading Model

Finance 925 – Analytical Finance Practicum

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June 5, 2005

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INTRODUCTION	3
TRADING STYLE	3
MODERN OPTIONS PRICING	4
<i>Calibration</i>	4
Curve Fitting	6
Implied Volatility Smile	7
<i>Assumptions and Persistent Smiles</i>	8
<i>Symmetric Smile</i>	8
<i>Smirk</i>	9
SPEED TRADING	10
THE MODEL	11
HISTORY	11
OVERVIEW	11
<i>The Parameters</i>	12
Snapshot parameters	13
Flow Parameters	15
Time Parameters	15
UNDERSTANDING THE MODEL	17
IMPLIED TRINOMIAL TREES--THEORY	18
DERMAN KANI IMPLIED TRINOMIAL TREE – MECHANICS	21
TESTING THE MODEL	23
DELTA AND GAMMA	23
<i>Understanding the Cause</i>	25
<i>A potential solution</i>	27
<i>Revised Finite Difference Calculation</i>	28
INPUT SMILE VS. OUTPUT SMILE	29
DIVERGENCE OF INPUT AND OUTPUT SMILE	31
ANALYSIS OF INPUT AND OUTPUT OPTION PRICE	34
SPEEDING CALIBRATION	39
EVALUATION OF THE MODEL	42
CURRENT USE	42
OTHER USES	42
<i>Fixed Income</i>	43
<i>Exotics</i>	43
FUTURE ANALYSIS IDEAS	44
Autofit	44
Hedging Performance	44
Parameter Time-Series Analysis	46
Optimal Tree Redrawing	46
Structured Equity Derivatives	46
BIBLIOGRAPHY	47

Introduction

Speedtrading is a proprietary trading firm based in Chicago, IL. They trade options and futures and have an institutional retail business. The firm's people include traders, sales people, developers, and support staff. Though particularly strong in market intuition and trading tool development, Speed lacks a quantitative analyst.

The focus of this paper is Speedtrading's process of and model for trading options on the Korean KOSPI index futures. We break our project into the following sections: 1) trading style and business needs, 2) the model and its role, 3) testing the model, 4) performance enhancement, 5) evaluation, and 6) suggested future work.

Trading Style

Market prices for the KOSPI index are frequently driven into inconsistent prices by investors who are at the same time unsophisticated and seeking excitement. Korea has a relatively large proportion of gamblers and these gamblers play the market based on hunches rather than particular pricing models. Sophisticated investors who understand modern finance theory can profit from bringing the market back into line.

Speedtrading's style can best be characterized as volatility arbitrage. They trade delta neutral strategies that allow them to profit from these temporary inconsistent market prices rather than directional moves.

In the following section, we explain volatility arbitrage in full detail, beginning with some background on options pricing.

Modern Options Pricing

Calibration

Nobody thinks the Black-Scholes formula prices all options correctly in every case. There are too many required assumptions that are violated in practice for it to hold. Examples of violated assumptions include lognormal returns, constant volatility dividends, and interest rates, frictionless trading, and continuous prices without the possibility of jumps. Furthermore, no single model has been created that perfectly accounts for all of the above violated assumptions in a manner that has been proven to be perfect. Finally, even if a perfect model did exist, different opinions about the parameters and different marginal investors could drive market prices out of line from time to time.

Therefore, any usable model will take into account information from the market but also allow input from knowledgeable market participants. In short, any

reasonable model must allow for calibration. For our purposes, we classify calibration into three forms: price, implied volatility, and fundamental.

The simplest form of calibration is to use price as an input. Most models could be modified to take price data as an input and return a full set of consistent market prices and Greeks. Unfortunately, with options constantly trading at different strikes and maturities, it's easy to see that this process would be cumbersome.

A similar but less cumbersome calibration parameter is *implied volatility*. Black Scholes Implied Volatility is the volatility input which--holding constant the futures price, strike, interest rate, and dividend amounts-- would yield the market price when entered into the Black Scholes formula. Inputting a different implied volatility for every traded strike serves the same purpose as inputting price, but it is far more useful for two reasons:

The first reason is implied volatility as an input parameter is stable. Since the Black Scholes formula automatically captures the passage of time, dividends, and moves of the underlying instrument, Black Scholes implied volatility remains constant as these factors change.

Second, the Black Scholes formula assumptions require actual volatility to be the same for all strikes (and equal to the volatility of the underlying instrument). Therefore, any difference in implied volatility across different strikes on the same option can be directly tied to a deviation from the Black Scholes assumptions. In fact, several important deductions can be made from calibrated implied volatility.

Another important method of calibration involves models that take into account variations on basic Black-Scholes. We could assume stochastic volatility, correlation between volatility and the level of the underlying, non-normal distributions of the underlying (including the possibility of jumps in the underlying price), investor preferences, and stochastic interest rates and vary those assumptions until we achieve our desired market prices. This method of calibration allows us to think critically about the deviations we see in the market and potentially to decide whether the deviations make sense, giving the trader the ability to consider market prices on a more fundamental level.

Curve Fitting

Regardless of the chosen calibration method, we expect to see similar amounts of required calibration across different strikes with the same maturity on a particular underlying. As a result, we expect to see a smooth curve if we plot our calibrated price or implied volatility versus strike. In other words, if ever a

smooth curve cannot be fit to the existing market, then a market inconsistency and potential trading opportunity exists.

Implied Volatility Smile

As previously stated, if the assumptions associated with the Black Scholes formula were correct, a plot of market implied volatility vs. strike for a group of options with the same expiration would be a straight line at the market prediction for the future volatility of the underlying.

Any deviation from the above must mean 1) one or more invalid assumptions, or 2) a market mispricing. Typically, we assume a model error would result in a persistent and smooth volatility smile while a temporary market mispricing would be temporary and rough.

There is a possibility that the market would form an incorrect consensus about a fundamental assumption thereby causing a persistent, smooth volatility-strike plot that nonetheless represented a trading opportunity. To accommodate such trading opportunities, a good model will need to allow calibration beyond smoothing-out market prices. In the long run, we assume that these trading opportunities will be limited due to powerful market forces--the market rewards with profits market participants who make the right assumptions and get the prices right and drives the rest of the market into bankruptcy.

Assumptions and Persistent Smiles

The smooth plot of market implied volatility versus strike is known as the volatility smile. Different markets exhibit different volatility smiles that have been tied to different assumptions about the underlying instrument. In this section we look at the causes of two different smiles.

Symmetric Smile

We begin with the symmetric smile. The symmetric smile appears just as it sounds—low implied volatility for the at the money option increasing with distance from the at-the-money strike.

Two important and related deviations from the Black-Scholes assumptions lead to this type of smile: kurtosis and jumps. Kurtosis refers to the size of the tails in a distribution. In general, traded options show “fat tailed” distributions indicating a higher than normal frequency of large returns and near-zero returns with a smaller than normal incidence of small to medium returns. Jumps are similar in that they also tend to entail larger than normal returns. However, discontinuous jumps cannot be hedged like smooth trades and therefore may induce a risk premium into options prices.

Starting from the assumption of normality, both jumps and kurtosis increase the conditional expected value of a low or high struck option conditional on that

option being in the money, so both would make those options worth more. Jumps may also require a risk-premium, making investors further unwilling to sell an option without further incentive to do so in the form of a higher price.

With kurtosis and symmetric jumps (up and down), we expect to see a symmetric smile reflecting a non-normal distribution of returns.

Smirk

Prior to the stock market crash in 1987, the symmetric smile was common. After the crash, we began to see what is familiarly known as a smirk, meaning low-striker options tend to trade at higher implied volatilities than their high-striker counterparts. The result is a tilted smile.

The primary cause of the smile turning into a smirk is negative correlation between the volatility and direction of moves of the underlying. A simple way of thinking about this correlation is: investors panic during market drops and become calm during rallies. If volatility increases as the underlying moves down, then a large move is likely to build on itself and result in even larger downside moves. As a result, the distribution of returns will be skewed to the downside and the value of options with downside strikes will be higher.

It may be helpful to note here that most of the smile/smirk causes we've mentioned here can be distilled down to and fully explained by the distribution of future returns. Shimko (1993) shows that in the absence of risk premia the relationship is direct. It is precisely this relationship that Speedtrading's model is meant to exploit. We will discuss this in detail shortly.

Speed Trading

With the above in place, Speedtrading's strategy becomes straightforward. Use market savvy to define the correct smile, and then sell everything above the smile, buy everything below, and hedge delta-neutral with futures. In expectation, one of two things will occur: 1) the market will fall back into line at which point the option can be sold for a profit, or 2) the profit and loss from delta hedging will be greater than the profit and loss from holding the option to maturity.

To see the second point, recall an option can be replicated by holding Delta shares of the underlying and B shares of borrowing and lending. If a long expensive option is hedged delta neutral to maturity, it is like being long the option and synthetically short the option at the same time. The P&L on the hedging will be equal to the value of the long option plus the expected value of the mispricing. See *Options Volatility and Pricing* by Sheldon Natenberg for a trader's look at this style of trading.

The Model

History

Speed's model has followed our project sponsors over the years through proprietary trading firms, investment banks, and was then brought to Spreadtrading. Through the years it has been modified and improved, but it was poorly documented. Though our sponsors were comfortable that the model did what it was supposed to do and had previously allowed them to trade profitably, they did not know the mechanics behind the model and therefore could not maintain the model or assess whether the model could be useful in other trading environments. Part of our goal was simply to make sense of the model and document it fully.

Overview

The goal of the model is to allow a trader to use his intuition to find trading opportunities as quickly as possible. To do that, the trader needs to input (as quickly as possible) his view of the market at any given time and wants an output of consistent market prices and Greeks. The parameters of the model use terms related to the volatility smile and indeed we do calibrate using implied volatility, but it is important to note that we could just as easily calibrate to match prices – it is the same thing.

To maximize fit and flexibility, the model is calibrated using one set of parameters for each different expirations (for example:1 month, 2 months, 3 months, 6 months, 1 year, and 2 years). With each set of parameters, we tell the model what the smile looks like today and, for reasons that will soon become clear, what the smile would look like for options that expired tomorrow, two days from now, etc. until the very last option expiration. Our goal is to give the model a clear picture of the volatility smile for options that expire at any time between now and the foreseeable future. There is clearly significant overlap here and room for flexibility. When calibrating the options that expire in one month, we may say very different things about tomorrow then we would when calibrating the model to price options expiring in two months. This flexibility must be used with caution.

The Parameters

In order to capture the smile now as well as capture changes in the smile we need three types of parameters: 1) snapshot parameters that capture the way things look today and 2) flow parameters that tell the smile how to change in response to time and moves in the underlying instruments, 3) parameters that tell the model about the impact of time on the skew.

Those parameters are explained below and illustrated for a sample market.

Snapshot parameters

Spot Price: The model begins bases all calculations on the volatility of the front month at the money strike. In order to know what that strike is, the model needs to know where the stock is trading.

Init Vol: This is the implied volatility of the front month at the money option. All implied volatilities are based on this initial volatility.

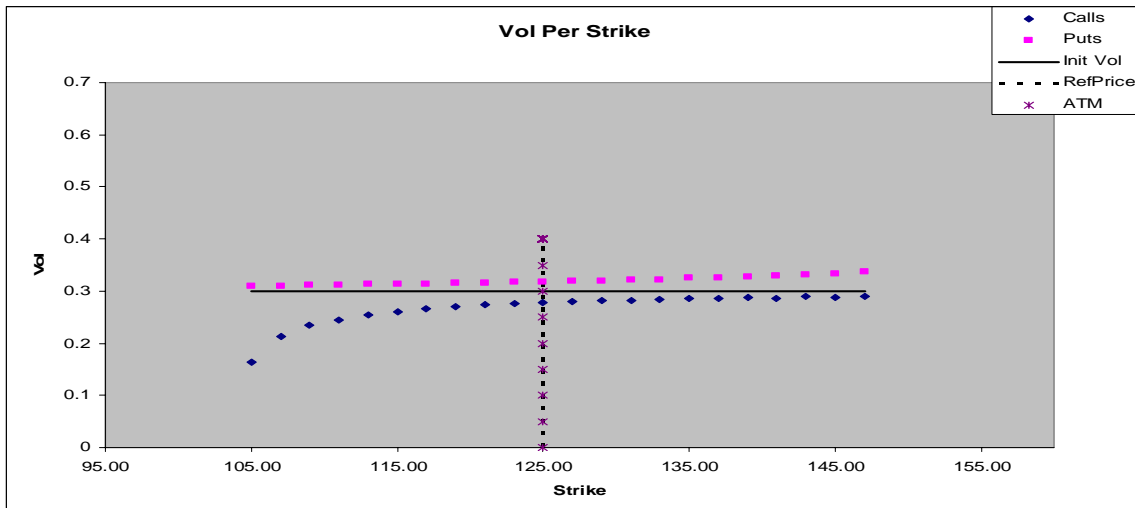
Curve: Curve generates the familiar smile shape. The curve parameter influences the second derivative of the strike-volatility function. More curve means more smile. Curve = 0 turns the smile into a straight line with slope equal to skew. Positive curve increases implied volatility of every strike it increases volatility of high and low strike options by more than at-the-money options.

Skew: Skew captures the fact that deep out of the money puts are generally more expensive than deep out of the money calls. A negative skew turns the smile into a smirk with low strike implied volatilities higher than high strike volatilities.

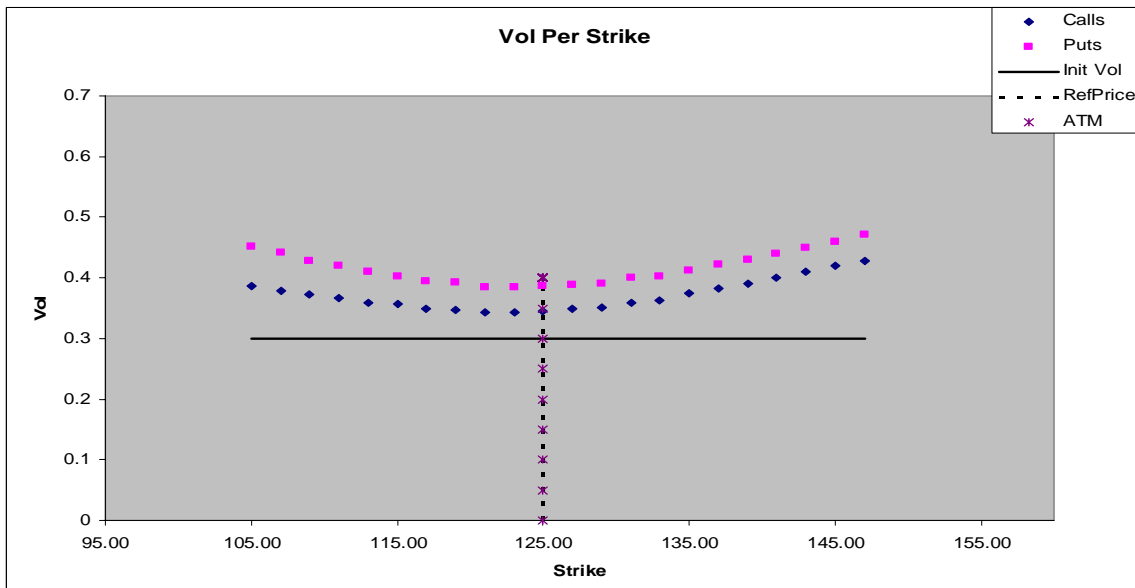
For negative skew, the model takes the difference between the current strike and the at the money strike and uses that to increase the volatility at low strikes and decrease the volatility at high strikes.

If skew is zero, you get a symmetrical smile. If skew is zero AND curve is zero, you get the same implied volatility at every strike and the model reduces to a simple trinomial model.

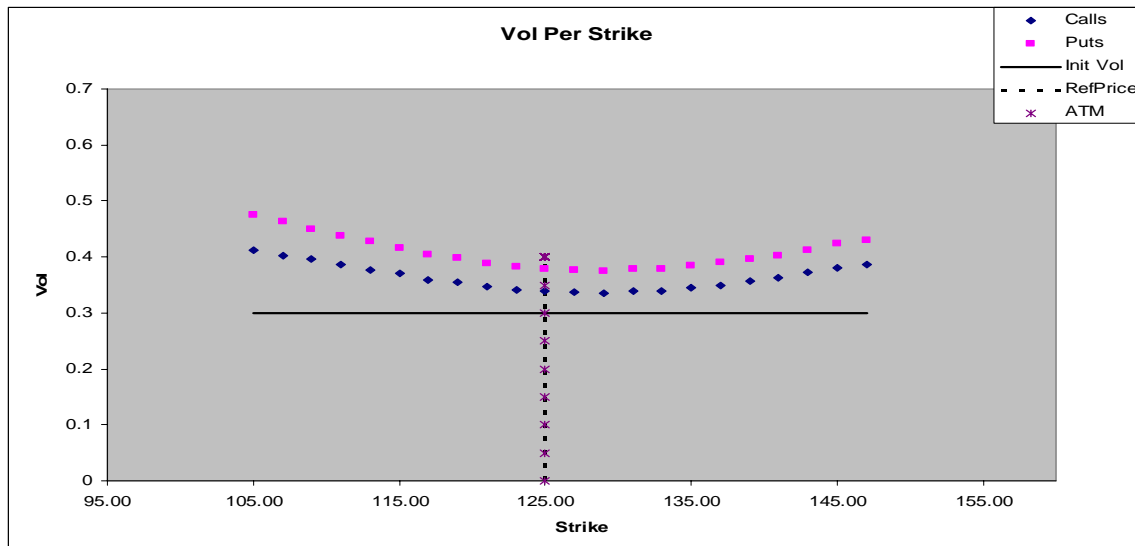
Base model: Skew, Curve = 0. Spot = Ref Price



Basic Smile: Curve = 1, Skew = 0.



Smirk (tilted curve): Curve = 1, Skew = -50



Flow Parameters

Reference Price: Reference price is the spot price when the model was fit. It is how the model knows the market has moved and knows to adjust the entire smile by changing the at the money volatility.

Slope: Slope is usually negative to reflect increased volatility during market downturns and vice versa. If slope is negative, the entire smile will increase when the market goes down and decrease when the market goes up.

Time Parameters

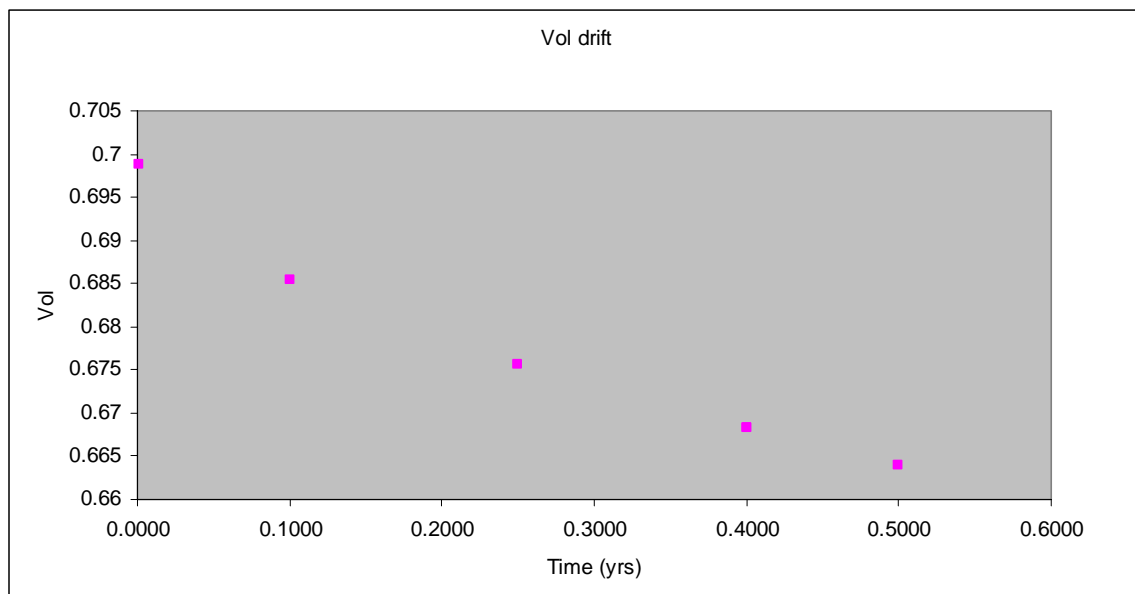
Two parameters change volatility with time. Informally, they are some future expected volatility for the underlying, and the length of time before that volatility is reached. Under this method the current month can have one

volatility and future months can have different volatilities. The parameters are as follows:

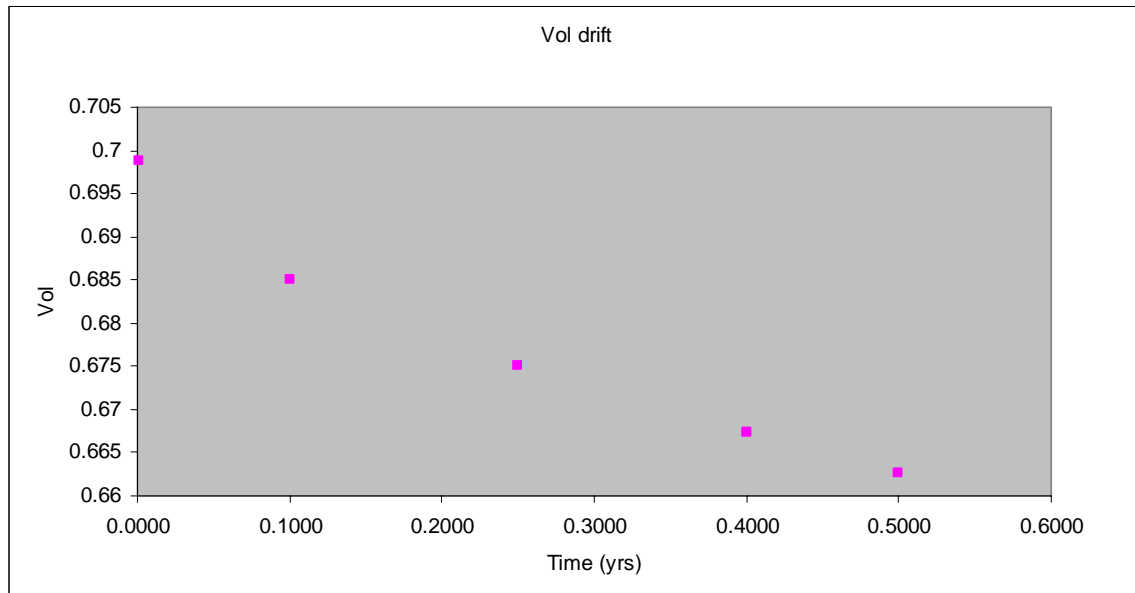
VolInf: This volatility is supposed to be based on the at-the-money volatility for the very back month. If VolInf is higher than the current at-the-money vol, then at-the-money volatility will increase between the front month and the back month. If VolInf is lower, the at-the-money volatility will decrease between the front month and back month.

Tau/Scalar: This parameter determines the speed of the move from the current volatility to the infinity volatility. Tau can be thought of as the number of days before the expected future volatility of the underlying equals the infinity volatility. If Tau is high, the middle maturity options will have an at-the-money volatility close to the front month at-the-money volatility. If Tau is low, middle maturity options will have an at-the-money volatility closer to vol infinity.

Example: current vol 65, VolInf 20, scalar 100 days



Example: Current Vol 65, VolInf 20, Scalar 20



The effect of scalar is subtle, but the back month volatility is lower in the second example.

Again, it is important to note that this functionality is included to ease in drawing a consistent trinomial tree. The model is calibrated separately for each set of expiring options to maximize the ability of a trader to fit the model to the market.

Understanding the Model

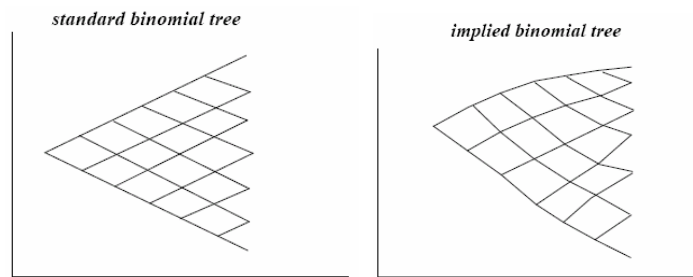
The underlying model that has been used option pricing program is the implied trinomial tree model. The main objective of using implied trinomial tree model is

that this model can ensure the consistency of untraded exotic options prices with the market prices of liquid standard options. It is worth noting that using implied trinomial tree with deterministic volatility function will also allow users to specify trading option price that the tree will try to match conveniently by inputting only one function. Following sections described part of implied tree model in detail.

Implied Trinomial Trees--Theory

In contrast to standard Cox-Ross-Rubinstein binomial tree which contain constant logarithmic spacing between each stock price node (e.g. constant volatility), the implied trinomial tree volatility of each node varies at each node. While option prices computed using the CRR tree will converge to the Black-Scholes continuous-time results in this limit, the option price calculated from implied tree will converge to input option price that we used to calculate volatility in each node at the limit. This makes it possible for users to tailor the tree to give option prices that are consistent with the market. In summary the idea is to construct the tree that will give consistent option price to the market price (by using price as an input).

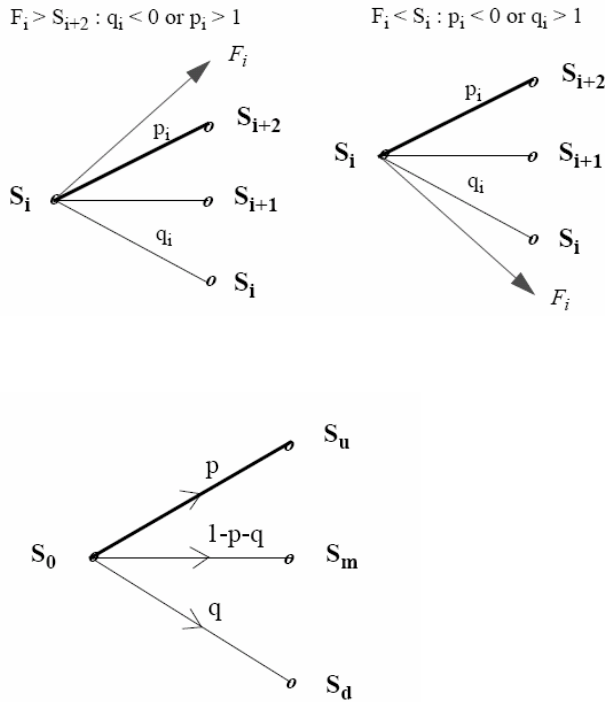
The theories assume that the stock price follows a preference-free process whose instantaneous volatility varies only with spot price and time. It also shown by Derman and Kani in 1994 that we can determine the local volatility function directly from the market prices of liquidly traded options.



Above pictures show difference between logarithmic spacing between nodes which is constant in standard binomial tree and that of implied binomial tree which is not constant.

The tree that we are trying to construct gives two properties: it is arbitrage free and it should give option price that we want. From now on, we will call option price that we want as output the "input option price".

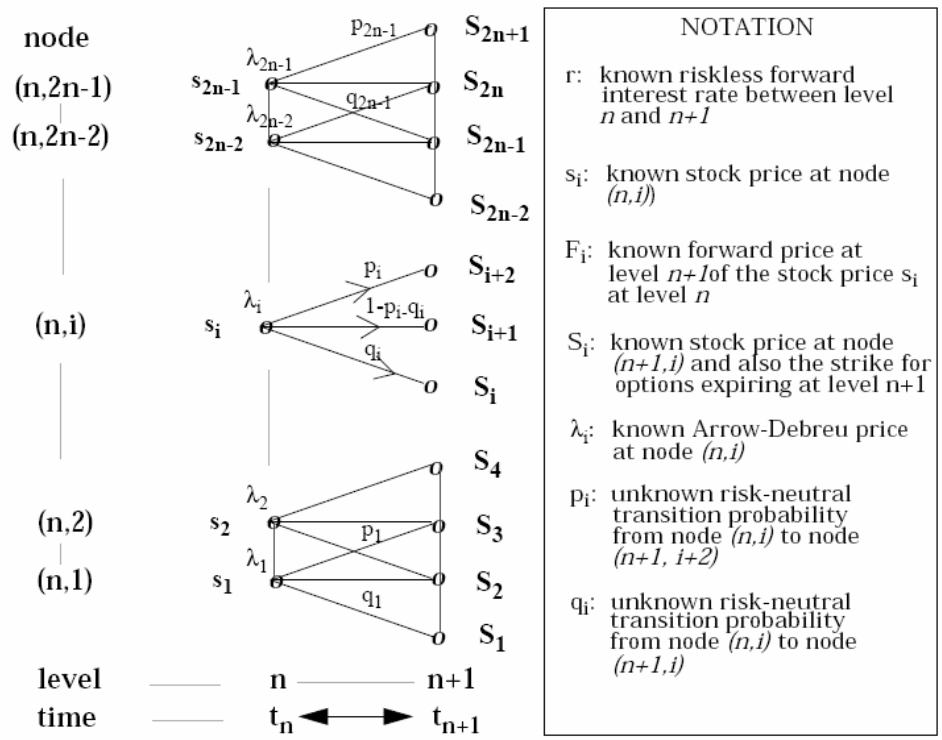
Besides, matching input option price, to make an arbitrage free tree, we have to make sure that the tree satisfied forward price condition which says that expected value of asset price is its forward price. To meet this condition, we first pick stock price in our tree in such the way that forward price is between high and low price node as shown in figure below. Note that it is possible to pick any stock price since we have our freedom to choose two probabilities give that our tree is trinomial rather than binomial. Then we can pick probabilities in such the way that we make forward price equal expected stock price.



In summary the tree construction process is as follow (detail construction detail can be found in Derman and Kani paper (1994 and 1996) :

- Select stock prices in each node in such the way that forward price of each node is in between high and low value of next node, therefore, calculated transition probability can be between 0 and 1.
- Calculate transition probabilities p and q as shown in figure above in such the way that expected stock price is the same as forward price
- Calculated option using current intermediate tree is the same as input option price.

Derman Kani Implied Trinomial Tree – Mechanics



To calculate transition probability of all node above central node we use call price given $C(K,T)$ is call price with strike equal K and time to maturity equal T . Following equation make transition probabilities consistent with two conditions described above and will be used to calculate transition probability of node above central node.

$$p_i = \frac{e^{r\Delta t} C(S_{i+1}, t_{n+1}) - \sum_{j=i+1}^{2n} \lambda_j (F_j - S_{i+1})}{\lambda_i (S_{i+2} - S_{i+1})}$$

$$q_i = \frac{F_i - p_i (S_{i+2} - S_{i+1}) - S_{i+1}}{S_i - S_{i+1}}$$

On another hand, to calculate transition probability of all node below central node (including the central node) we use put price given $P(K,T)$ is call price with strike equal K and time to maturity equal T .

$$q_i = \frac{e^{r\Delta t}P(S_{i+1}, t_{n+1}) - \sum_{j=0}^{i-1} \lambda_j(S_{i+1} - F_j)}{\lambda_i(S_{i+1} - S_i)}$$

$$p_i = \frac{F_i + q_i(S_{i+1} - S_i) - S_{i+1}}{S_{i+2} - S_{i+1}}$$

However, although we choose our stock price in such the way that we can easily satisfy our forward condition, it is not always possible to find feasible p and q (e.g. between 0 and 1 and make $1 - p - q$ between 0 and 1 as well) that will fit all option price. These option prices will be virtually considered as arbitrage price and will be throw away from the model. The probability that will be assigned instead of the probabilities that will provide such option prices is as follow (note that these probabilities will not provide input option price):

If forward price in between top (S_{i+2}) and middle node(S_{i+1}):

$$p_i = \frac{1}{2} \left[\frac{F_i - S_{i+1}}{S_{i+2} - S_{i+1}} + \frac{F_i - S_i}{S_{i+2} - S_i} \right]$$

$$q_i = \frac{1}{2} \left[\frac{S_{i+2} - F_i}{S_{i+2} - S_i} \right]$$

And if forward price is the middle node (S_{i+1}) or between middle node (S_{i+1}) and low node (S_i):

$$p_i = \frac{1}{2} \left(\frac{F_i - S_i}{S_{i+2} - S_i} \right)$$

$$q_i = \frac{1}{2} \left[\frac{S_{i+2} - F_i}{S_{i+2} - S_i} + \frac{S_{i+1} - F_i}{S_{i+1} - S_i} \right]$$

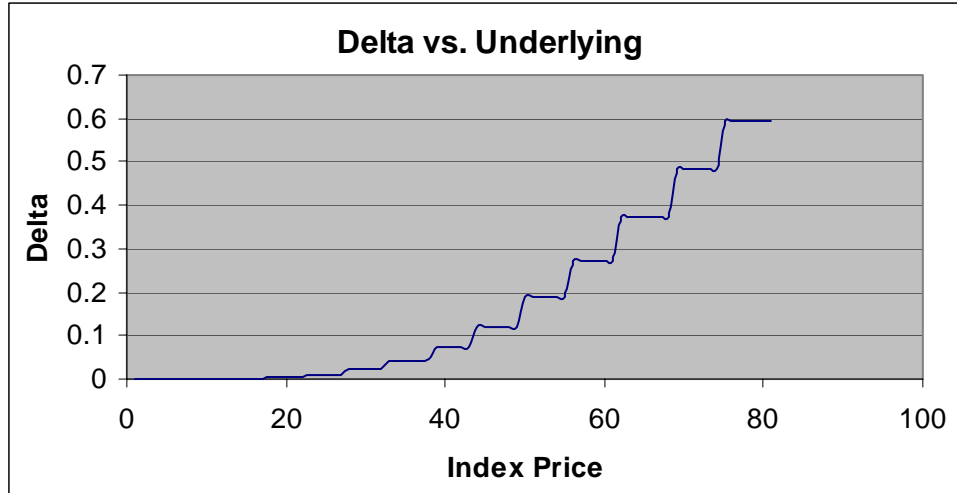
Testing the Model

Part of our assignment was to check the model against our understanding, verify its accuracy, and fix any known problems. Our efforts focused around two potential areas. The first area was the calculation of the risk sensitivities (Greeks), which Speedtrading suspected had a major problem. The second area was verifying a correct link from the input parameters to the model's output.

Delta and Gamma

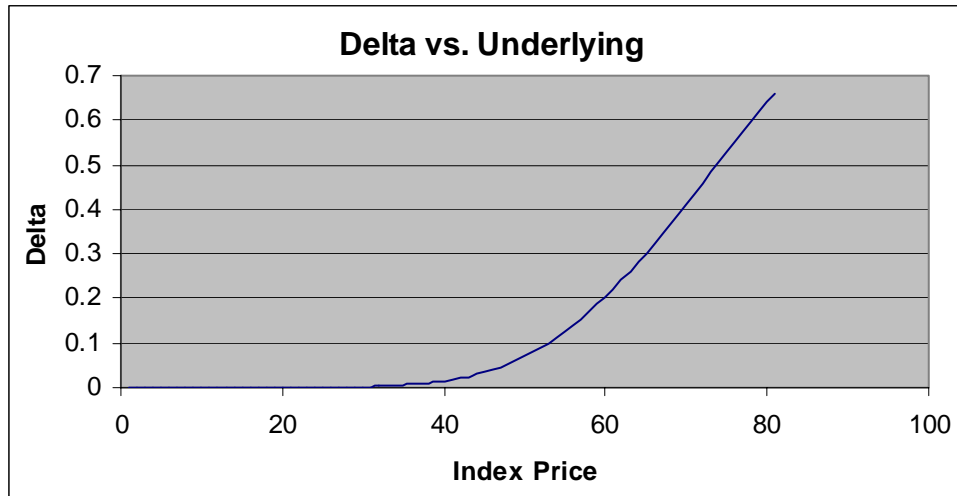
A problem with the calculation of Delta and Gamma was seriously limiting Speedtrading's ability to trade profitably. Specifically, the model was calculating deltas that looked like a step function when graphed versus the price of the underlying.

Example: Derman-Kani generated Deltas for a range of Futures values



Given the importance to the trading strategy of relatively continuous delta hedging, it is clear to see this was a problem. In the graph above, as the underlying moved between 55 and 60, the model would make the trader continually over and under hedge.

Our goal was to produce deltas that increased smoothly with the underlying as in the graph below:



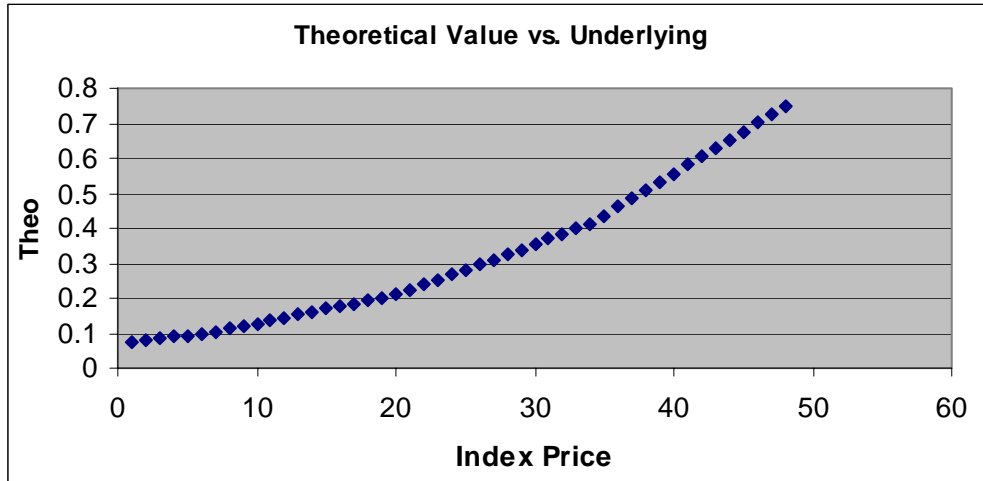
Speedtrading felt that the general level of the deltas was good. As such our goal was to find a smoother delta that was on average the same as the one calculated by the model.

Understanding the Cause

Delta was calculated using the finite difference method as follows:

- calculate the option price at one underlying price
- add a small amount to the price of the underlying and recalculate the option price
- Divide the difference in calculated option prices by the amount of the underlying price change.

Simple testing determined that when calculating Delta in the same manner using a simple binomial tree model, the same problem existed.



While this is a good approximation resulting in very little error in prices, there is a problem when calculating Delta.

The problem is, any tree-based model is a discrete approximation of a continuous process. The price-strike function is a piecewise linear approximation to a curve, which means the first derivative, if not taken carefully, will be alternately constant and jumpy.

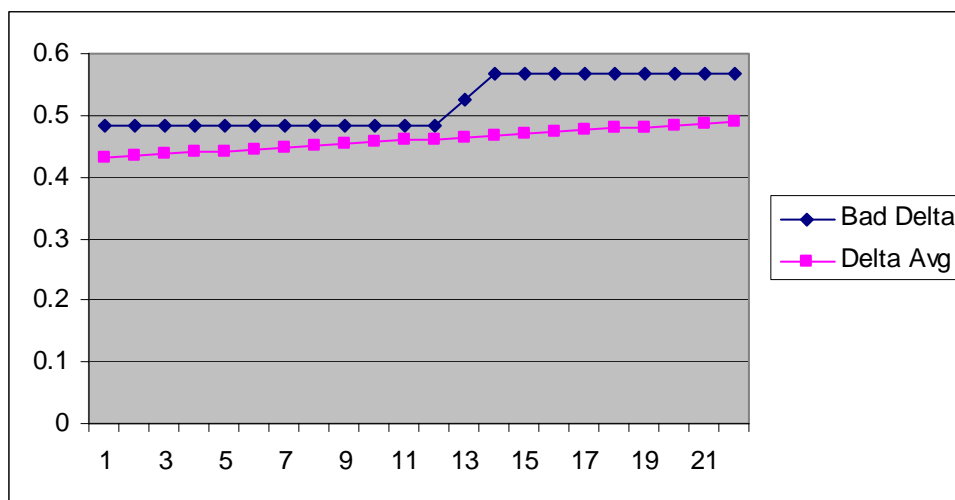
The problem occurs because, as the value of the underlying increases, the value of the option increases at a constant rate as all nodes on the option tree increase in value at some combined constant rate. When the stock price increases so that a previously out of the money node moves into the money--becomes greater than the strike price--the option value at that node begins to increase in value along

with the others. The value from that new node is added to the increase in value of the other nodes, which changes the slope of the Option-Underlying relationship.

A potential solution

The first set of solutions we investigated involved calculating delta directly from the tree. In a binomial tree the basic delta calculation is $(C_u - C_d) / (S_u - S_d)$. This calculation provides a smooth and unbiased calculation of delta. In the trinomial tree, there are 3 nodes (C_u , C_o , and C_d), so the calculation is not as simple. We tried $(C_u - C_o) / (S_u - S_o)$, $(C_u - C_d) / (S_u - S_d)$, and the average of $(C_u - C_o) / (S_u - S_o)$ with $(C_o - C_d) / (S_o - S_d)$. All fixed the smoothness problems but were systematically biased in one direction or another from the original calculation.

In the following graph it is clear that the delta calculation avoids the discontinuity but is systematically too low to be useful.



Both Wilmott (1998) and Hull (2004) state that this method of calculation is not specifically defined for trinomial trees. They do not, however, warn of this kind of bias. One possibility is that the asymmetry of the nodes in the implied tree was enough to bias our results so significantly. Either way, our only hope was to use the finite difference method in a more careful manner.

Revised Finite Difference Calculation

The only way to overcome the jumps in delta due to the discreet nature of the model is to make the difference between the two calculations large enough to span more than one linear segment. To accommodate the larger difference, we needed to take both up and down moves to avoid the upward bias in the derivative created by the increasing function. It turns out, Hull has very nice calculations for delta and gamma using this method:

$$\text{Delta} = [V(S+e) - V(S-e)] / 2e$$

$$\text{Gamma} = [V(S+e) - 2V(S_0) + V(S-e)] / e^2$$

Speedtrading implemented these new calculations and added the necessary tree-calculations required to do so.

Testing revealed that this approximation was always better than the prior approximation but still had room for improvement. Specifically, the success of

the results depended closely on the time to maturity and volatility. We were able to further improve the calculation by making the amount of the difference proportional to the relevant standard deviation adjusted for time (σ / \sqrt{t}) for delta and $\sigma/t^{1/4}$ for gamma.

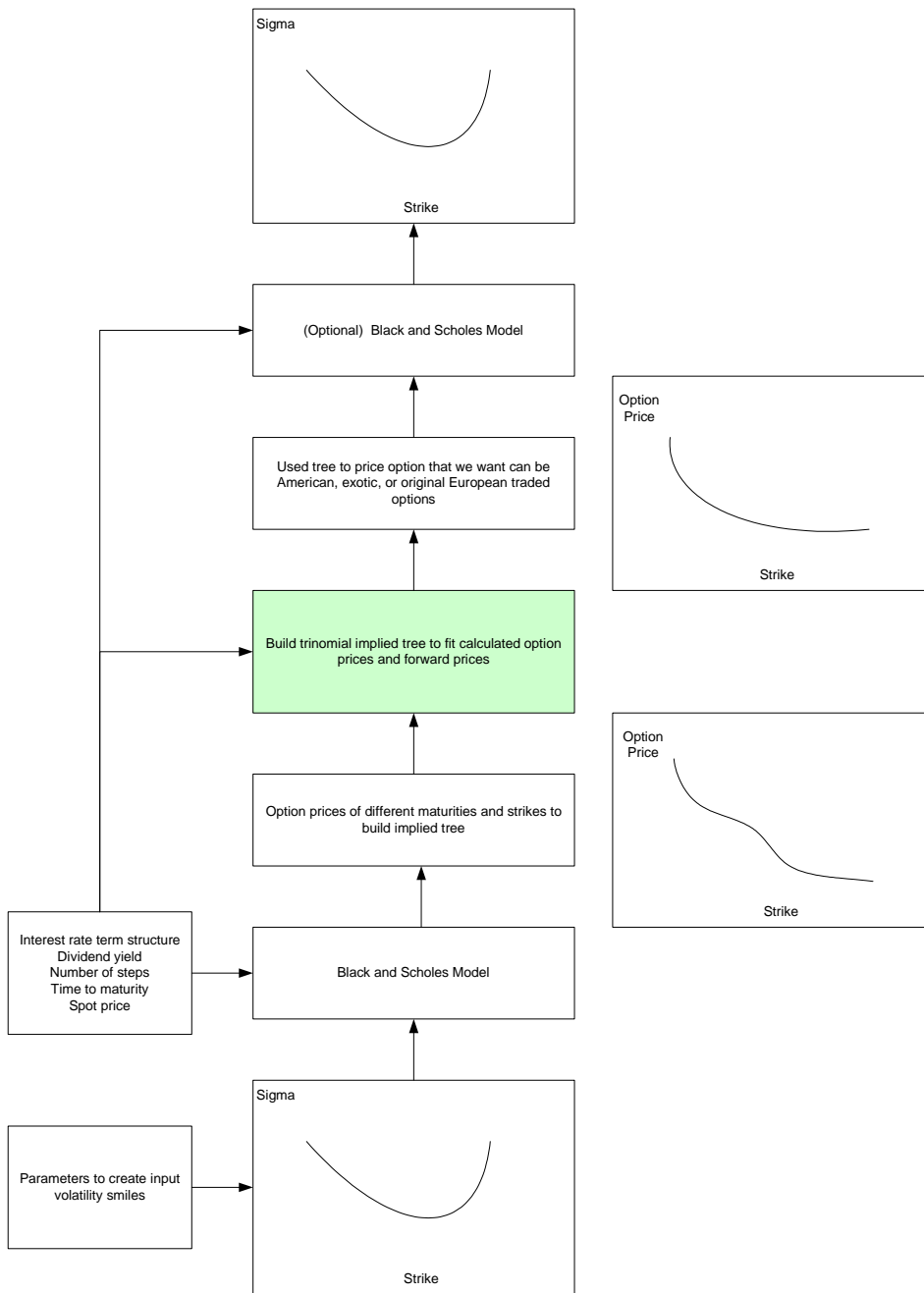
Speedtrading has been operating with their new Greeks and is happy with the new results.

Input Smile vs. Output Smile

We were originally convinced that the input volatility smile with the same maturity as option attempted to price should be the same as output implied volatility smile. This should actually be true given if there is no exception in tree construction process (as discussed in the background section). However, we found that there are a lot of exception that is calculated probability in many nodes are not between zero and one, and as discussed in the background section, we have to disregard option price used to calculate such probability and satisfied only forward equation. This suggests that the more we found exception, the more we will have divergence between input and output smile. As will be discussed in following sections, we finally found that the model will match option price (that it choose to) in such the way that create certain kind of option price per strike rather than certain smile shape. Also, we found strong correlation between numbers of exception to the level of nonlinearity of volatility function. Therefore, it is

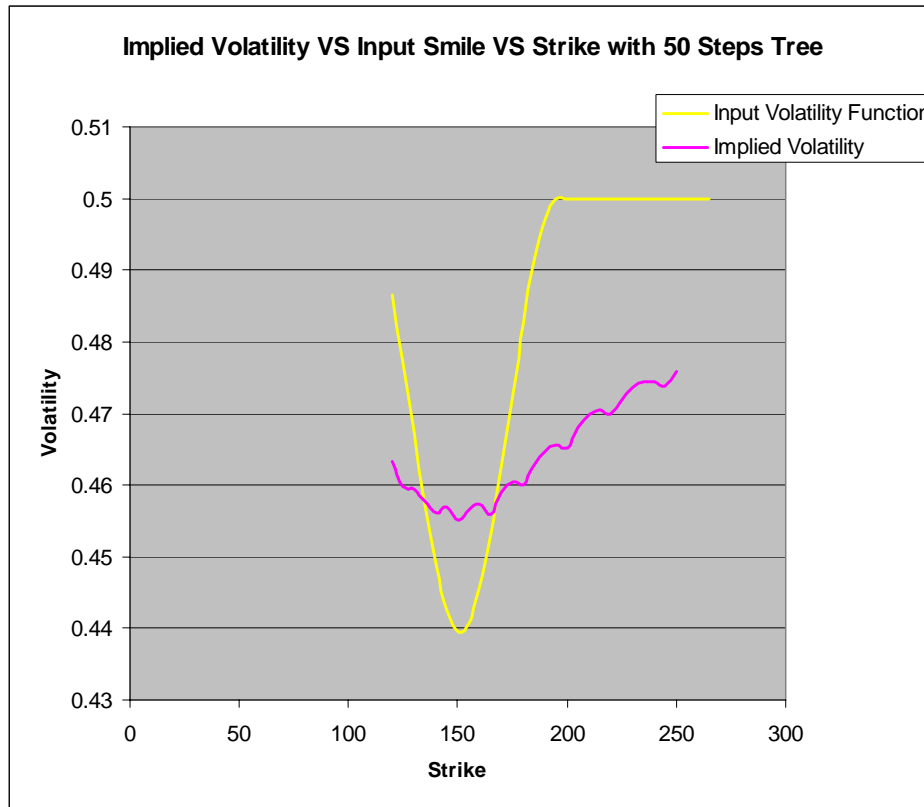
expected to find bigger difference as level of nonlinearity increase (e.g. curvature and skewness). This will be shown later that more curvature in input volatility may give strange looking input option price that derived from given smile using Black-Scholes formula.

The main implication of this divergence between input and output smile is that input smile is very sensitive to input parameters and its shape can be controlled very effectively by input parameters (since these parameters are designed to control the shape of this smile). However, since input and output smiles are not the same, controlling input smile does not necessarily give decided output smile. Moreover, our analysis shows that output smiles are normally quite insensitive to input parameters. In next section, we describe our finding in the level of divergence between input and output smiles.



Divergence of Input and Output Smile

Input and output smile should be in theory the same since both of them are same exchanged traded option. However, as shown in figure below shape of input and output volatility smiles are not the same in the model.



There are several potential causes of this divergence. Following is the list of our hypothesis and initial analysis:

- Not enough tree steps so that output price still not converged to true price
 - We rejected this hypothesis because we construct our tree to fit input option prices. Therefore, if we construct our tree correctly without any error or exception, the result output option price from the tree should be the same as input prices no matter what is the given tree size. However, as will be shown in the next section, tree steps play

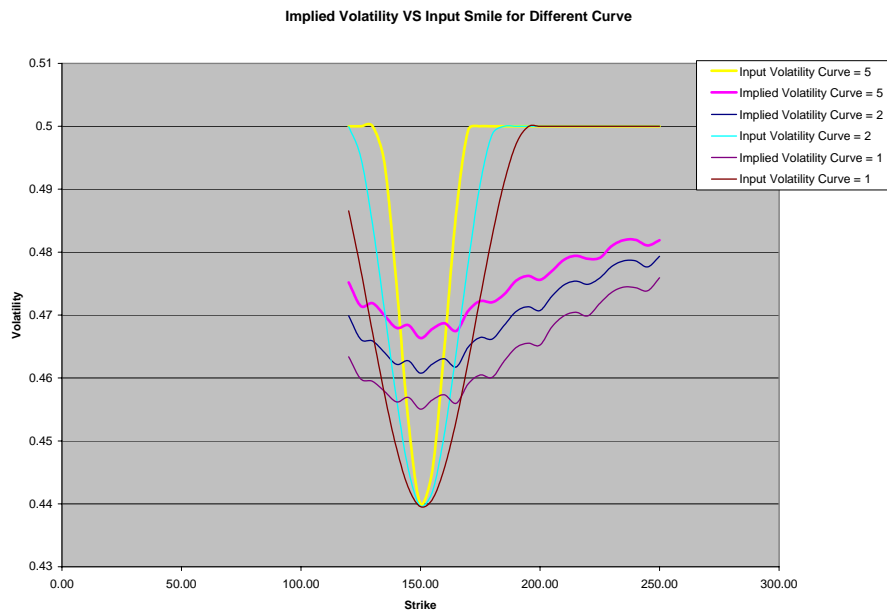
critical role in shape of output as well as its sensitivity to input parameters.

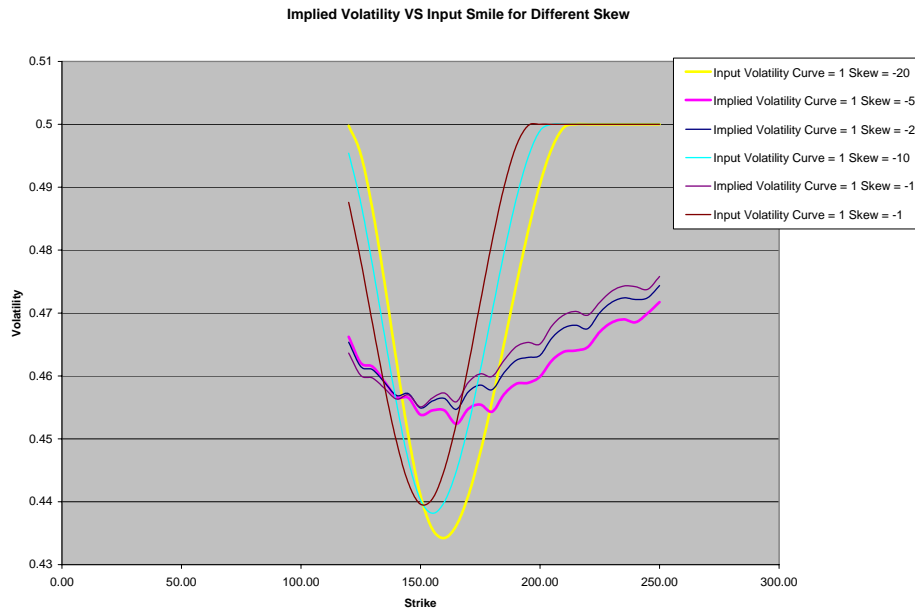
- Error in source code and misrepresentation of Derman Kani model in the program.
 - This hypothesis is still in pending and more future work should be done to verify. We found some inconsistent pieces of code with the original paper equations. However, changing these pieces did not change output price dramatically. Therefore, we think it provides only minor contributions to the divergence. On another hand, this suggests that there is a room for ones to verify source code. The correctness of algorithm will play very critical role if the model is used to price non-traded exotic options or rarely traded American options.
- Model constrain in fitting implied tree
 - We found that this hypothesis sounds very compelling as discussed in the background section, if calculated probability is not between 0 and 1, we will just throw away option price from the smile that we tried to fit and make sure that we satisfied only forward constrain in each node. Given that in our test, we found that rate of option price being threw away as up to 40% – 50%, this could potentially be the

main cause of the big divergence between input and output option price.

Analysis of Input and Output Option Price

In this section, we discuss the rationale of looking at option price per strike instead of only looking at volatility smile. In previous section, we discuss about the divergence between input and output smile. In fact, we found that the change in input smile parameter make it easier to control shape of input smile, however, output smile is pretty insensitive to this change in parameters. Following is some figure that shows insensitivity of change in parameters on output smile.

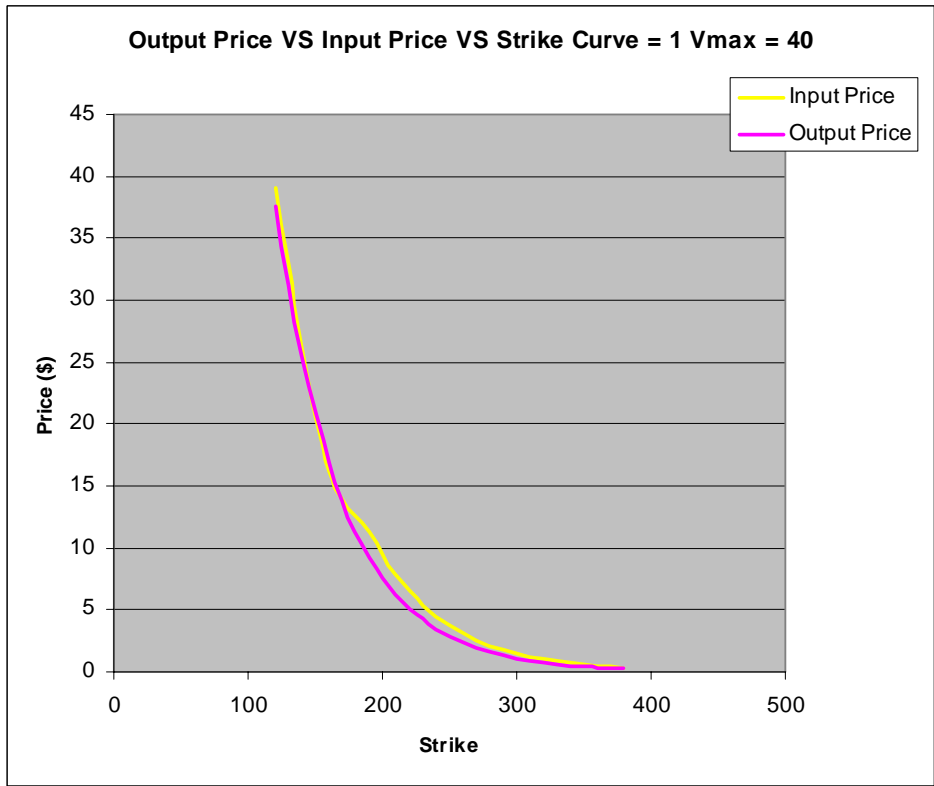
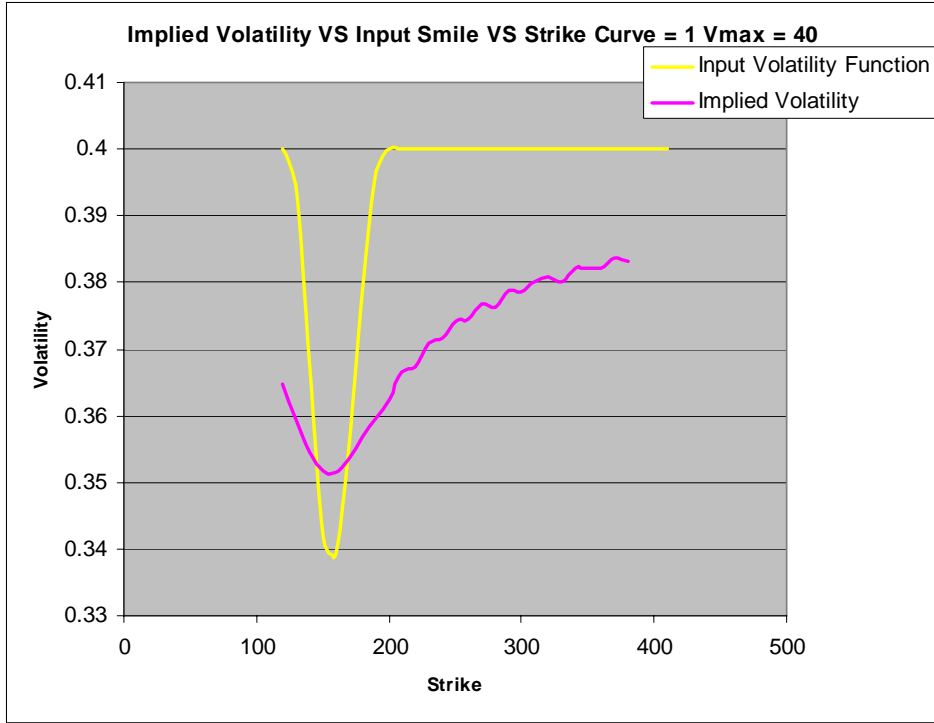




Above figures illustrated the effect of change in two major parameters in input smile, which are curve and skew. These changes has effect on input smile as we expected, however, its effect on output smile is ambiguous. To verify the reason of this incidence, we looked at input option price and output option price instead. Input option price is the price that we use Black-Scholes formula to calculate using input smile as volatility for each strike, while output price is the result from using implied tree to calculate option price.

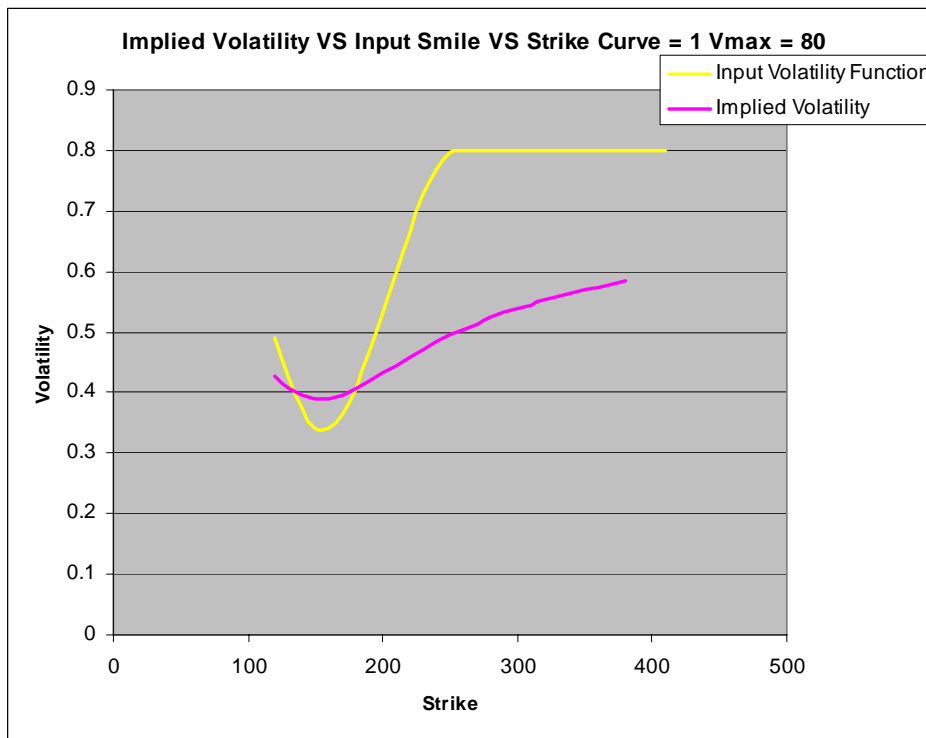
Following figure shows input and output price and smile with different curve and maximum volatility parameters.

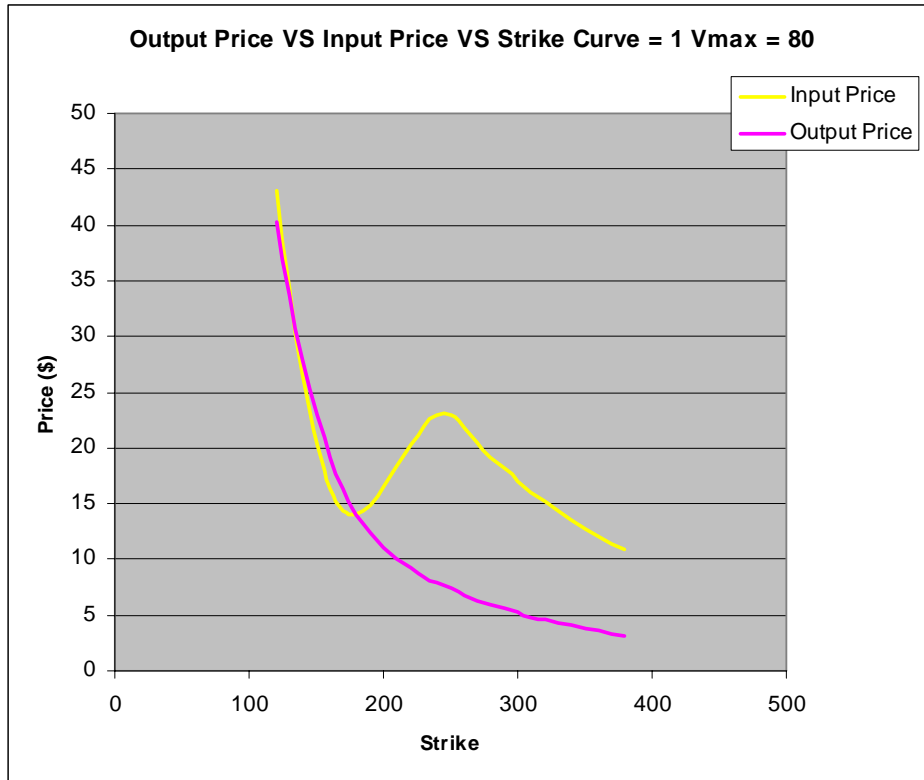
Scenario 1: Curve = 1 and maximum volatility = 40%



In this scenario, we can find big different in volatility graph and unsmooth implied volatility (the purple curve). However, when we look at input price derived from input smile, its shape looks odd. It is both convex and concave depends on level of strike price. This represent arbitrage option price. However, for the input option price (the purple curve), its shape looks much smoother and represent no arbitrage option price. Also, it worth noting that represents convex shape.

Scenario 2: Curve = 1 and maximum volatility = 80%



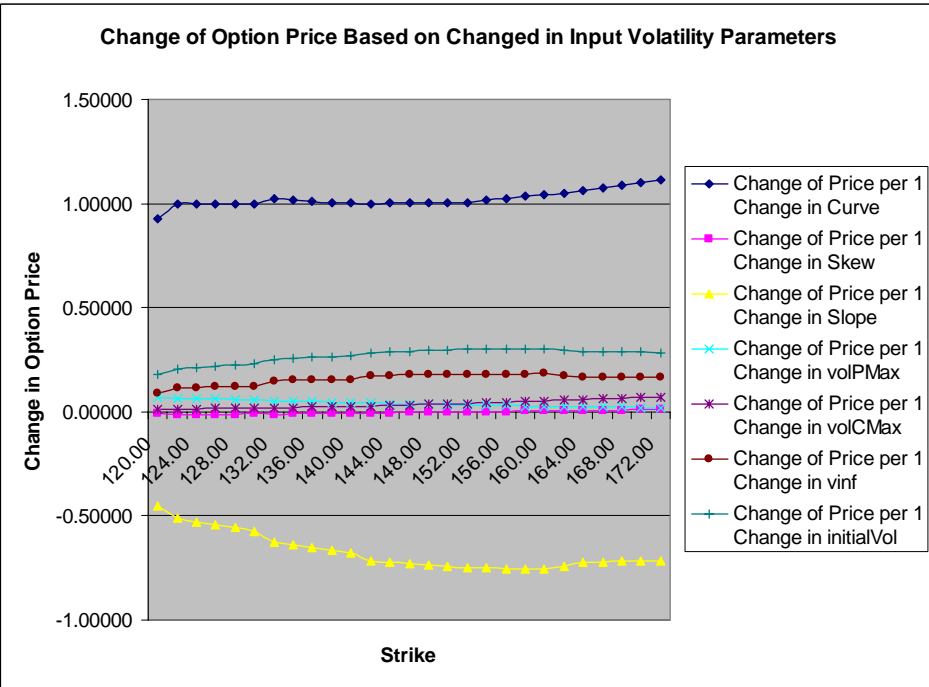
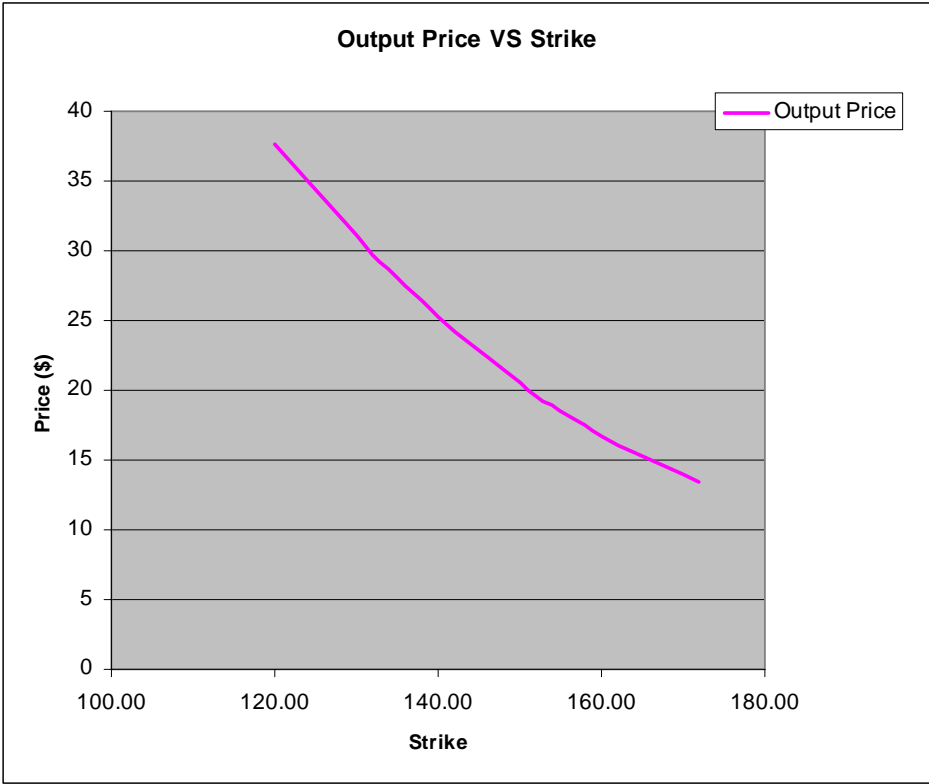


Above figures show that by changing parameters of input smile, we can get very odd looking price curve (yellow curve). However, the model will always throw away odd option price and make its output to have nice looking convex curve. This suggests the reason that changing in parameters of input smile will not necessary give same output smile since the model will filter input price and use only non-arbitrage price. This may create difficulty for trader to adjust output price in the way that they want but on another hand the model help them make sure that the entered price is reasonable. In the following sections, we discuss the sensitivity of input parameters on output option price since using this sensitivity analysis, traders can fit output price to their thought quicker than purely trial and error.

Speeding Calibration

Given above analysis, the implied tree will give smooth out option price per strike. Although this may make sometime hard for traders to adjust output put price based on input parameters but they can be sure that output price is a no arbitrage price. However, trader still need to know the effect of change in each input parameters on output price in order to adjust model to fit their intuition. To facilitate trader to do the task quickly, we have come up with a set of functions to calculate average change of output price per 1 unit change in each input parameters. Following figure shows change in output call price (of each strike) based on 1 unit change of input parameters.

Initial Parameters	
spot	146
treesize	50
initvol	20
volinf	50
scalor	1
slope	0
skew	0
curve	1.5
volpmax	50
volcmax	50
refprice	195
exp	1
zerorate	0.035
strike	120



Based on above figure, ones can change of slope and curve has high effect on output option price. However, this may due to the fact that one unit change in curve parameter is much bigger than 1% change in maximum volatility. To get desirable shape of input, traders should focus on shape of each graph and multiply the effect by number of unit change. It also worth noting, this graph is approximation and is subject to change based on initial parameters. To facilitate comparison, we delivered the Visual C++ code to calculate these sensitivities.

As shown in above figure, one can use *vinf* and *initilaVol* to change the level of output price, this will also change shape of output price curve a little bit. To offset the change in shape, we can use either *volCmax* or *volPmax* to adjust convexity of right and left side of output price from spot price. Changing *curve* will also shift the level of output, as well as change convexity. Changing in *slope* will virtually change reference price and will shift the entire curve as well as some convexity.

In summary, given that all of these inputs will directly manipulate input smile curve but not directly related to input price. The effect of changing in these parameters on output price may not be as expected. Traders can use provided spread sheet to approximate change in output option price based on change in input parameters. The graph such as above can guide traders on the effect of parameters on output price and would be more accurate than rely on intuition alone.

Evaluation of the Model

Current Use

Our next step was to determine whether the model was appropriate for its current use. Since the theoretical values are taken from trader intuition and market prices, we were unsure what value the model provided. On one hand, the model is slightly slower than a simple Black Scholes would be when calibrated in the same way. In addition, the key output of the model—the implied tree, which is designed for use in pricing exotics—is left unused. On the other hand, the volatility surface works well, does a great deal to prevent arbitrageable prices, and is far too tightly linked to the Derman-Kani back end to be separated. In the end, this model is better than anything that could be quickly built from scratch and will serve its purpose well.

Other Uses

One major question was to think about other potential markets to trade using the model. Clearly with the unique calibration ability, the model is a fit for Speed's style of trading in any equity options market. We fear, however, outside Korea, trading opportunities might be limited. We investigated fixed income and exotic options as potential areas for further use.

Fixed Income

We do not recommend trading bond options with this model. A key feature of any usable bond option pricing model is stochastic interest rates. Derman-Kani simply doesn't allow for those in a suitable manner.

Exotics

Where the Derman-Kani model supposedly shines is in the pricing of structured or exotic equity derivatives. For untraded exotic options with no market for to rely on for pricing, there is great difficulty knowing how to price and hedge. Derman Kani can't change the fact that there is no market, but it can and indeed it does infer the market's estimate of the volatility of the underlying asset from traded options. For exotic derivatives on assets that have traded options, the Derman Kani implied tree can be used to price exotics consistently.

To the extent that Speedtrading can find customers for exotic options, they would have an advantage in pricing those options and could charge a premium. Perhaps Redsky's institutional customers would be interested special position insurance based on various contingencies that could be implemented with exotic derivatives.

In order to implement anything with the tree, the tree will need to be output and stored. We worked with one of their developers, Matt Otto, to output the tree. The code is written but would need to be made live before it could be used.

Future Analysis Ideas

Unfortunately there was far more to do on this project than time allowed. We feel there are several other interesting projects that can and should be undertaken. They are as follows.

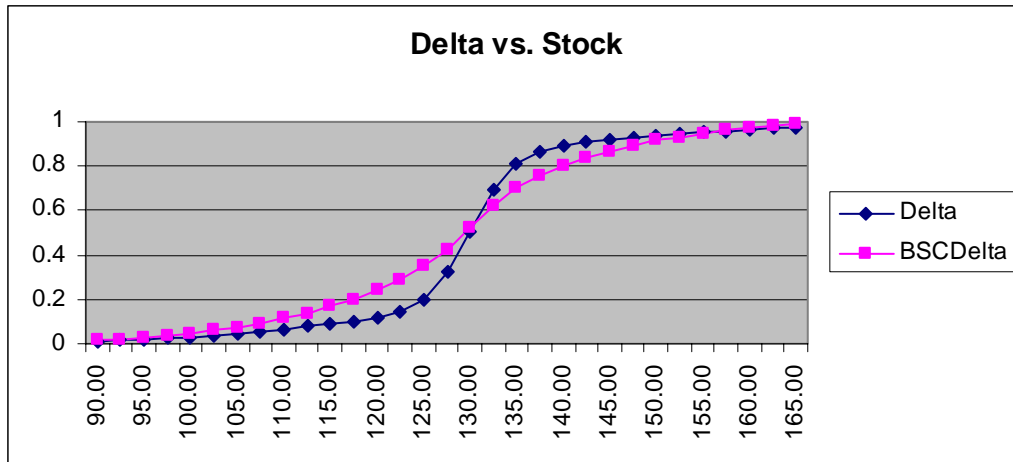
Autofit

Perhaps the easiest place to make significant improvement is to develop an algorithm to automatically select the parameters that best fit a given market. Excel's solver tool proved unsuitable for this task, but we're confident that it can be done with a bit of logic. Not only will automatic calibration make trading more efficient, but it will also allow future analysts to undertake more interesting projects that involve calibrating the model to large amounts of data.

Hedging Performance

If the model could be back-calibrated quickly, one important question we could answer is whether this model produces better hedges than Black Scholes would. A preliminary analysis reveals that Derman Kani produces different hedge ratios than Black Scholes does, which is not surprising. Calibrating Black Scholes involves increasing the volatility in a normal distribution in order to match

market prices, whereas Derman Kani infers the distribution of the underlying directly from the market. Derman Kani should be better, but we would like to see someone prove it.



We attempted this analysis. Our goal was to create typical trading and hedging strategies and run them over time using both models. We were then planning to compare the returns from the portfolios and check to see which model led to higher and less volatile returns.

The major difficulty with the analysis was calibrating the model to the market with enough frequency to be realistic. With an automatic calibration routine, this analysis would be far more efficient.

Parameter Time-Series Analysis

With the calibration parameters stored as a time series along with market prices, we think interesting regression analysis could be undertaken. Specifically, we would like to see how Curve and Skew relate to time to maturity, level of the underlying, and prior returns. We would also see how Curve and Skew at time t relate to Curve and Skew at some future time T . It is possible that such tight calibration results in over-fitting the market, and an analysis of the fit over time would provide valuable insight to that question.

Optimal Tree Redrawing

Since each tree calculation takes time, we would like to see more analysis done on when to recreate the tree and when to use the existing tree. Careful analysis could be undertaken to determine when it is optimal.

Structured Equity Derivatives

If Speedtrading is to sell exotics to their customers, as we recommended, they will need someone to develop the pricing methods, hedging strategies, and marketing material to support that process.

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