A FRAMEWORK FOR ASSESSING THE DIVERSIFICATION BENEFITS OF ADDITIONAL SECURITIES ON PORTFOLIO RISK

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Abstract

The purpose of this paper is to establish a framework to assess the diversification benefits of adding more securities into a portfolio. Using the traditional mean-variance framework to generate efficient frontiers for portfolios comprising different asset classes (equities, bonds and derivatives), we are able to measure the diversification benefits of additional securities and compare them against investors' characteristics, namely (i) target level of portfolio returns, (ii) risk appetite and (iii) level of aggressiveness in the portfolios. In addition, the paper evaluates the adequacy of the mean-variance framework in measuring portfolio risk, and suggests using Value-at-Risk ("VaR") as a supplemental risk measurement tool under certain circumstances.

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1. PROJECT OBJECTIVES AND METHODOLOGY

Introduction

How many assets do investors need in order to have a well-diversified portfolio? Evans and Archer (1968) concluded that approximately ten stocks will suffice. More recently, Statman (1987) argued that a well-diversified portfolio of randomly chosen stocks must include at least thirty stocks for a borrowing investor and forty stocks for a lending investor. This has led us to question whether there is indeed a general rule (i.e. an "X" number of assets) that is applicable to *all* portfolios. However, this alone is not a well-posed question since different investors have different characteristics, as well as different asset classes/universes with which they are dealing. Thus, it is difficult to answer the question of "how many" without being more specific. For this reason, in this study, we have chosen instead to develop a framework to assess the *benefit of portfolio expansion* given different investor characteristics and assets involved.

Literature Review

Modern portfolio theory was introduced by Markowitz with his paper "Portfolio Selection" which appeared in the 1952 *Journal of Finance*. Essentially, Markowitz demonstrated how investors can construct portfolios that minimize market risk for a given level of expected return. Based on different combinations of the assets, an "efficient frontier" of optimal portfolios can be constructed. Each portfolio on the efficient frontier offers the maximum possible expected return for a given level of risk.

Portfolio theory has provided a broad context for understanding the trade-offs and interactions between systematic risk and returns. Subsequently, there have been a number of academic papers seeking to answer the question of how many stocks constitute a diversified portfolio. The first of such studies was conducted by Evans and Archer (1968), who measured portfolio risk by the standard deviation of returns from average returns. The study assumed equal investment in all securities in the portfolio, which is optimum if the investor has no information about future returns variances and co-variances. Evans and Archer concluded that for a randomly selected and equally weighted portfolio, there is very little diversification benefits beyond eight to ten stocks.

While this conclusion has been widely cited, Bird and Tippet (1986) questioned the methodology adopted by Evans and Archer. Instead, they derived an exact parametric relationship between portfolio standard deviation and size. Their approach showed that the ordinary least squares ("OLS") parameters employed by Evans and Archer were biased. As such, there were considerable benefits obtained by expanding the portfolio size beyond the levels suggested by Evans and Archer. Using a totally different methodology, Statman (1987) stated that a well-diversified portfolio must contain at least thirty stocks for a borrowing investor and forty stocks for a lending investor. Statman adopted the principle that marginal costs should be compared to marginal benefits in determining the optimal levels of diversification.

While mean-variance optimization is useful in helping investors to achieve their target returns with maximum diversification, the use of standard deviation alone may not provide an adequate measure of risk. This is particularly pertinent if the portfolios contain assets that exhibit "non-linear" returns, for instance derivatives. Hence, there is an increasing need to supplement the use of standard deviation to measure risk. Perhaps the most notable of private-sector initiatives toward better risk management is that of J.P. Morgan, which unveiled its RiskMetrics system in October 1994. Jorian (2001), an advocate of VaR, stated that the greatest benefit of VaR lies in the imposition of a structured methodology for critically thinking about risk.

Methodology Overview

The literature review demonstrated that there is a need to establish a robust framework which would critically evaluate the benefits obtained by expanding portfolio size. The literature review also showed that modern portfolio research did not involve non-linear securities nor investors' characteristics. Our study therefore seeks to build an analytical framework incorporating investors' characteristics that would be able to handle different asset classes ranging from equities to non-linear securities like derivatives. In addition, we also assess the usefulness in measuring the VaR of portfolios as a supplemental risk measure to portfolio standard deviation.

For the purpose of this study, we set up diverse portfolios comprising of equities, bonds and derivatives, which will enable us to examine the diversification benefits of expanding the portfolio size. For our initial dataset, we divided twenty equity indices that are currently traded in the form of exchange-traded funds ("ETFs") into two "baskets" equally[1], namely a basket comprising "*broad-based*" indices ("Basket #1") and a basket comprising "*sector-specific*" indices ("Basket #2"). In other words, each basket contains ten equity ETFs. Within each basket, portfolios comprising Equities Only, Equities & Bonds, and Covered Calls (long equities and short calls) were created. This would allow us to investigate the diversification benefits across different asset classes as follows:

(a) "A" portfolios: An Equities Only portfolio containing five equities to an Equities Only portfolio containing ten equities (of which five equities were from the former portfolio);

(b) "B" portfolios: An Equities & Bonds portfolio containing five equities and two Government Treasuries[2] to an Equities & Bonds portfolio containing ten equities and the same two Government Treasuries; and

(c) "C" portfolios: A Covered Calls portfolio containing five out-of-the-money ("OTM") covered calls to a Covered Calls portfolio containing ten OTM covered calls (of which five OTM covered calls were from the former portfolio).

See Appendices I and II for details on the composition of the portfolios.

Ten-year price data on the equity indices were readily available, which was not the case for Government Treasuries and calls. Therefore, we were required to simulate the prices of

[1] It is important that the indices are tradable as this would ensure that the impact of transactions costs would be minimal. Although the ETFs on many of the indices did not begin trading until the late 1990s, we were able to collect raw index data over the past ten years and perform the necessary computations.
[2] Two- and ten-year on-the-run US Government Treasuries were used.

Government Treasuries and calls. For the Government Treasuries, we collected weekly yields and prices were computed by using modified duration and convexity of the bonds. As for calls, given that they are not delivered weekly in the market, the prices of calls were computed by using the Black-Scholes formula[3]. The table below outlines the details for computing the weekly returns and standard deviation for each asset class. See Appendix III for specifics regarding data collection methodology and Appendix IV for the asset characteristics.

Table 1: Returns and Standard Deviation Computation Methodology

Asset Class	**Weekly Returns / Standard Deviation[4]**	**Remarks**
Equities	• Weekly mean returns and standard deviation computed based on weekly continuously compounded returns	NONE
Bonds	• Converted yields to weekly bond returns by using $dB/B = -$ Modified Duration * $dy + 0.5$ * convexity * dy^2 • Weekly mean returns and standard deviation computed based on weekly continuously compounded returns	• Current duration and convexity were used to compute bond prices
OTM Covered Calls	• Covered calls priced based on prevailing long equity price and short OTM 90-day call, and marked to market each week • OTM 90-day calls were priced using Black-Scholes formula, with strike price at 1 standard deviation above the stock price • Weekly mean returns and standard deviation computed based on weekly continuously compounded returns	• Each week, the existing OTM call would be sold, while a *new* OTM call with 90 days to expiration would be bought • Volatility of underlying equity was assumed to be constant • 90-day T-Bill rate was used as the risk-free rate • Dividend yield of 1% was assumed for ETFs that did not have data on dividend yields.

Similar to the method used by Markowitz, we constructed an efficient frontier for each portfolio by varying the weights on each security in the portfolio and minimizing the portfolio risk (i.e. standard deviation) for expected *annual* returns ranging from 1-30%. We then compared the results against investors' characteristics such as target level of portfolio returns, risk appetite and level of aggressiveness in the portfolios. See Appendix V for the asset correlation matrix.

The final part of our methodology involves computing VaR at a certain confidence level over a specific horizon. We have decided that a suitable confidence level (one-tail) is 99% confidence over a horizon of one week. The computation of VaR was done using the following methods:

(a) Normal assumption: This method assumes that the portfolio returns are normally distributed and computes the z-value at the 99% confidence level.

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[3] Black-Scholes formula for a European call option is: $Se^{-\delta T} N(d_1) - Ke^{-rT}N(d_2)$ where $d_1 = (\ln(S/K)+(r-\delta+\sigma^2/2)T/\sigma T^{0.5}$ and $d_2=d_1- \sigma T^{0.5}$

[4] Weekly returns and standard deviations were annualized using a factor of 52 and square root of 52, respectively.

(b) Historical simulation: This method does not assume any distribution of the portfolio returns. Instead, it assumes that the historical weekly returns are representative of future returns. By running each portfolio through the historical returns for the past ten years, we can compute the corresponding portfolio weekly returns. We then sort the portfolio weekly returns in ascending order such that they are arranged in percentiles. The portfolio weekly return at the 1st percentile would thus correspond to the VaR with 99% confidence level.

(c) Monte Carlo simulation: This method is used to generate future equity and bond prices (which are correlated), which can then be used to compute the covered call returns. This method is particularly suitable when non-linear securities such as derivatives are involved.

2. RESULTS AND ANALYSIS

Part A: "Broad-Based" Basket (Basket #1)

In order to determine the risk reduction benefit of adding more securities to a portfolio, we first looked at the efficient frontiers generated through optimization of the portfolios at different levels of expected return. Figures 1, 2 and 3 show these efficient frontiers for the Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. The inefficient parts of the frontiers are discarded. See Appendix VI for details on the optimized portfolios.

In all cases, the larger portfolio clearly exhibits lower risk for every level of return. However, the frontiers also show that the extent of this risk reduction is not uniform, and appears to increase as expected return increases. Therefore, based on this result, the question of the benefit of adding more securities to a portfolio cannot be asked in isolation, and must be considered in the context of returns.

However, to consider expected returns alone would be ignoring the element of risk involved. Higher returns come with higher risk, and to simply say that "higher returns will provide more risk reduction benefit of having additional securities" is insufficient, because there is also a higher risk level in the first place. Otherwise, all investors will simply aim for the highest return with the highest risk reduction benefit.

Different investors have different risk appetite characteristics, and we therefore wanted to assess the risk reduction benefit of having additional securities in this context. Thus, we introduced into our analysis the concept of utility value. A standard utility function is defined as follows:

$$U = E(r) - \frac{A \times \sigma^2}{2}$$

Where:
E(r) = Portfolio Expected Return
A = Risk Aversion Coefficient[5]
σ = Portfolio Standard Deviation

Each investor can assign a utility value to competing investment portfolios based on the expected portfolio returns and risk. Higher utility values are assigned to portfolios with higher expected returns or lower volatility. In addition, volatility's role in the utility value is dependant on the specific investor's risk appetite characteristic. This is represented by the "risk aversion coefficient" ("*A*") – a lower *A* indicates an investor who is less risk averse or more aggressive, and vice versa. For instance, for a given expected return, a more risky portfolio held by a more aggressive investor could yield the same utility value as a less risky portfolio held by a less aggressive investor.

[5] An investor with zero risk aversion coefficient is a risk-neutral investor. An investor with a high risk aversion coefficient is more risk averse (i.e. less aggressive) compared to one with a low risk aversion coefficient.

Since we can compare these utility values to risk-free returns, we can interpret the utility value ("U") as a "certainty equivalent rate of return" ("R^{CE}"). In other words, the R^{CE} of a portfolio is the rate that risk-free investments would need to offer with certainty to be considered equally attractive as risky portfolios.

For a given risk aversion coefficient, we can plot an "indifference curve" by varying the portfolio expected return and standard deviation, while holding the R^{CE} constant. For an investor with a certain risk aversion coefficient, he/she would rationally choose to maximize R^{CE} for any given portfolio. (Graphically, this is represented by the highest indifference curve that meets the portfolio efficient frontier, and that meeting point is the optimum portfolio for that particular investor.) By comparing the maximized R^{CE} for the smaller portfolio against the maximized R^{CE} for the larger portfolio, we are therefore able to quantify the benefit of additional securities for that investor.

Figures 4, 5 and 6 plot the investor's risk aversion coefficient against the benefit of additional securities (R^{CE} for the larger portfolio minus R^{CE} for the smaller portfolio) for Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. The results consistently indicate that as the risk aversion coefficient increases, the benefit of having additional securities decreases until it eventually plateaus.

The following table summarizes the benefit at risk aversion coefficients of 1 vs. 2 vs. 3, across the three asset classes:

Table 2: Diversification Benefits across Risk Aversion Coefficients and Asset Classes (Basket #1)

	Risk Aversion Coefficient		
	1	**2**	**3**
Equities Only	7.89%	4.33%	3.25%
Equities & Bonds	8.27%	4.14%	2.76%
Covered Calls	8.30%	4.37%	3.10%

See Appendix VII for full data generated for investor's risk aversion coefficient vs. the benefit of additional securities.

The data clearly shows that as the investor becomes less aggressive (i.e. higher risk aversion coefficient), the benefit of having additional securities decreases. Furthermore, there does not appear to be a great deal of difference in the level of this benefit between the Equities Only, Equities & Bonds and Covered Calls portfolios. At a risk aversion coefficient of 2, for instance, the diversification benefit of adding 5 securities to a portfolio is in the 400bps range for Basket #1 regardless of asset class.

While the above results are interesting, in practice, it is unlikely that an investor would know his/her risk aversion coefficient. Thus, we wanted to find a more observable way to measure investor aggressiveness against the benefit of additional securities. One way to do this is to ask the investor how aggressive is the portfolio that he/she is starting out with prior to adding on more securities.

To measure a portfolio's aggressiveness ("AGR"), we can take the absolute value of each security weight in the portfolio and then compute the average value. AGR is thus defined as follows:

$$\text{AGR} = \frac{\sum |w|}{n}$$

Where:
w = individual security weights
n = number of securities in portfolio

We can then compare the AGR of the starting portfolio along the efficiency frontier against the benefit of having additional securities. In this case, the benefit can be measured in terms of the percentage point reduction in standard deviation in moving from the smaller portfolio to the larger portfolio.

Figures 7, 8 and 9 plot the base portfolio AGR against the benefit of additional securities for Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. The results consistently indicate a positive relationship between the aggressiveness of the base portfolio and the benefit of having additional securities. Thus, an investor starting out with a more aggressive portfolio would be able to obtain more benefit from having additional securities.

We were also able to assess the sensitivity of benefit to base portfolio aggressiveness by running regressions and looking at the slopes of the plots. The following table summarizes the slopes of the plots across the three asset classes:

Table 3: Base Portfolio Aggressiveness vs. Diversification Benefit Regression Results (Basket #1)

	Slope	**p-value**	**R^2**
Equities Only	0.1366	0.0000%	99.78%
Equities & Bonds	0.2205	0.0000%	99.37%
Covered Calls	0.0876	0.0000%	99.91%

See Appendix VIII for further details on the regression results.

The data clearly shows a positive relationship between the aggressiveness of the base portfolio and the benefit of having additional securities. Furthermore, there is some variation in the sensitivity of benefit to base portfolio aggressiveness across different asset classes, with Equities & Bonds being most sensitive, followed by Equities Only then Covered Calls.

Part B: "Sector-Specific" Basket (Basket #2)

Having established our initial results and conclusions, we proceeded to apply the same analytical framework to Basket #2 in order to test whether same conclusions would hold for a completely different universe of assets as well. Recall that while Basket #1 consists of broad-based equity indices, Basket #2 consists of sector-specific equity indices.

Figures 10, 11 and 12 show the efficient frontiers for the Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. (See Appendix VI for details on the optimized portfolios.) Again, there is a general trend whereby the risk reduction benefit of additional securities appears to increase as expected return increases. However, notice that this time the trend is not as consistent since at the lower part of the curves, risk reduction actually decreases initially as return increases.

Figures 13, 14 and 15 plot the investor's risk aversion coefficient against the benefit of additional securities (R^{CE} for the larger portfolio minus R^{CE} for the smaller portfolio) for Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. As we saw before, the results consistently indicate that as risk aversion coefficient increases, the benefit of having additional securities decreases. The following table summarizes the benefit at risk aversion coefficients of 1 vs. 2 vs. 3, across the three asset classes:

Table 4: Diversification Benefits across Risk Aversion Coefficients and Asset Classes (Basket #2)

	Risk Aversion Coefficient		
	1	**2**	**3**
Equities Only	4.53%	1.88%	1.25%
Equities & Bonds	5.89%	2.97%	2.00%
Covered Calls	13.58%	6.37%	4.07%

See Appendix VII for full data generated for investor's risk aversion coefficient vs. the benefit of additional securities.

Our previous conclusion that as the investor becomes less aggressive (i.e. higher risk aversion coefficient) the benefit of having additional securities decreases, therefore continues to hold with Basket #2. However, while before there did not appear to be a great deal of difference in the level of this benefit between the Equities Only, Equities & Bonds and Covered Calls portfolios, this is no longer the case. In the case of Basket #2, Covered Calls provide the highest level of benefit for every level of risk aversion coefficient by quite a large degree, followed by Equities & Bonds then Equities Only. At a risk aversion coefficient of 2, for instance, the diversification benefit of adding 5 securities to a portfolio is 637bps for Covered Calls, 297bps for Equities & Bonds and 188bps for Equities Only. This stands in comparison to the benefit of around 400bps across all asset classes for Basket #1 at the same risk aversion coefficient.

Figures 16, 17 and 18 plot the base portfolio AGR against the benefit of additional securities for Equities Only, Equities & Bonds and Covered Calls portfolios, respectively. As we saw before, the results consistently indicate a positive relationship between the aggressiveness of the base portfolio and the benefit of having additional securities. The following table summarizes the slopes of the plots across the three asset classes:

Table 5: Base Portfolio Aggressiveness vs. Diversification Benefit Regression Results (Basket #2)

	Slope	p-value	R^2
Equities Only	0.0677	0.0000%	93.94%
Equities & Bonds	0.1012	0.0000%	98.77%
Covered Calls	0.1071	0.0000%	99.26%

See Appendix VIII for further details on the regression results.

Our previous conclusion that there is a positive relationship between the aggressiveness of the base portfolio and the benefit of having additional securities, therefore continues to hold with Basket #2. There continues to be variation in the sensitivity of benefit to base portfolio aggressiveness across different asset classes, but with Basket #2, Covered Calls is the most sensitive, followed by Equities & Bonds then Equities Only.

Part C: Standard Deviation vs. Value-at-Risk Comparison

Mean-variance optimization, as performed in the previous section, is useful in helping investors to determine asset allocation such that they can achieve targeted returns with maximum diversification. However, using the portfolio standard deviation alone does not provide investors with a good appreciation of their risk profile. Investors need to view their total capital at risk on a portfolio basis. As such, over the past ten years, there has been an increasing trend of using VaR as a more comprehensive measure in capturing the maximum downside risk of the portfolio based on historical returns.

VaR has gained popularity because of its simplicity and applicability. In fact, in recent years VaR has seen more applications outside of the financial industry as treasurers and financial officers in large corporations realize the advantages of VaR over conventional risk measurement methods. One commonly-used method is to generate VaR based on historical returns over a specified time period. This method provides a historical benchmark of the portfolio value that can be lost should past historical trends be representative of future trends. This method appears to give a better sense of risk compared to merely assuming that portfolio returns are normally distributed and using standard deviation as a risk benchmark.

Using selected optimized portfolios generated by the mean-variance approach (as outlined in the previous section)[6], we can compute the VaR at 99% confidence level (one-tail) by either using historical returns or assuming that portfolio returns are normally distributed[7]. Our findings show that at 99% confidence level (one-tail), VaR risk level predicted by historical returns ("*historical* VaR") is consistently larger than that predicted by assuming normal portfolio returns ("*normal* VaR"). (See Figures 19-30.) Historical VaR has produced significantly more conservative

[6] Portfolios with annual target returns of 5%, 10%, 15% and 20% were selected.
[7] The purpose of this exercise is not to optimize portfolios based on mean-VaR approach.

results given the dataset of portfolios A, B and C. This is not surprising since asset returns and hence portfolio returns are not normally distributed in reality.

What was of particular interest was the difference in historical VaR and normal VaR across the different portfolios. While we expected portfolio C to exhibit the largest difference between the estimations, portfolios A and B were also affected as much. Given that portfolios A and B consist of only stocks and bonds, one would have intuitively thought that the VaR predicted by assuming that portfolio returns are normal would provide a good risk measure. Although portfolio B contains bonds and in certain instances bond returns will exhibit non-linear return attributes, we did not see a substantial difference in the absolute change in loss between the portfolios A and B, which would indicate these two portfolios have in general similar risk characteristics.

On the other hand, portfolio C exhibited large fluctuations between historical VaR and normal VaR estimates. This result was in line with our initial expectations. Once derivative positions were introduced, the inability of normal distribution function to fit outlier positions severely impedes the effectiveness of the normal distribution assumption. Historical VaR, however, is able to capture the significant oscillations in the value of the derivatives since it does not pre-suppose any distribution function, and produces what seems to be consistently higher change in observed risk as we move from five to ten securities in respective portfolios.

Finally, we were able to observe a consistent pattern of growing difference in risk estimation with higher targeted return. Investors with a higher targeted return would have a larger misinterpretation of VaR if they had assumed that portfolio returns were normal (when compared to VaR computed based on historical data) than investors with a lower targeted return. This reflects the growing risks associated with the portfolio as one takes more taxing targeted returns and requires more aggressive positions in the portfolio assets.

Monte Carlo Simulation

The greatest criticism of using historical data to compute VaR is that the historical dataset, especially if it is limited, may not be representative. While there is no correct answer to this criticism other than gathering sufficient amount of data, we have to note that this downside may still be better than assuming that portfolio returns are normally distributed. That said, to provide a counter-check to the historical VaR estimates, we proceeded to set up a Monte Carlo simulation. We also chose to focus on the "C" portfolios since they contain non-linear assets (i.e. covered calls) and we were expecting to see the most significant difference due to the non-linearity of derivative returns.

Setting up a model that will produce robust results is the single most important issue in Monte Carlo simulation. While we were confident that our data will be consistent simply because we were using the same reference points from the previously developed portfolios, we wanted to ensure that all simulated values of the returns will be correlated and dispersed under the same probability density function ("PDF") implied by the historical information. In other words, we need to generate correlated random variables in order to perform Monte Carlo simulations.

This requires a 2-step process which first involves using the Cholesky decomposition of the correlation matrix as follows:

$$a_{i,j} = \frac{1}{a_{j,j}}\left[\rho_{i,j} - \sum_{k=1}^{j-1} a_{j,k}a_{i,k}\right] \text{ when } i > j$$

$$a_{i,i} = \sqrt{1 - \sum_{k=1}^{i-1} a_{i,k}^2} \text{ otherwise}$$

Where:
$a_{i,j}$ = Cholesky Decomposition Coefficients
$\rho_{i,j}$ = Correlation between i^{th} and j^{th} Securities

Once the Cholesky Decomposition coefficients have been computed, it is relatively straightforward to perform the second step (i.e. to generate correlated random variables) using the following equation:

$$Z(i) = \sum_{j=1}^{i} a_{i,j}\varepsilon_j$$

Where:
Z(i) = Correlated Random Variable for the i^{th} Security
ε_j = Uncorrelated Random N(0,1) Variable[8]

To ensure our data was correctly simulated, we ran several goodness-of-fit ("GOF") tests. The distribution with the smallest error was selected for simulation. We used Chi-square for GOF as it most consistently returned p-values in the 0.5 range or greater[9]. We did not use Kolmogorov-Smirnov test because the largest vertical distance between the two cumulative distribution was skewing the results between some distributions that had extreme outliers and the resulting GOFs were not statistically acceptable. Anderson-Darling test was more accurate than Kolmogorov-Smirnov because it placed weights on the differences between the distributions at the tails rather than mid-ranges. This produced a more acceptable GOF but due to the consistently high Chi-square and given wide acceptability of such tests in statistical literature, we decided to proceed with the curves produced in GOF with Chi-square rankings[10].

[8] N(0,1) indicates a normal distribution with zero mean and standard deviation of one.
[9] Generally, such p-values in Chi-square indicate a close fit.
[10] We acknowledge that our Monte Carlo simulation used was built on GOF curves that imply returns on certain asset classes are normally distributed. However, given that the simulation separately derives the theoretical value of the option price for use in derivative valuation, this should compensate for the non-linearity of the portfolio C returns.

In principle, all our GOF tests yielded normal PDFs except for the bond simulations which were based on Extreme Value Distribution and Gamma Distribution (see Appendix IX). While Weibull and Logistic distributions were close seconds to many of the distributions, we suspect this is due to the small dataset and decided not to pursue these findings further.

To run the Monte Carlo simulation, we created 10,000 trials using the aforementioned distributions and *historical* correlations. Weekly returns on the equity and bond portions but not the derivative portions of the portfolio were simulated. While the GOF tests recognized the non-linearity of derivative returns, it was not clear to us whether repeated simulations would force those returns in a more recognizable pattern. As such, using the returns on equities and bonds, we could indirectly generate the returns of derivative positions in the portfolio by computing the prices of the underlying equities and bonds, and then computing the prices of the derivatives.

The Monte Carlo VaR simulations ("Monte Carlo VaR") yielded results that were closely aligned with our initial expectations (see Figures 31-34). Again, as targeted returns increase, the difference in risk estimation between with Monte Carlo VaR and normal VaR also increases.

In portfolios 1C and 2C, we found that relative to Monte Carlo estimates, historical VaR estimates had overestimated the magnitude of the losses at the 99% confidence level. On the other hand, in portfolios 3C and 4C, the results were opposite in direction. Monte Carlo VaR loss estimates were higher than those of historical VaR. While we cannot conclude whether VaR generated by historical data would consistently be lower or higher than VaR generated by Monte Carlo simulations[11], it is clear that computing VaR based on the assumption that returns are normal is dangerous. The magnitude of the difference in VaR estimates depends on the target portfolio returns and type of asset classes in the portfolios.

Monte Carlo simulation has provided us with an alternative to computing VaR based on historical data. However, this is not to say that limited historical data precludes us from drawing conclusions on which method is more applicable in measuring portfolio risks. It appears that assuming portfolio returns are normally distributed is sub-optimal for measuring VaR regardless of the portfolio composition. VaR computed based on Monte Carlo simulations and historical data would provide a more accurate estimate of the potential for loss.

[11] One reason for the differences in estimates could be due to the fact that we did not model stock jumps in the Monte Carlo simulations. Incorporating stock jumps in Monte Carlo simulations would likely lead to a higher loss magnitude at the 99% confidence level.

3. CONCLUSIONS

Our results have shown that there is no general rule that can be used to answer the question of how many assets are required to form a well-diversified portfolio. Our approach in trying to answer this question has highlighted that the diversification benefits of adding more securities to a portfolio cannot be asked in isolation. The *precise* diversification benefits would depend on investors' characteristics such as portfolio target returns, risk appetite and level of aggressiveness in the portfolio as follows:

(a) *Portfolio Target Returns.* In general, diversification benefits increase as investors increase their target returns. However, the extent of the diversification benefits is not uniform across asset classes or asset universes. In addition, solely focusing on target returns ignores the corresponding risk level which should be taken into account. As such, using target returns as a basis to quantify risk reduction benefits is imprecise and insufficient. The positive relationship between target returns and diversification benefits is at best a rule of thumb for investors.

(b) *Risk Appetite.* An investor who has a lower risk aversion coefficient (i.e. is more aggressive) enjoys greater diversification benefits from adding securities to his/her portfolio, relative to an investor who has a higher risk aversion coefficient. However, the extent of benefit depends on the investor's asset universes (e.g. Basket #1 or Basket #2). For example, an investor with a risk aversion coefficient of 2 would enjoy about 400bps in diversification benefits by adding 5 securities if his/her portfolio is constructed from assets in Basket #1. On the other hand, the same investor who chooses to construct his/her portfolio from assets in Basket #2 would enjoy diversification benefits ranging from 200-600bps, depending on the type of asset classes in his/her portfolio.

(c) *Aggressiveness.* Unlike portfolio target returns and risk aversion, aggressiveness of portfolios is an observable characteristic since one can easily observe portfolio asset weights. Our results have shown that an investor starting off with a more aggressive base portfolio enjoys greater benefit from adding securities to his/her portfolio. While this held consistently across different asset classes (Stocks Only, Stocks & Bonds and Covered Calls), as well as different asset universes (Basket #1 and Basket #2), the exact diversification benefits again vary across different asset classes and universes.

In addition, since portfolio optimization is based on looking at standard deviation as a measure of risk, we also attempted to assess whether the use of VaR would provide additional useful risk information. Our results show that investors should compute VaR of their mean-variance optimized portfolio to have a greater confidence in their risk exposure, especially when the tolerance for risk is low. In particular, investors need to be aware of the following:

(a) In most cases, it is dangerous to assume that portfolio returns are normally distributed in computing VaR, even when the portfolios contain equities only. At best, this assumption may suffice for equity-only portfolios that have lower target returns. However, it would break down when the assets within the portfolios are re-allocated to increase the target returns, and when bonds are introduced into the portfolios.

(b) The normal assumption would not hold when investors have derivatives in their portfolios. In our controlled experiment environment, VaR generated based on historical returns appears to produce a much more robust set of results for asset classes with non-linear returns but it may suffer from historical data limitations. Using Monte Carlo simulations is one way to compensate for insufficient historical data, and provide a counter-check on the results generated by using historical data.

In summary, there is no specific answer to the question “what is the benefit of adding more securities to a portfolio” per se, since the levels and extents of benefit varies depending on the asset classes and universes concerned, and investors’ characteristics. However, we believe our work provides a clear methodology and framework for assessing and quantifying the diversification benefits that can be applied to different situations. It has also proven that there is strong positive relationship between targeted returns, risk appetite, base portfolio aggressiveness, and the benefit of adding securities to a portfolio. Finally, for certain types of portfolios, it is essential for investors to look at VaR to supplement standard deviation as a risk measure.

FIGURES

Figure 1: Efficient Frontiers of Portfolio 1A (5 Equities) vs. Portfolio 2A (10 Equities)

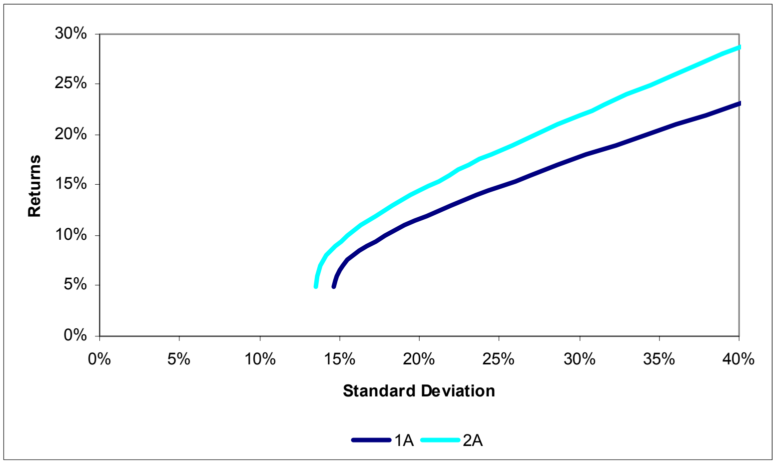


Figure 2: Efficient Frontiers of Portfolio 1B (5 Equities+ 2 Bonds) vs. Portfolio 2B (10 Equities+ 2 Bonds)

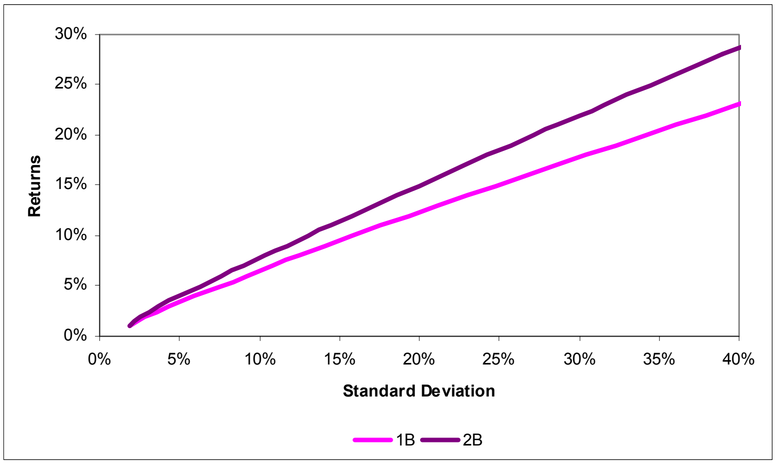


Figure 3: Efficient Frontiers of Portfolio 1C (5 Covered Calls) vs. Portfolio 2C (10 Covered Calls)

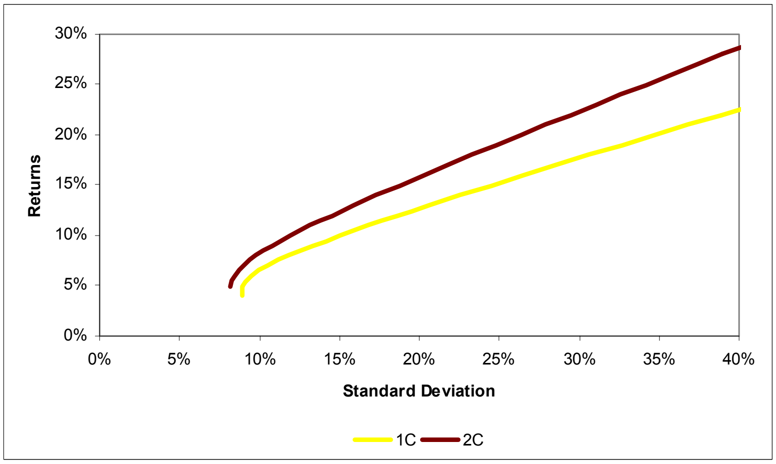


Figure 4: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(2A) - R^{CE}(1A)$)

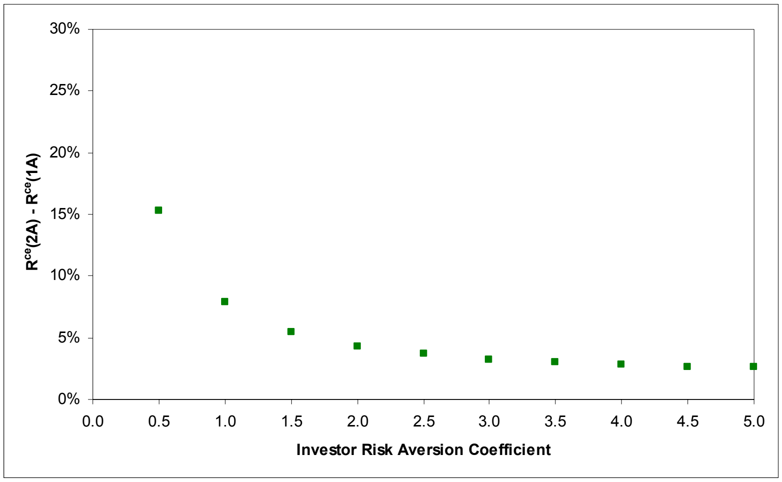


Figure 5: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(2B) - R^{CE}(1B)$)

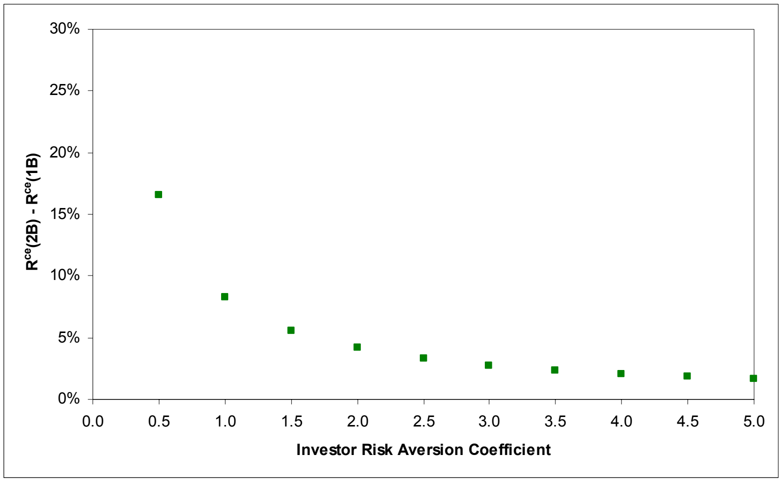


Figure 6: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(2C) - R^{CE}(1C)$)

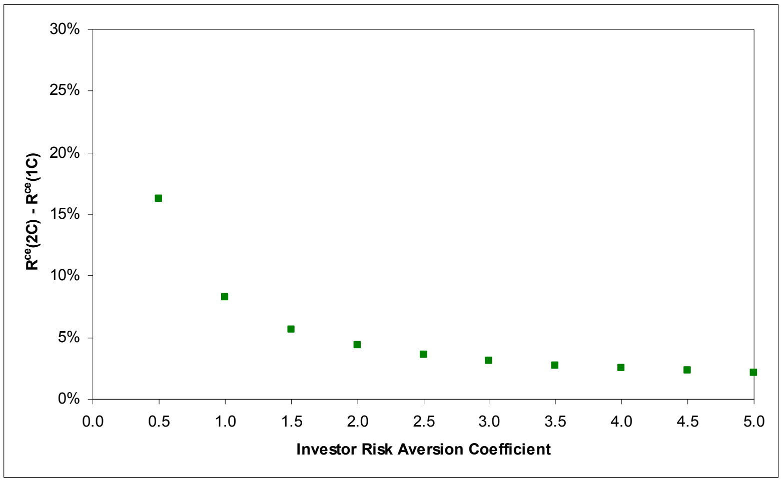


Figure 7: Aggressiveness of Portfolio 1A vs. % Point Risk Reduction (SD(1A) - SD(2A))

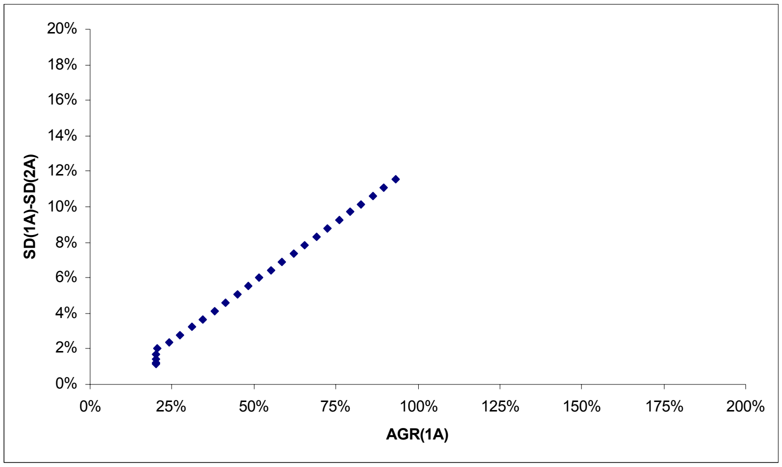


Figure 8: Aggressiveness of Portfolio 1B vs. % Point Risk Reduction (SD(1B) - SD(2B))

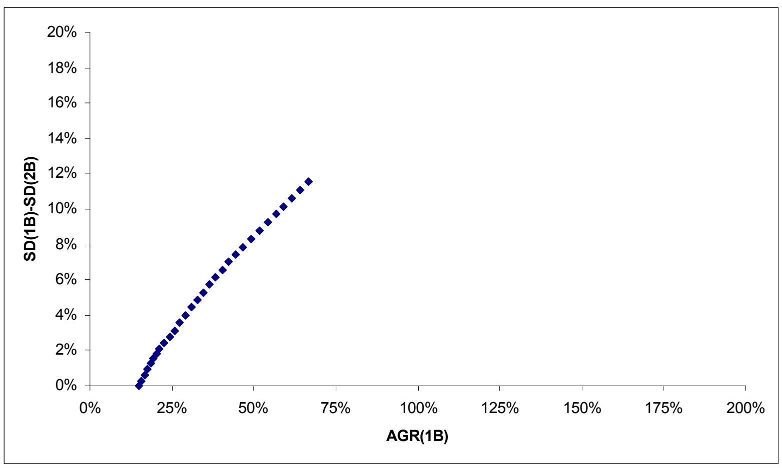


Figure 9: Aggressiveness of Portfolio 1C vs. % Point Risk Reduction (SD(1C) - SD(2C))

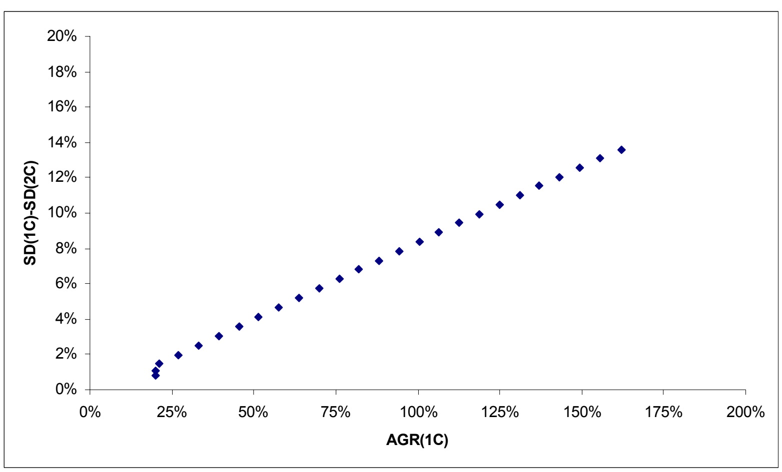


Figure 10: Efficient Frontiers of Portfolio 3A (5 Equities) vs. Portfolio 4A (10 Equities)

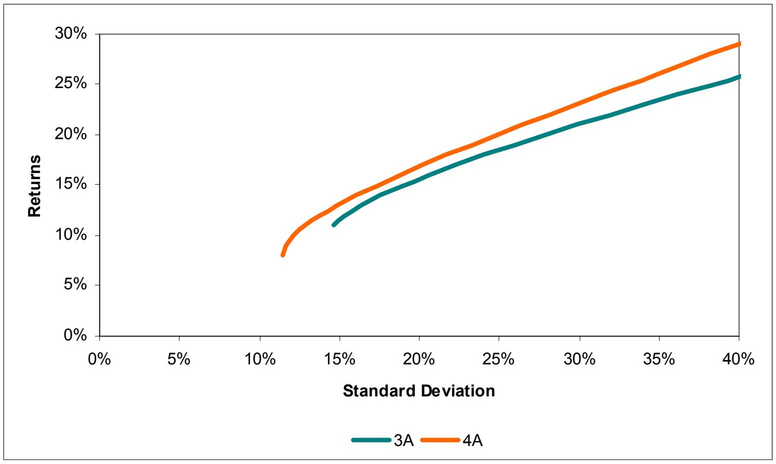


Figure 11: Efficient Frontiers of Portfolio 3B (5 Equities+ 2 Bonds) vs. Portfolio 4B (10 Equities+ 2 Bonds)

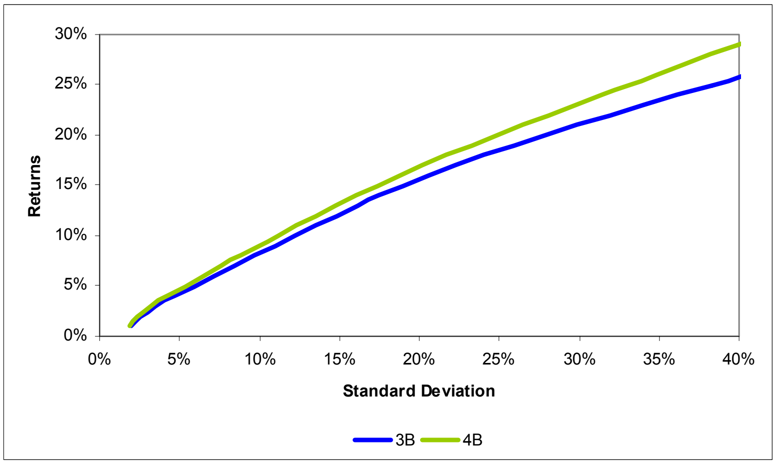


Figure 12: Efficient Frontiers of Portfolio 3C (5 Covered Calls) vs. Portfolio 4C (10 Covered Calls)

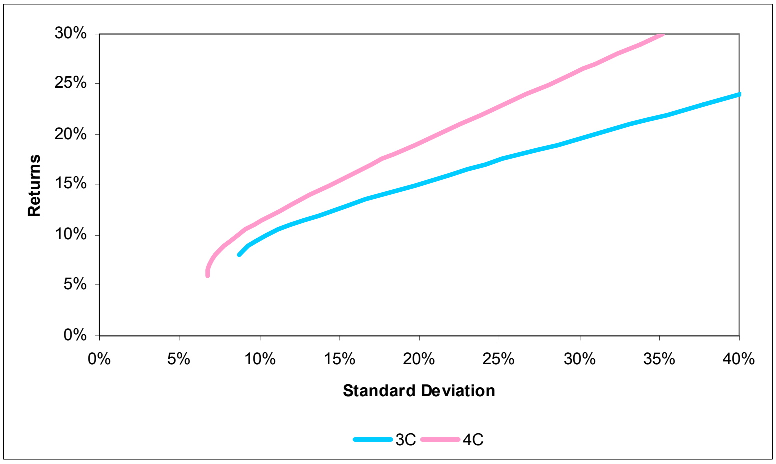


Figure 13: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(4A)$ - $R^{CE}(3A)$)

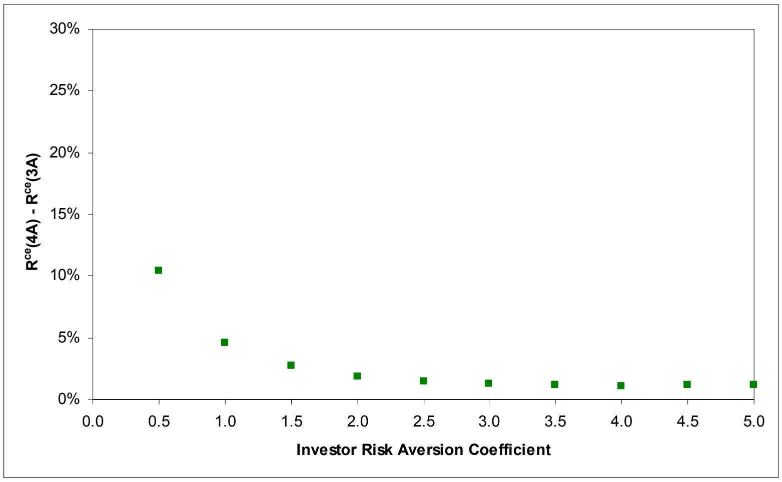


Figure 14: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(4B)$ - $R^{CE}(3B)$)

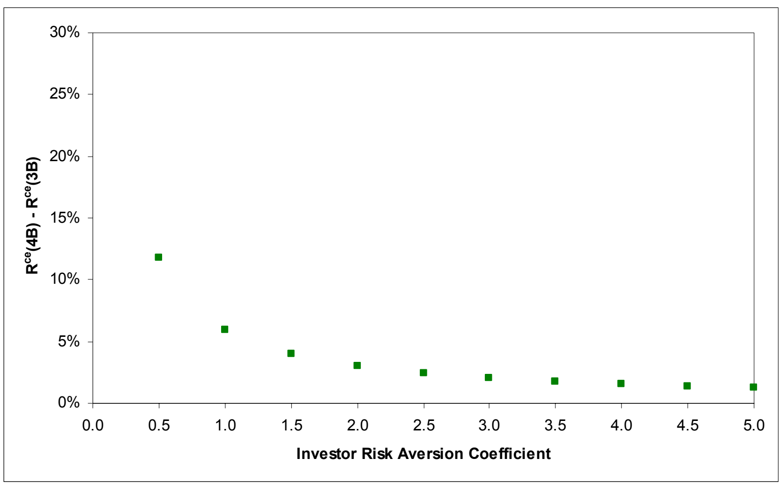


Figure 15: Investor Risk Aversion Coefficient vs. Benefit of Additional Securities ($R^{CE}(4C)$ - $R^{CE}(3C)$)

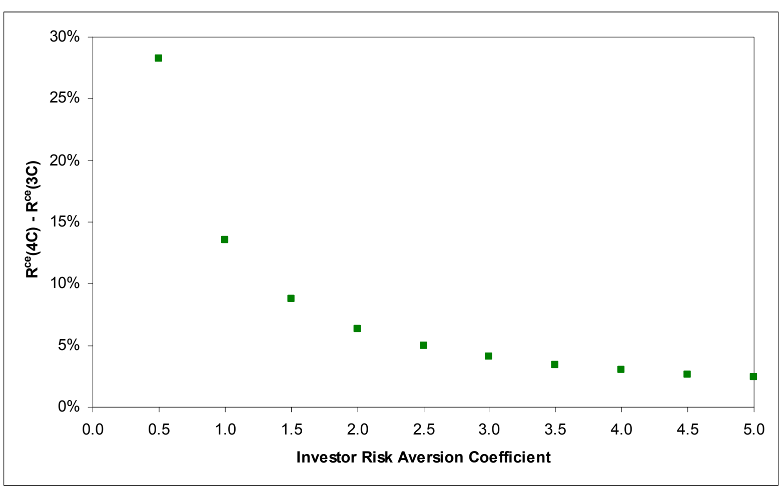


Figure 16: Aggressiveness of Portfolio 3A vs. % Point Risk Reduction (SD(3A) - SD(4A))

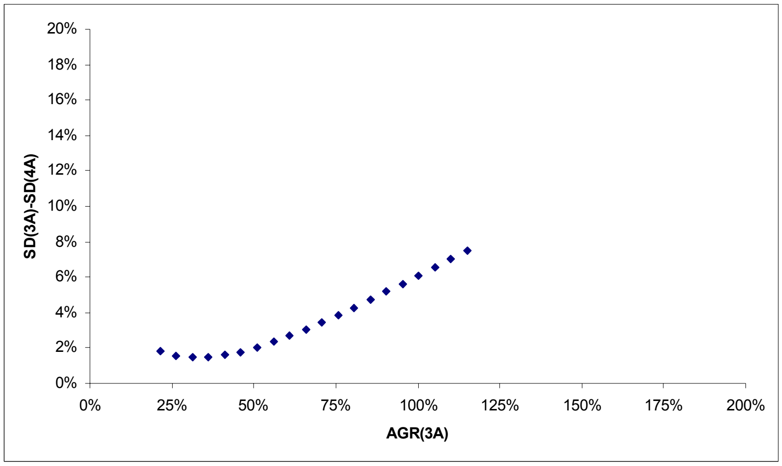


Figure 17: Aggressiveness of Portfolio 3B vs. % Point Risk Reduction (SD(3B) - SD(4B))

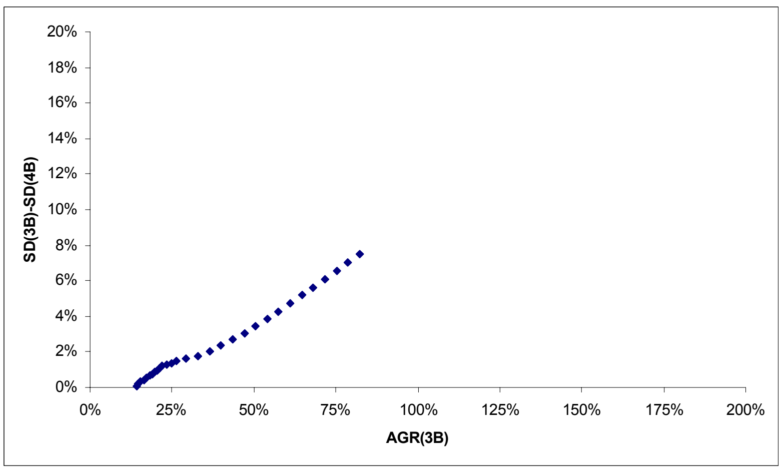


Figure 18: Aggressiveness of Portfolio 3C vs. % Point Risk Reduction (SD(3C) - SD(4C))

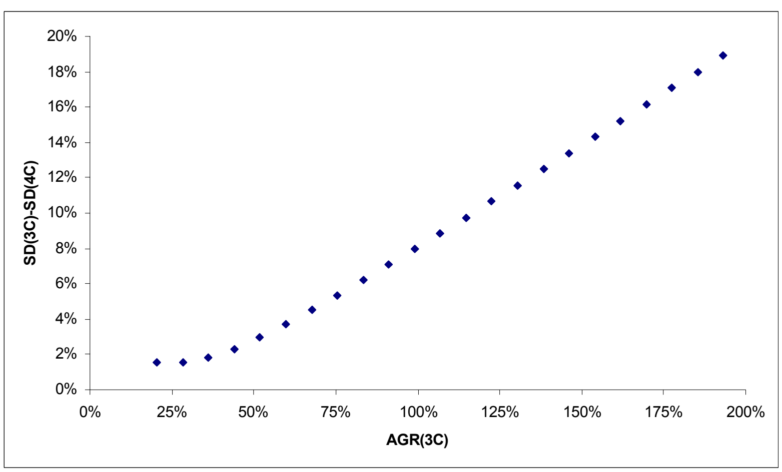


Figure 19: Expected Weekly Loss for Portfolio 1A at 99% Confidence Level

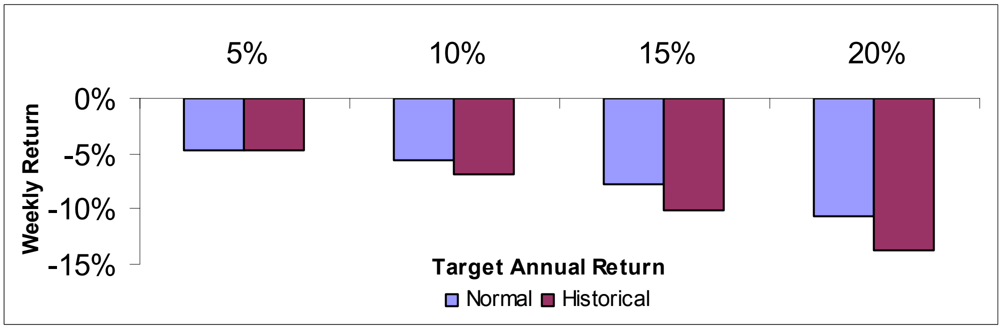


Figure 20: Expected Weekly Loss for Portfolio 2A at 99% Confidence Level

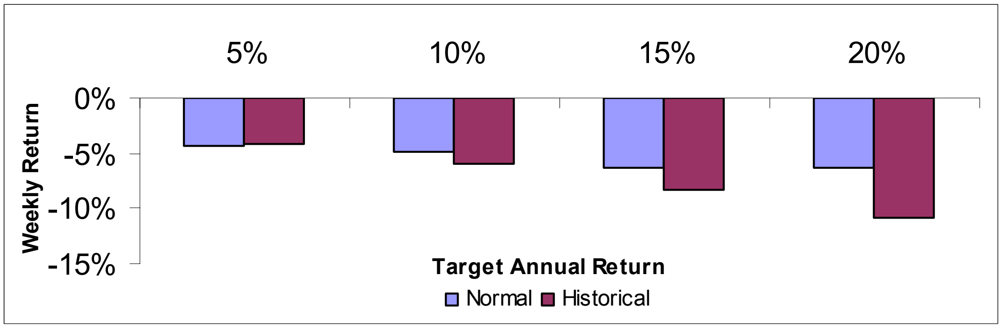


Figure 21: Expected Weekly Loss for Portfolio 3A at 99% Confidence Level

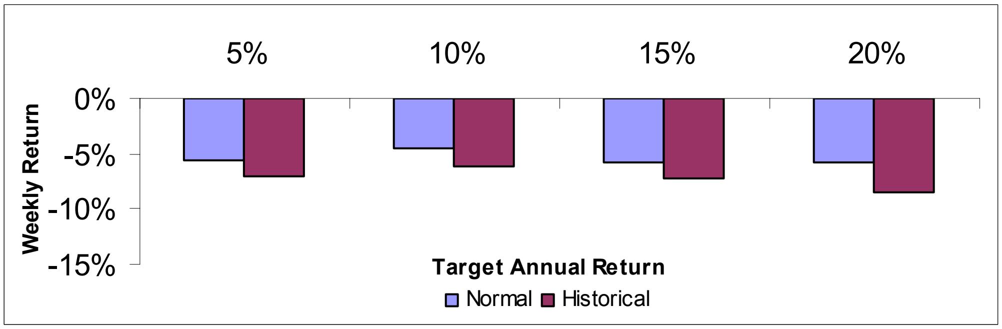


Figure 22: Expected Weekly Loss for Portfolio 4A at 99% Confidence Level

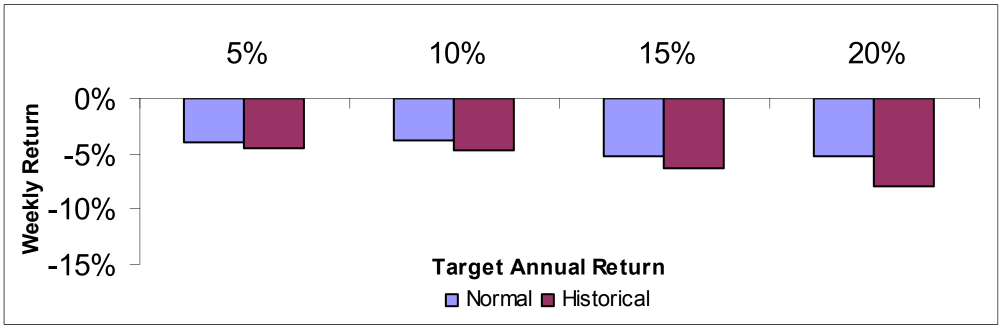


Figure 23: Expected Weekly Loss for Portfolio 1B at 99% Confidence Level

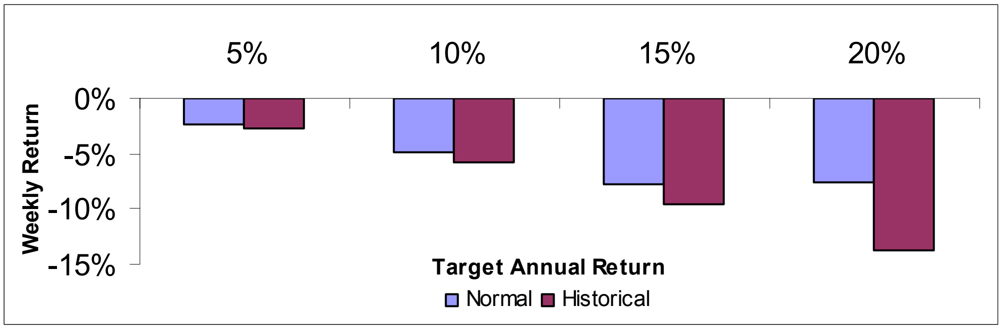


Figure 24: Expected Weekly Loss for Portfolio 2B at 99% Confidence Level

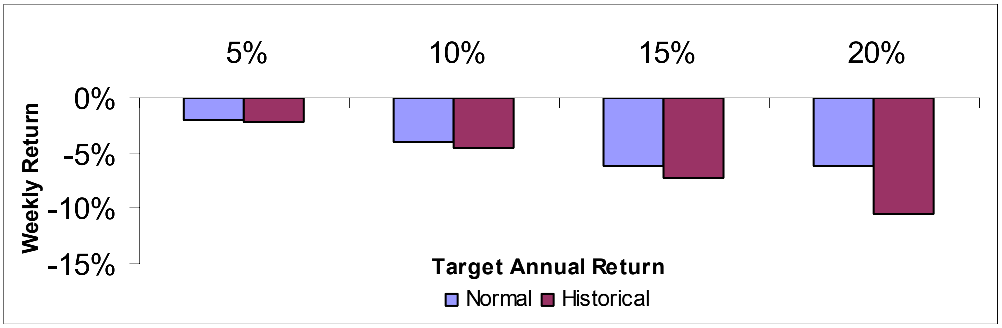


Figure 25: Expected Weekly Loss for Portfolio 3B at 99% Confidence Level

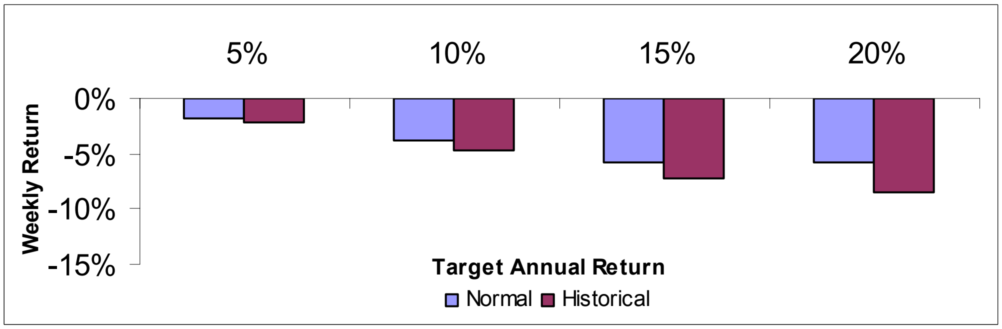


Figure 26: Expected Weekly Loss for Portfolio 4B at 99% Confidence Level

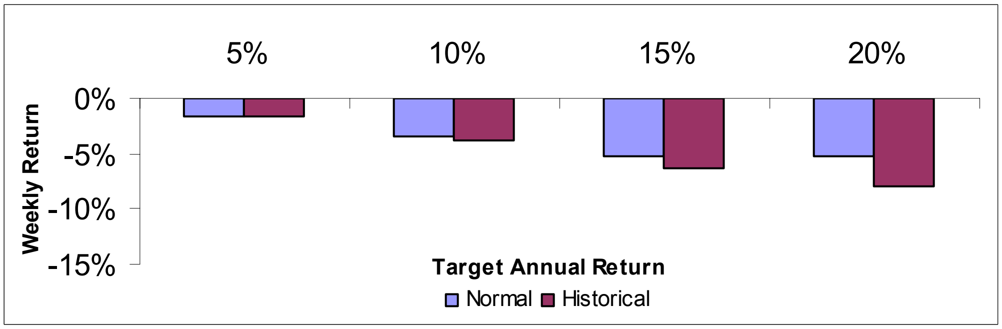


Figure 27: Expected Weekly Loss for Portfolio 1C at 99% Confidence Level

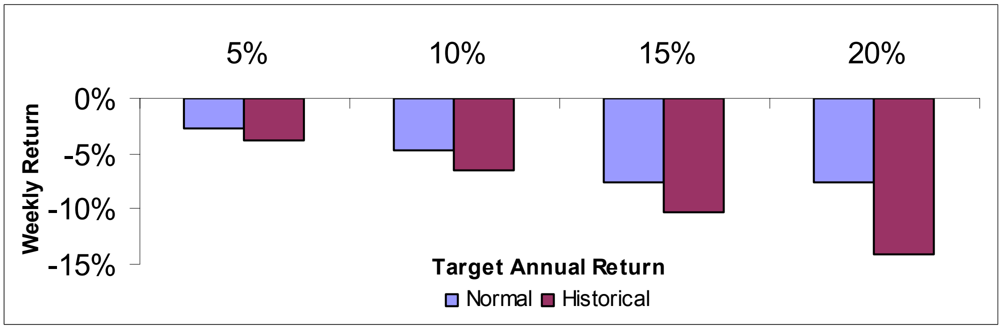


Figure 28: Expected Weekly Loss for Portfolio 2C at 99% Confidence Level

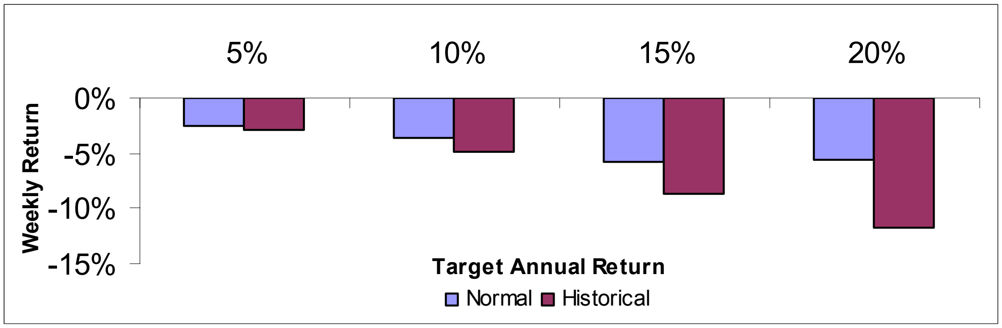


Figure 29: Expected Weekly Loss for Portfolio 3C at 99% Confidence Level

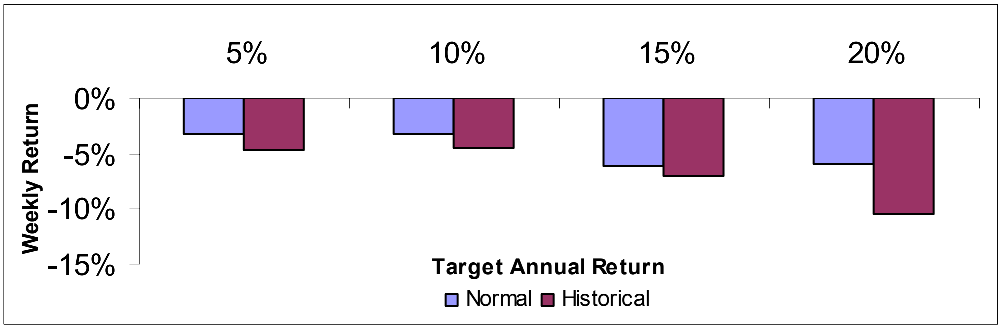


Figure 30: Expected Weekly Loss for Portfolio 4C at 99% Confidence Level

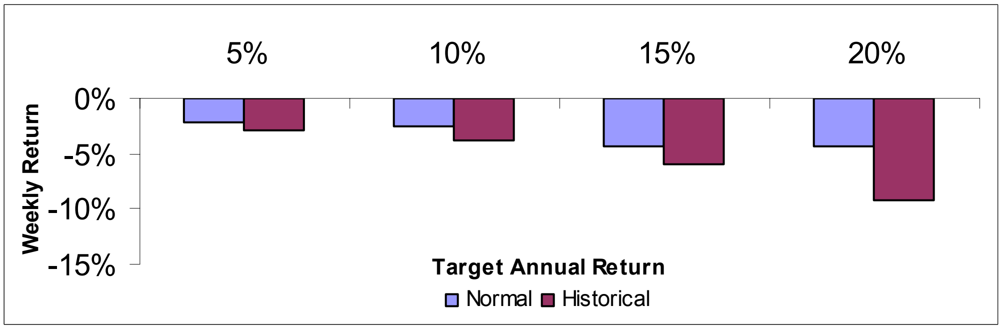


Figure 31: Expected Weekly Loss for Portfolio 1C at 99% Confidence Level

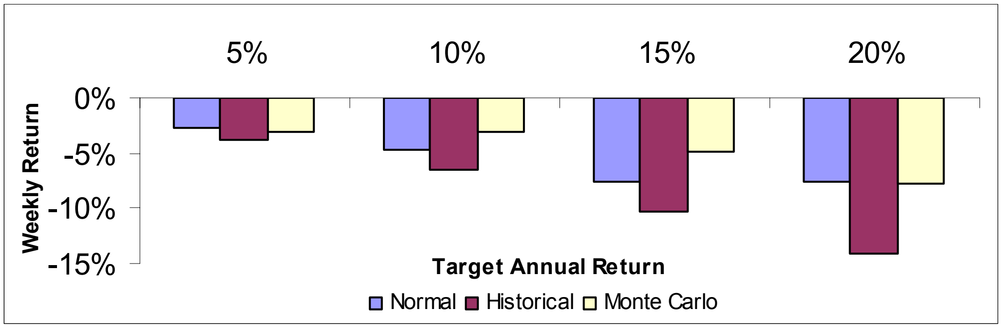


Figure 32: Expected Weekly Loss for Portfolio 2C at 99% Confidence Level

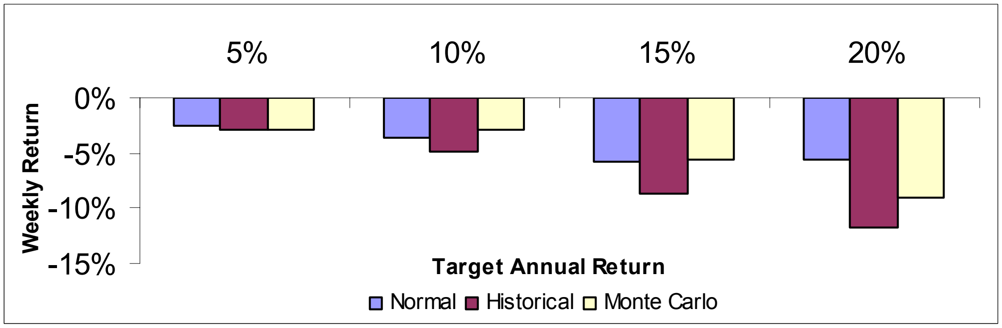


Figure 33: Expected Weekly Loss for Portfolio 3C at 99% Confidence Level

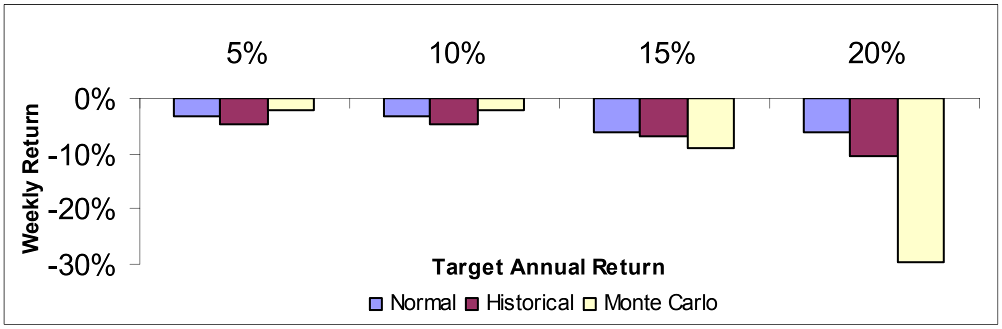
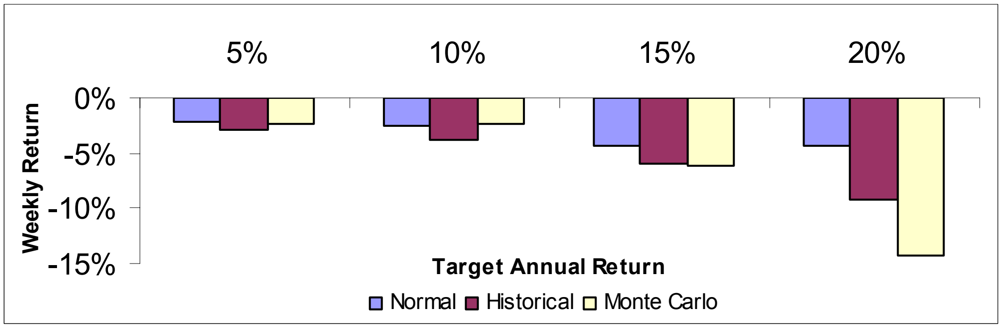


Figure 34: Expected Weekly Loss for Portfolio 4C at 99% Confidence Level



APPENDICES

Appendix I: Asset Legend

Asset Name	Legend
Equities	
DJ US Basic Materials	DJUSBM
DJ US Consumer Cyclical	DJUSCY
DJ US Consumer Non-Cyclical	DJUSNC
DJ US Energy	DJUSEN
DJ US Healthcare	DJUSHC
DJ US Real Estate	DJUSRE
DJ US Transportation	TRAN
GS Semiconductor	GSM
GS Software	GSO
MSCI Australia	MSDUAS
MSCI Brazil	MSEUSBR
MSCI Canada	MSDUCA
MSCI Japan	MSDUJN
MSCI Pacific Ex-Japan	MSDUP
MSCI South Africa	MSEUSSA
NASDAQ Biotech	CXBT
Russell 2000	RTY
S&P 500 / Barra Growth	SGX
S&P 500 / Barra Value	SVX
S&P Europe 350	SPEU
Bonds	
2-Year Government Treasury	GT2
10-Year Government Treasury	GT10
Derivatives	
Covered Calls on DJ US Basic Materials	DJUSBMcc
Covered Calls on DJ US Consumer Cyclical	DJUSCYcc
Covered Calls on DJ US Consumer Non-Cyclical	DJUSNCcc
Covered Calls on DJ US Energy	DJUSENcc
Covered Calls on DJ US Healthcare	DJUSHCcc
Covered Calls on DJ US Real Estate	DJUSREcc
Covered Calls on DJ US Transportation	TRANcc
Covered Calls on GS Semiconductor	GSMcc
Covered Calls on GS Software	GSOcc
Covered Calls on MSCI Australia	MSDUAScc
Covered Calls on MSCI Brazil	MSEUSBRcc
Covered Calls on MSCI Canada	MSDUCAcc
Covered Calls on MSCI Japan	MSDUJNcc
Covered Calls on MSCI Pacific Ex-Japan	MSDUPcc
Covered Calls on MSCI South Africa	MSEUSSAcc
Covered Calls on NASDAQ Biotech	CXBTcc
Covered Calls on Russell 2000	RTYcc
Covered Calls on S&P 500 / Barra Growth	SGXcc
Covered Calls on S&P 500 / Barra Value	SVXcc
Covered Calls on S&P Europe 350	SPEUcc

Appendix II: Portfolio Composition

	Basket #1 (Broad-Based)		Basket #2 (Sector-Specific)	
	Portfolio 1A	**Portfolio 2A**	**Portfolio 3A**	**Portfolio 4A**
A-series Portfolios (Equities Only)	• SGX • SVX • RTY • SPEU • MSDUJN	• SGX • SVX • RTY • SPEU • MSDUJN • MSDUP • MSDUCA • MSEUSBR • MSDUAS • MSEUSSA	• DJUSBM • DJUSCY • DJUSNC • DJUSHC • DJUSEN	• DJUSBM • DJUSCY • DJUSNC • DJUSHC • DJUSEN • DJUSRE • TRAN • CXBT • GSM • GSO

	Basket #1 (Broad-Based)		Basket #2 (Sector-Specific)	
	Portfolio 1B	**Portfolio 2B**	**Portfolio 3B**	**Portfolio 4B**
B-series Portfolios (Equities & Bonds)	• SGX • SVX • RTY • SPEU • MSDUJN • GT2 • GT10	• SGX • SVX • RTY • SPEU • MSDUJN • MSDUP • MSDUCA • MSEUSBR • MSDUAS • MSEUSSA • GT2 • GT10	• DJUSBM • DJUSCY • DJUSNC • DJUSHC • DJUSEN • GT2 • GT10	• DJUSBM • DJUSCY • DJUSNC • DJUSHC • DJUSEN • DJUSRE • TRAN • CXBT • GSM • GSO • GT2 • GT10

	Basket #1 (Broad-Based)		Basket #2 (Sector-Specific)	
	Portfolio 1C	**Portfolio 2C**	**Portfolio 3C**	**Portfolio 4C**
C-series Portfolios (Covered Calls)	• SGXcc • SVXcc • RTYcc • SPEUcc • MSDUJNcc	• SGXcc • SVXcc • RTYcc • SPEUcc • MSDUJNcc • MSDUPcc • MSDUCAcc • MSEUSBRcc • MSDUAScc • MSEUSSAcc	• DJUSBMcc • DJUSCYcc • DJUSNCcc • DJUSHCcc • DJUSENcc	• DJUSBMcc • DJUSCYcc • DJUSNCcc • DJUSHCcc • DJUSENcc • DJUSREcc • TRANcc • CXBTcc • GSMcc • GSOcc

Appendix III: Detailed Data Collection Methodology

To define measures of risk and benefits of diversification accurately, data gathered for simulations have to be current, unbiased and span a period of at least ten years back. In addition, three asset classes with a wide density of returns to represent a typical pallet of securities available to investors, namely equities, bonds and derivatives, were chosen.

Equities

For our initial dataset, we collected weekly historical prices for twenty equity indices that are currently traded in the form of ETFs. It is important for the securities to be tradable and liquid so that transaction costs can be minimized. Although the ETFs on many of the equity indices did not begin trading until late 1990s, we were able to collect raw index data over the past ten years which was subsequently used as a basis for computing continuously compounded returns.

Bonds

Ideally, we would have included both corporate bonds and Government Treasuries in our portfolios. It was important to gather data from bond indices (such as Lehman or Morgan Stanley indices) rather than bond spreads or historical credit information because index data would incorporate correlations amongst credit spreads and would be a more representative measure of returns available at the time. Unfortunately, such data was unavailable and even after multiple searches we could not obtain reliable data. Whatever data that was available was interspersed with gaps.

Gathering data on Treasury bonds was more straightforward. We collected weekly yields for two to thirty year Treasuries over the past ten years. Yields gathered were for the on-the-run securities and prices were computed by using modified duration and convexity of the current issues. Once the historical prices were determined, we could easily compute the continuously compounded returns.

Derivatives

Collecting data on puts, calls and futures was much more involved and presented a myriad of problems. We adopted a model in which an investor would purchase a derivative security with n days to expiration, which in our project was 90 days months from time t=0. Consequently, at time t+1 (where 1 represents the time format in days and in our case t+1 is 1 week after the purchase date), the investor sells the original derivative and buys a new derivative with n days to expiration. The return can then be computed by taking the natural logarithm of the ratio of sale price to purchase price.

Since derivatives are not delivered weekly in the market, we decided to use the Black-Scholes formula to simulate the theoretical price of derivatives. The implicit assumptions are: (i) liquidity and consequently bid-ask spreads are non-consequential; and (ii) the volatility of the underlying equity does not follow a stochastic process and remains constant. Also, to compute derivative prices, we collected historic dividend yields and volatility or each equity index. The assumption in this case is that the historical dividend yield and volatilities are representative of future dividend yields and volatilities, respectively. While we did not perform extensive sensitivity analysis on the aforementioned assumptions, we do not believe changes in these variables would significantly affect the robustness of our data.

Data Collection

Several other important 'best practices' are worth mentioning in the data collection methodology. For example, if daily data is to be collected, it is imperative to exclude non-working days and holidays from the dataset. If we either leave those days open or transfer the Friday closing price over the weekend, correlation between different asset classes will be significantly affected because approximately 2/7 of the data will have correlation coefficient of 1 which will certainly make results unreliable. Also, to ensure that the correlations of the dataset are semi-positive definite, we must ensure the same number of observations is available for all the data in the dataset. For example, in our case, September 11, 2001, was a Tuesday and a working day throughout the world but tragic events in the US brought the US financial markets to a halt. Hence, US data on the following three days was unavailable. However, data on our international index remained available for all of the above mentioned dates and if these were not removed, we would have erroneously identified a correlation effect that could generate a non-positive-definite covariance matrix.

Simulation of Efficient Frontiers

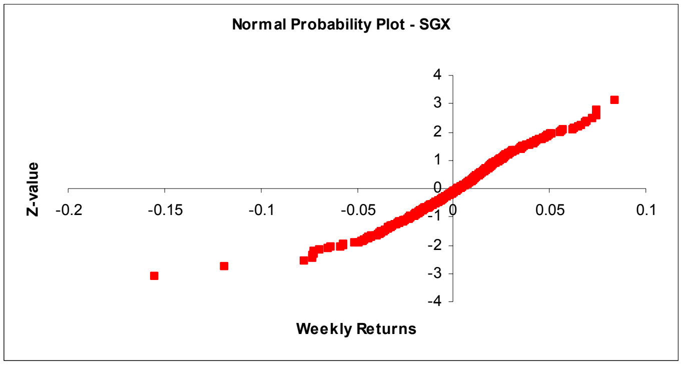
Compared to Excel, MatLab is a more efficient application to simulate optimized portfolios and construct efficient frontiers. Based on the portfolios created, initial results were discouraging. This was because some of the equity indices were a subset of other equity indices data which were also in the same portfolio. For example, both S&P 500 and DJI were included in the same portfolio, which created issues of singularity and forced the optimization algorithm to produce invalid results. We had to re-look at our equity indices and filter out those that were sub-sets of other equity indices. Replacement equity indices had to be carefully selected to avoid the same problem.

Furthermore, the MatLab algorithm allowed investors to short risk-free assets at risk-free rate – an otherwise implausible market scenario. In addition, if the portfolio contained calls, puts and the underlying equities, the MatLab algorithm would end up creating a synthetically risk-free portfolio (based on put-call parity). Alongside this shortcoming, largely negative standard deviations of derivatives forced the algorithm to continually place large short weights on the diametrically opposite positions effectively eliminating the entire benefit of diversification and plotting a payoff of the risk free asset, but without taking into account the bid-ask spread of options.

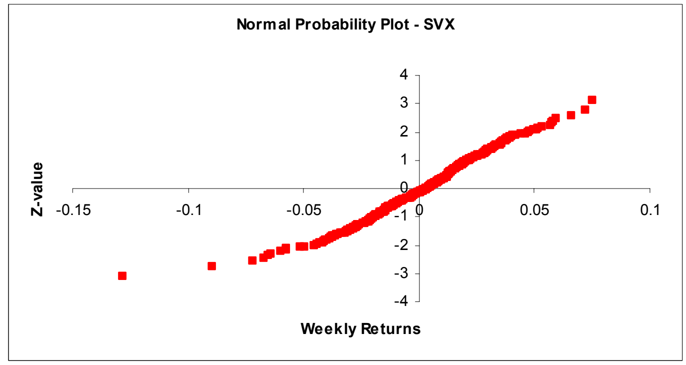
To control for the problems witnessed, we redesigned the MatLab algorithm to prevent shorting of bonds and stop the possibility of synthetic risk free asset creation. Furthermore, we reassembled the portfolios eliminating multi-collinearity among the assets and devised a derivative simulation to include only covered calls based on weekly data points to improve the non-linearity of the expected returns.

Appendix IV: Asset Characteristics

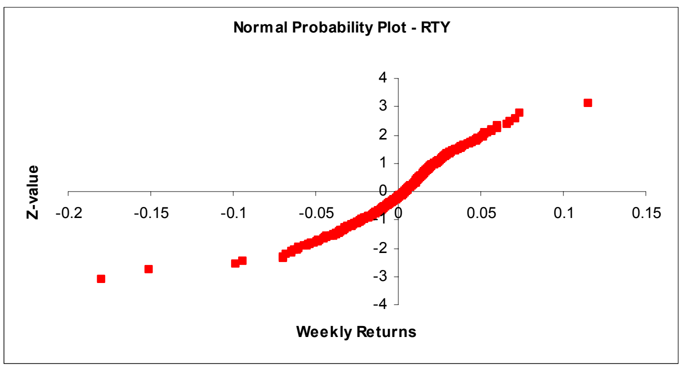
Basket #1 Assets



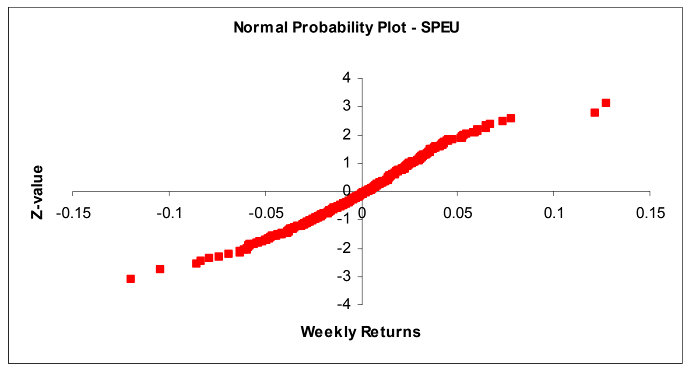
	Weekly	Annual
Minimum Return	-15.488%	-805.357%
Mean Return	0.190%	9.905%
Median Return	0.332%	17.257%
Maximum Return	8.378%	435.657%
Standard Deviation of Returns	2.644%	19.069%



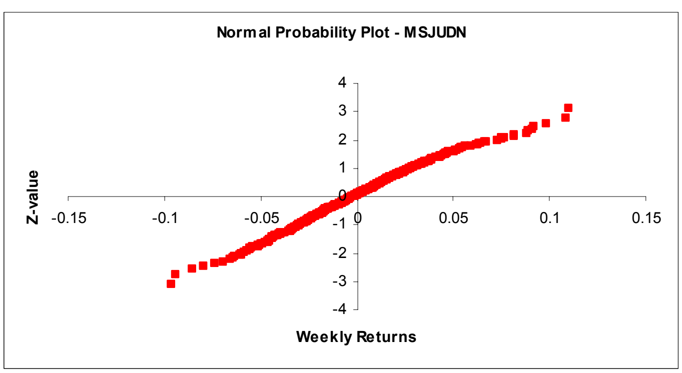
	Weekly	Annual
Minimum Return	-12.829%	-667.103%
Mean Return	0.161%	8.364%
Median Return	0.361%	18.786%
Maximum Return	7.512%	390.613%
Standard Deviation of Returns	2.304%	16.618%



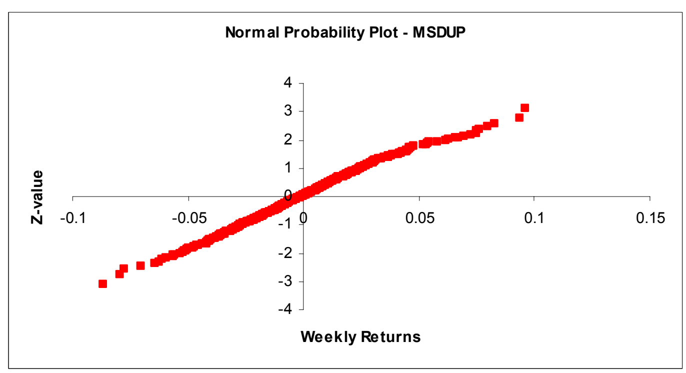
	Weekly	Annual
Minimum Return	-17.961%	-933.975%
Mean Return	0.166%	8.610%
Median Return	0.448%	23.292%
Maximum Return	11.484%	597.176%
Standard Deviation of Returns	2.714%	19.571%



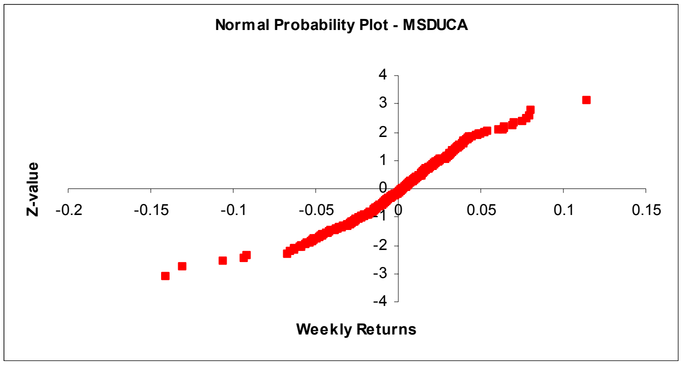
	Weekly	Annual
Minimum Return	-11.915%	-619.576%
Mean Return	0.142%	7.407%
Median Return	0.329%	17.122%
Maximum Return	12.775%	664.324%
Standard Deviation of Returns	2.824%	20.364%

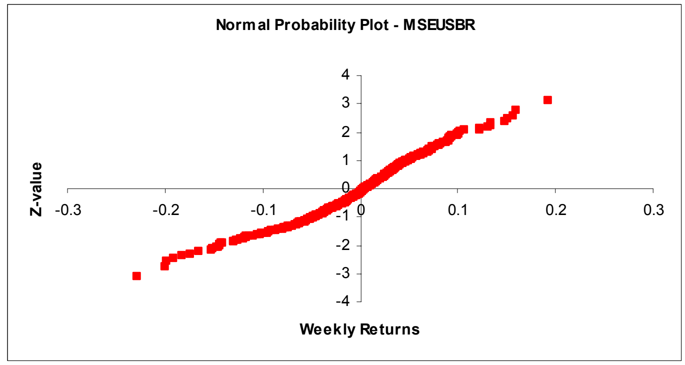


	Weekly	Annual
Minimum Return	-9.605%	-499.465%
Mean Return	-0.054%	-2.818%
Median Return	-0.183%	-9.506%
Maximum Return	11.016%	572.823%
Standard Deviation of Returns	3.123%	22.518%



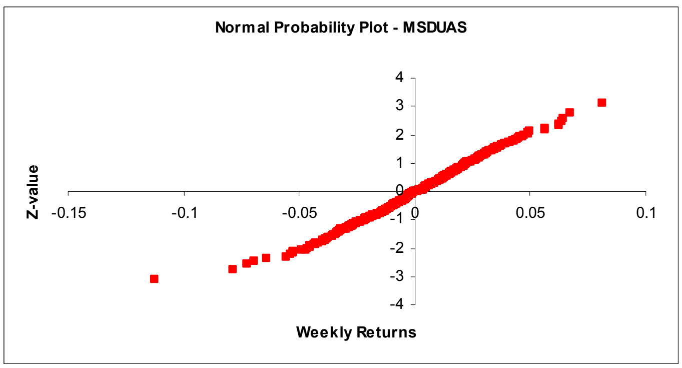
	Weekly	Annual
Minimum Return	-8.660%	-450.310%
Mean Return	-0.039%	-2.032%
Median Return	-0.103%	-5.373%
Maximum Return	9.636%	501.076%
Standard Deviation of Returns	2.725%	19.649%

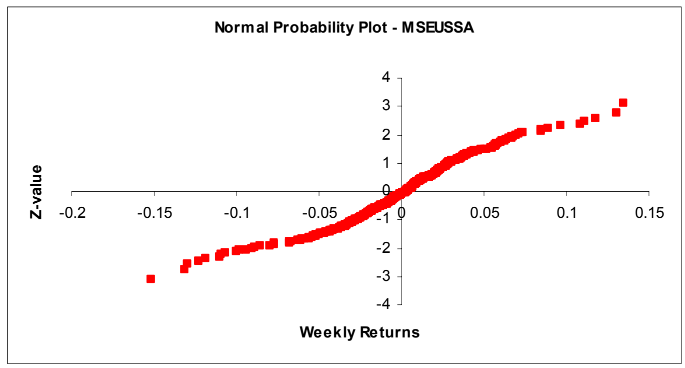




	Weekly	Annual
Minimum Return	-14.063%	-731.301%
Mean Return	0.179%	9.310%
Median Return	0.317%	16.487%
Maximum Return	11.457%	595.765%
Standard Deviation of Returns	2.756%	19.875%

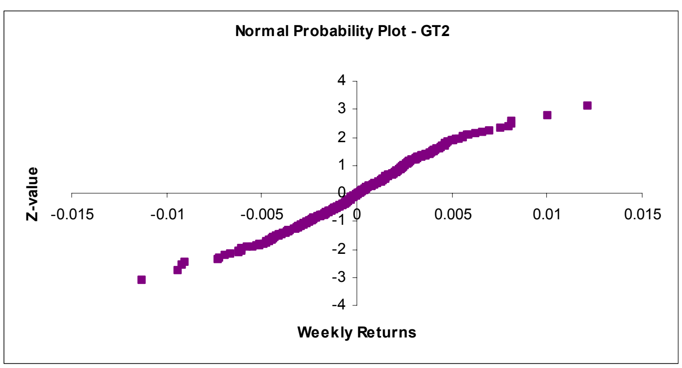
	Weekly	Annual
Minimum Return	-22.879%	-1189.718%
Mean Return	0.059%	3.092%
Median Return	0.484%	25.182%
Maximum Return	19.257%	1001.371%
Standard Deviation of Returns	5.725%	41.282%

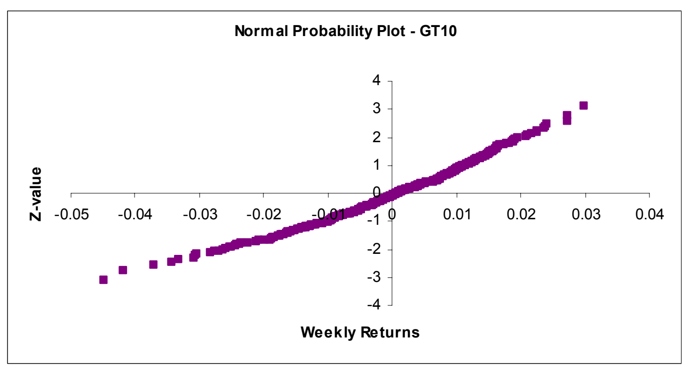




	Weekly	Annual
Minimum Return	-11.230%	-583.962%
Mean Return	0.099%	5.133%
Median Return	0.092%	4.798%
Maximum Return	8.114%	421.914%
Standard Deviation of Returns	2.375%	17.125%

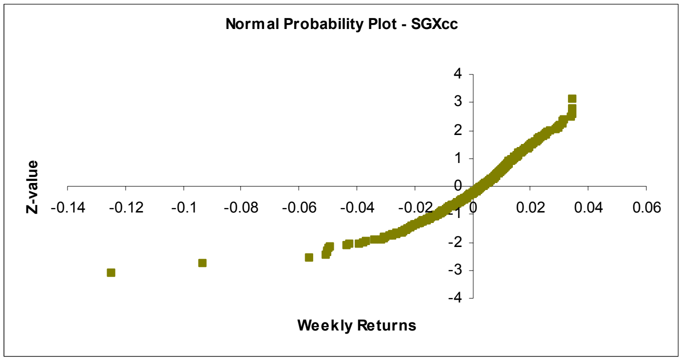
	Weekly	Annual
Minimum Return	-15.168%	-788.747%
Mean Return	0.090%	4.696%
Median Return	0.315%	16.361%
Maximum Return	13.517%	702.861%
Standard Deviation of Returns	3.586%	25.855%



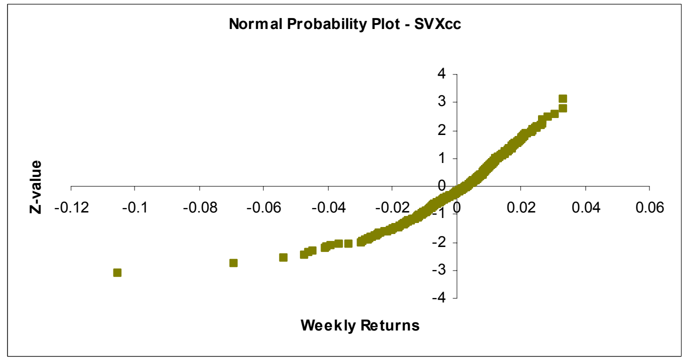


	Weekly	Annual
Minimum Return	-1.130%	-58.741%
Mean Return	0.013%	0.661%
Median Return	0.015%	0.797%
Maximum Return	1.218%	63.316%
Standard Deviation of Returns	0.280%	2.018%

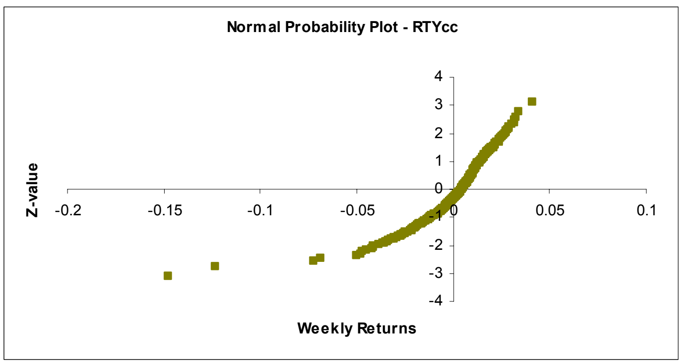
	Weekly	Annual
Minimum Return	-4.480%	-232.982%
Mean Return	0.037%	1.918%
Median Return	0.081%	4.219%
Maximum Return	2.991%	155.518%
Standard Deviation of Returns	1.139%	8.211%



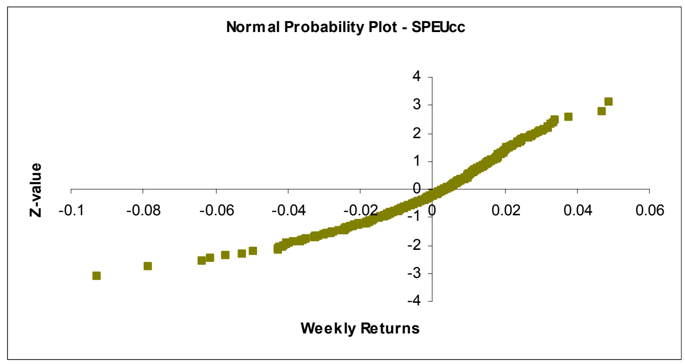
	Weekly	Annual
Minimum Return	-12.453%	-647.535%
Mean Return	0.144%	7.494%
Median Return	0.366%	19.029%
Maximum Return	3.487%	181.311%
Standard Deviation of Returns	1.610%	11.609%



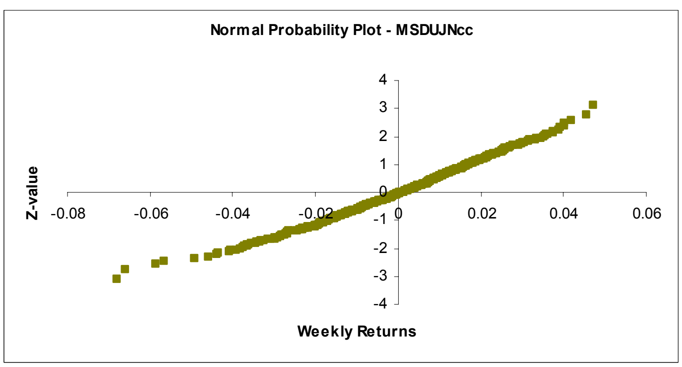
	Weekly	Annual
Minimum Return	-10.527%	-547.407%
Mean Return	0.119%	6.190%
Median Return	0.363%	18.888%
Maximum Return	3.344%	173.911%
Standard Deviation of Returns	1.411%	10.173%



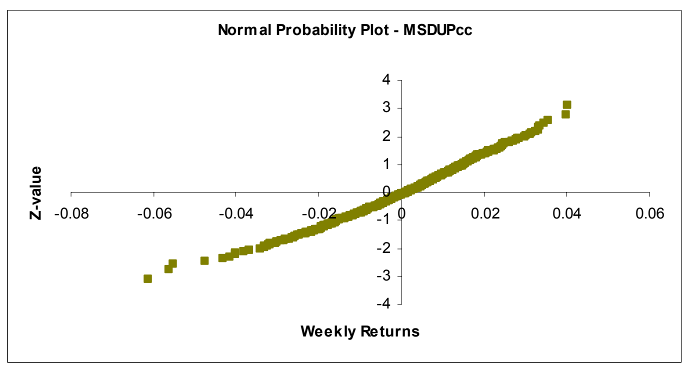
	Weekly	Annual
Minimum Return	-14.781%	-768.603%
Mean Return	0.137%	7.124%
Median Return	0.439%	22.815%
Maximum Return	4.119%	214.198%
Standard Deviation of Returns	1.712%	12.348%



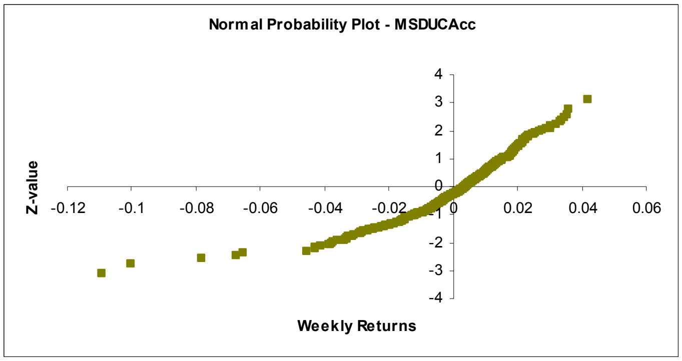
	Weekly	Annual
Minimum Return	-9.280%	-482.557%
Mean Return	0.107%	5.545%
Median Return	0.363%	18.872%
Maximum Return	4.889%	254.208%
Standard Deviation of Returns	1.721%	12.412%



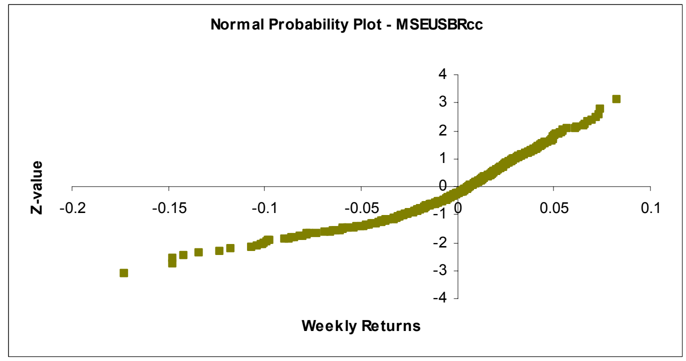
	Weekly	Annual
Minimum Return	-6.798%	-353.510%
Mean Return	0.008%	0.392%
Median Return	0.106%	5.518%
Maximum Return	4.711%	244.994%
Standard Deviation of Returns	1.787%	12.890%



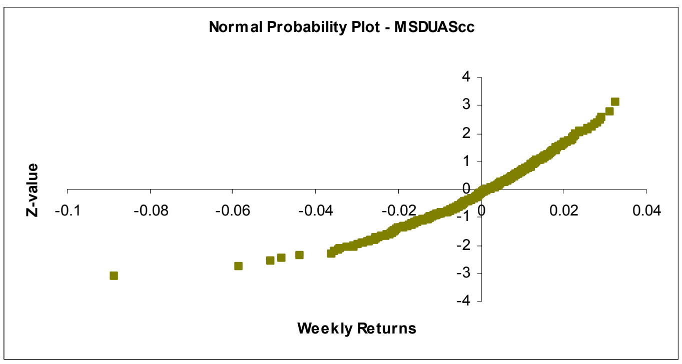
	Weekly	Annual
Minimum Return	-6.128%	-318.669%
Mean Return	0.015%	0.773%
Median Return	0.129%	6.705%
Maximum Return	4.038%	209.980%
Standard Deviation of Returns	1.564%	11.276%



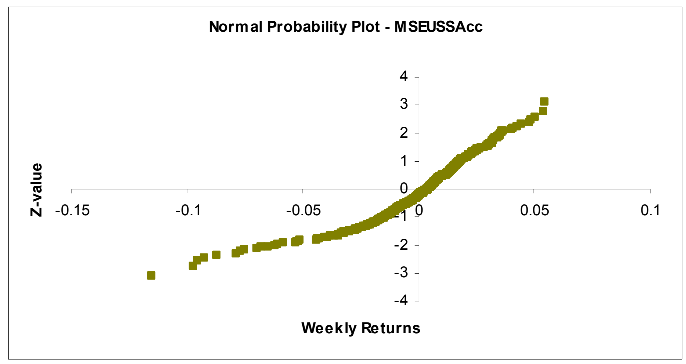
	Weekly	Annual
Minimum Return	-10.938%	-568.754%
Mean Return	0.142%	7.392%
Median Return	0.374%	19.432%
Maximum Return	4.171%	216.882%
Standard Deviation of Returns	1.667%	12.017%



	Weekly	Annual
Minimum Return	-17.296%	-899.383%
Mean Return	0.105%	5.464%
Median Return	0.648%	33.691%
Maximum Return	8.307%	431.967%
Standard Deviation of Returns	3.571%	25.749%

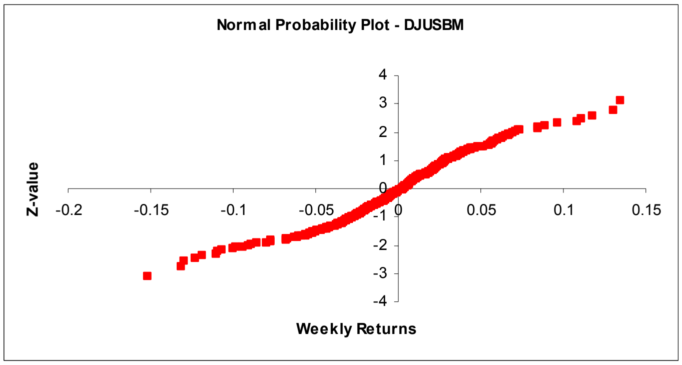


	Weekly	Annual
Minimum Return	-8.851%	-460.273%
Mean Return	0.094%	4.887%
Median Return	0.208%	10.827%
Maximum Return	3.262%	169.634%
Standard Deviation of Returns	1.398%	10.081%

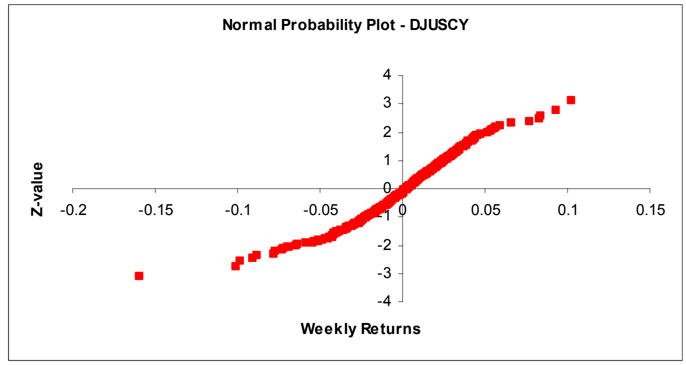


	Weekly	Annual
Minimum Return	-11.545%	-600.338%
Mean Return	0.100%	5.185%
Median Return	0.418%	21.711%
Maximum Return	5.437%	282.731%
Standard Deviation of Returns	2.176%	15.694%

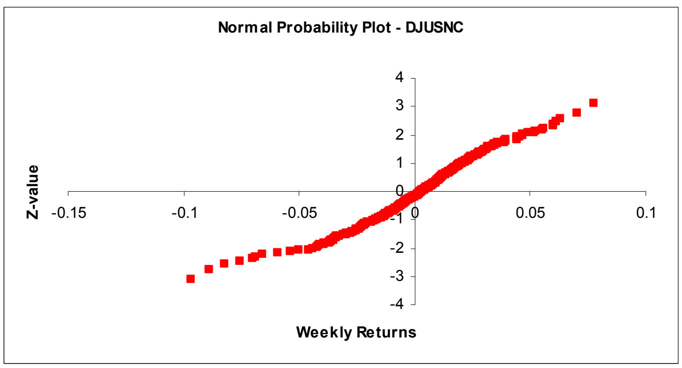
Basket #2 Assets



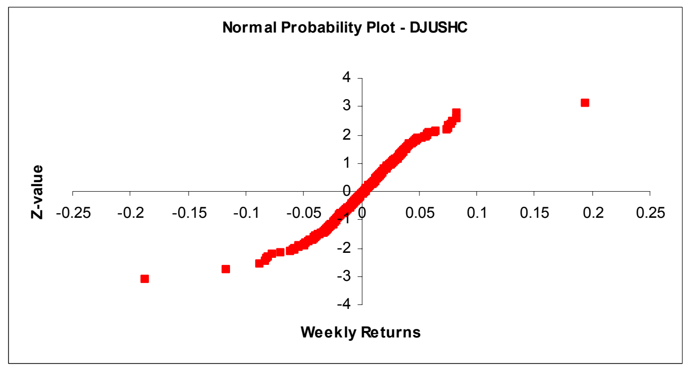
	Weekly	Annual
Minimum Return	-16.329%	-849.090%
Mean Return	0.080%	4.152%
Median Return	0.240%	12.503%
Maximum Return	15.095%	784.945%
Standard Deviation of Returns	2.969%	21.412%



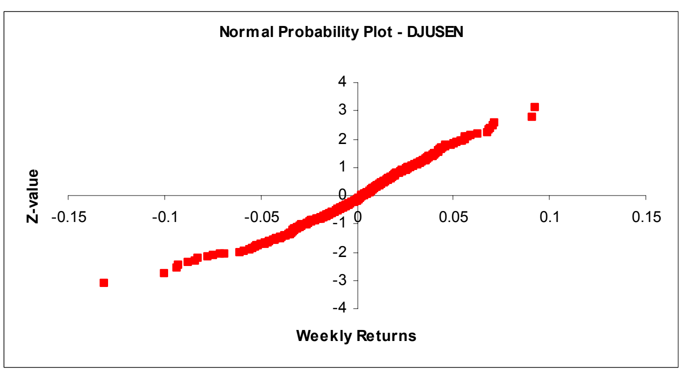
	Weekly	Annual
Minimum Return	-15.969%	-830.383%
Mean Return	0.154%	8.004%
Median Return	0.303%	15.780%
Maximum Return	10.235%	532.216%
Standard Deviation of Returns	2.727%	19.662%



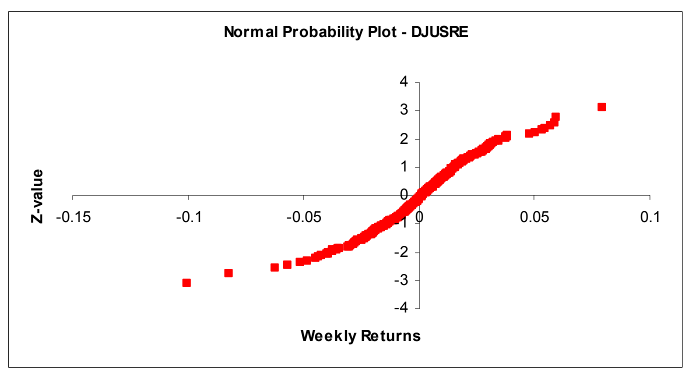
	Weekly	Annual
Minimum Return	-9.656%	-502.127%
Mean Return	0.202%	10.487%
Median Return	0.310%	16.102%
Maximum Return	7.785%	404.802%
Standard Deviation of Returns	2.194%	15.821%



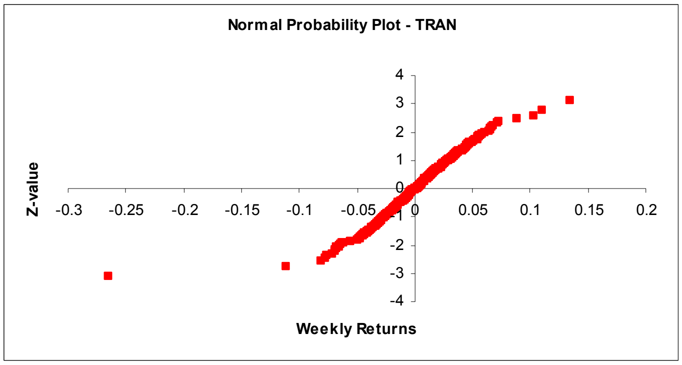
	Weekly	Annual
Minimum Return	-18.748%	-974.908%
Mean Return	0.273%	14.192%
Median Return	0.356%	18.535%
Maximum Return	19.380%	1007.736%
Standard Deviation of Returns	2.872%	20.712%



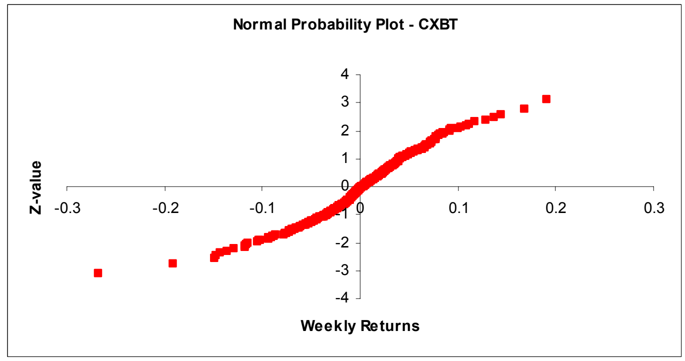
	Weekly	Annual
Minimum Return	-13.118%	-682.127%
Mean Return	0.170%	8.842%
Median Return	0.311%	16.159%
Maximum Return	9.291%	483.146%
Standard Deviation of Returns	2.865%	20.662%



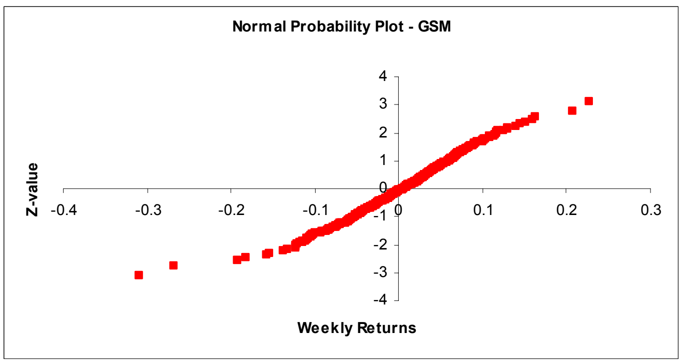
	Weekly	Annual
Minimum Return	-10.019%	-520.988%
Mean Return	0.107%	5.579%
Median Return	0.124%	6.473%
Maximum Return	7.965%	414.159%
Standard Deviation of Returns	1.795%	12.941%



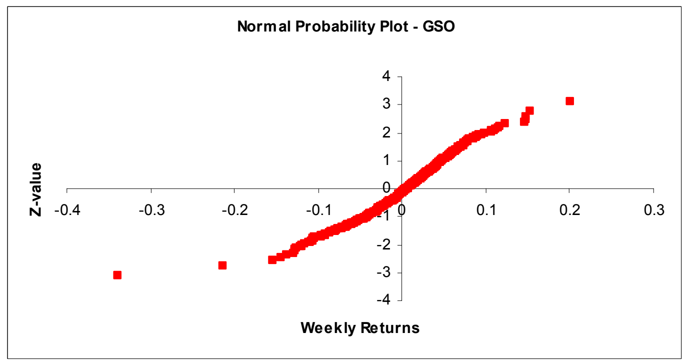
	Weekly	Annual
Minimum Return	-26.431%	-1374.403%
Mean Return	0.114%	5.932%
Median Return	0.130%	6.745%
Maximum Return	13.507%	702.365%
Standard Deviation of Returns	3.170%	22.860%



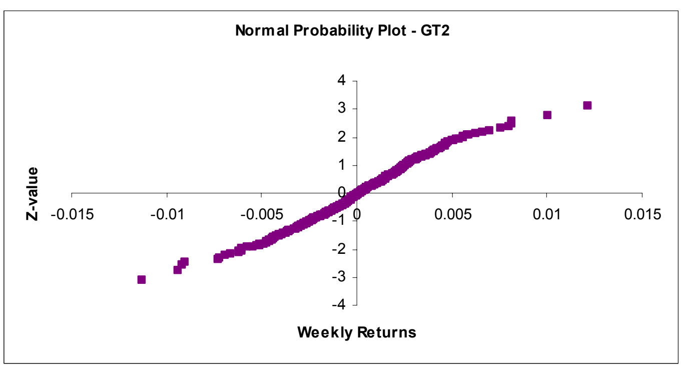
	Weekly	Annual
Minimum Return	-26.811%	-1394.176%
Mean Return	0.310%	16.120%
Median Return	0.284%	14.759%
Maximum Return	19.160%	996.341%
Standard Deviation of Returns	4.730%	34.110%



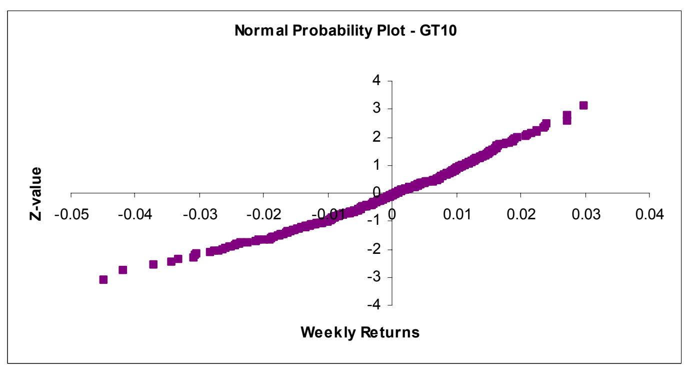
	Weekly	Annual
Minimum Return	-30.976%	-1610.746%
Mean Return	0.278%	14.468%
Median Return	0.583%	30.319%
Maximum Return	22.774%	1184.222%
Standard Deviation of Returns	6.090%	43.917%



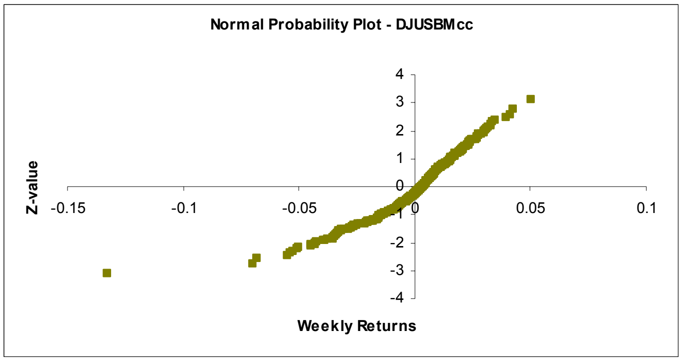
	Weekly	Annual
Minimum Return	-33.972%	-1766.528%
Mean Return	0.247%	12.821%
Median Return	0.526%	27.371%
Maximum Return	20.068%	1043.537%
Standard Deviation of Returns	5.285%	38.107%



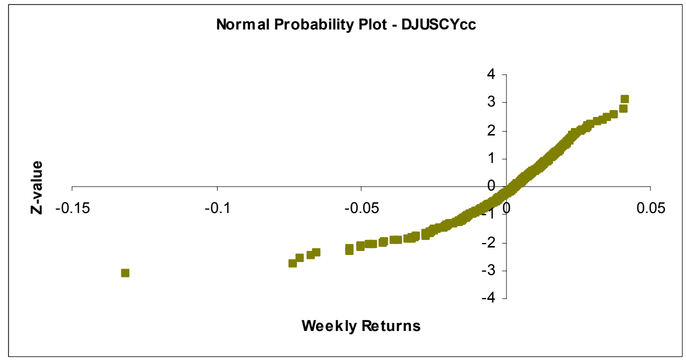
	Weekly	Annual
Minimum Return	-1.130%	-58.741%
Mean Return	0.013%	0.661%
Median Return	0.015%	0.797%
Maximum Return	1.218%	63.316%
Standard Deviation of Returns	0.280%	2.018%



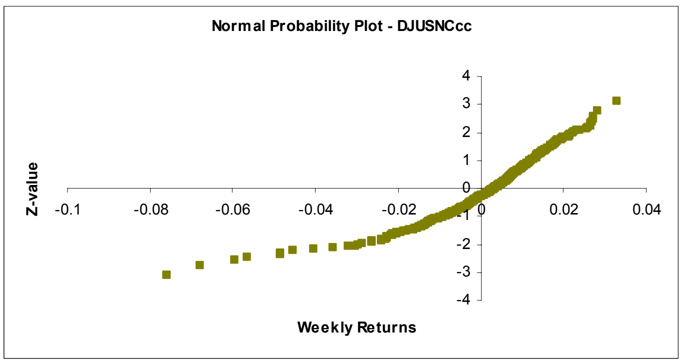
	Weekly	Annual
Minimum Return	-4.480%	-232.982%
Mean Return	0.037%	1.918%
Median Return	0.081%	4.219%
Maximum Return	2.991%	155.518%
Standard Deviation of Returns	1.139%	8.211%



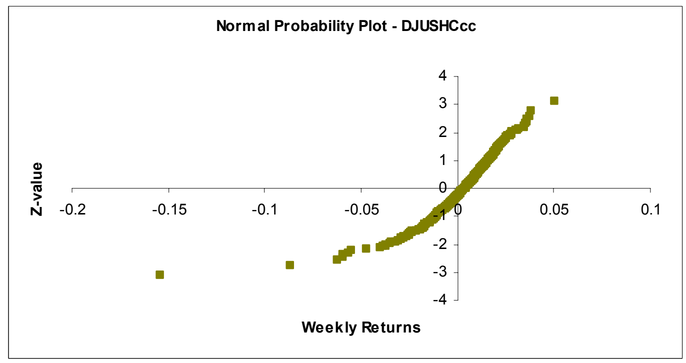
	Weekly	Annual
Minimum Return	-13.278%	-690.469%
Mean Return	0.087%	4.510%
Median Return	0.323%	16.818%
Maximum Return	5.027%	261.381%
Standard Deviation of Returns	1.777%	12.811%



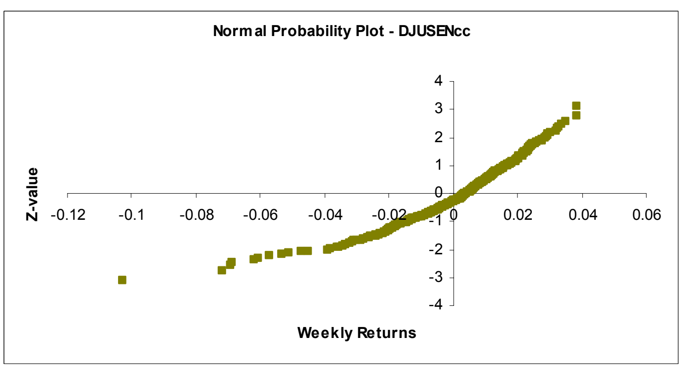
	Weekly	Annual
Minimum Return	-13.132%	-682.842%
Mean Return	0.127%	6.613%
Median Return	0.356%	18.509%
Maximum Return	4.126%	214.569%
Standard Deviation of Returns	1.673%	12.067%



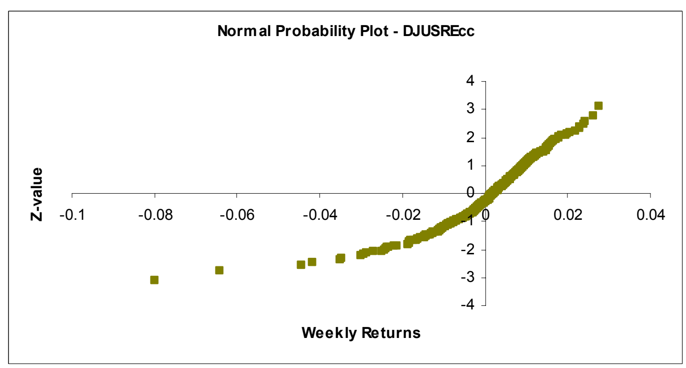
	Weekly	Annual
Minimum Return	-7.588%	-394.585%
Mean Return	0.149%	7.728%
Median Return	0.328%	17.035%
Maximum Return	3.312%	172.203%
Standard Deviation of Returns	1.300%	9.373%



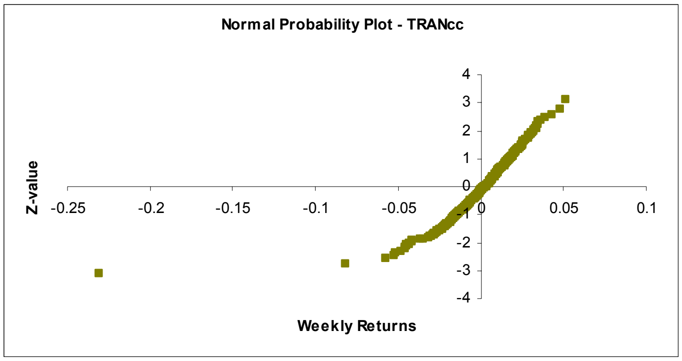
	Weekly	Annual
Minimum Return	-15.423%	-802.013%
Mean Return	0.196%	10.186%
Median Return	0.383%	19.929%
Maximum Return	5.024%	261.224%
Standard Deviation of Returns	1.701%	12.269%

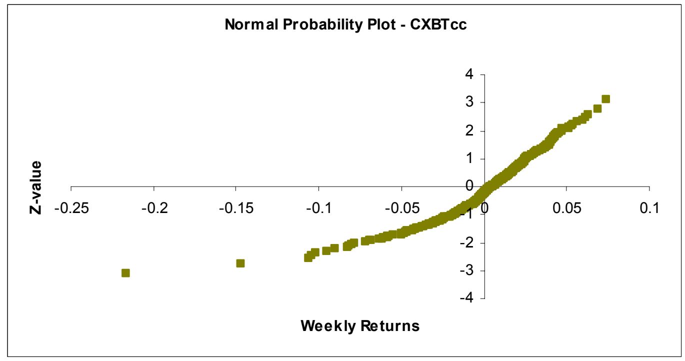


	Weekly	Annual
Minimum Return	-10.292%	-535.188%
Mean Return	0.135%	7.010%
Median Return	0.392%	20.378%
Maximum Return	3.836%	199.478%
Standard Deviation of Returns	1.732%	12.490%



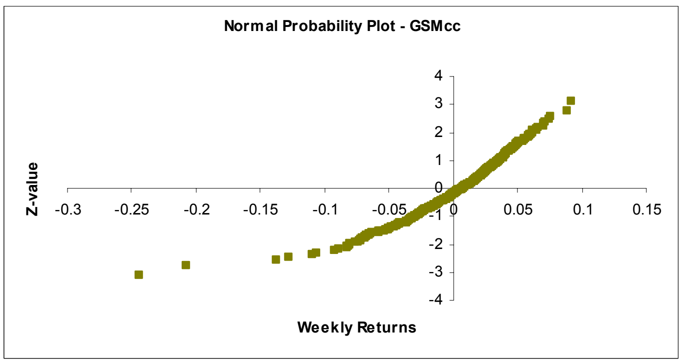
	Weekly	Annual
Minimum Return	-7.985%	-415.223%
Mean Return	0.096%	4.982%
Median Return	0.210%	10.906%
Maximum Return	2.768%	143.934%
Standard Deviation of Returns	1.061%	7.649%

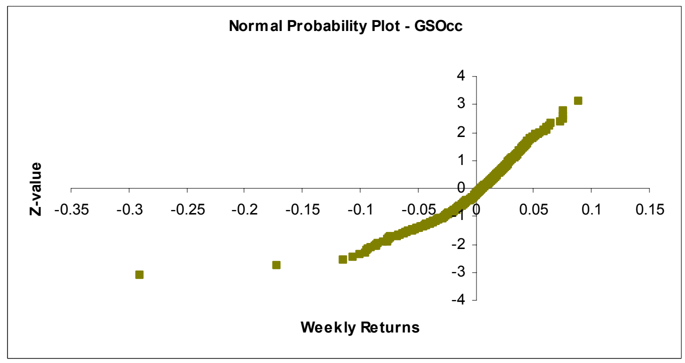




	Weekly	Annual
Minimum Return	-23.045%	-1198.342%
Mean Return	0.112%	5.818%
Median Return	0.282%	14.640%
Maximum Return	5.123%	266.409%
Standard Deviation of Returns	1.978%	14.264%

	Weekly	Annual
Minimum Return	-21.641%	-1125.323%
Mean Return	0.237%	12.348%
Median Return	0.448%	23.317%
Maximum Return	7.367%	383.095%
Standard Deviation of Returns	2.902%	20.928%





	Weekly	Annual
Minimum Return	-24.434%	-1270.584%
Mean Return	0.243%	12.625%
Median Return	0.737%	38.328%
Maximum Return	9.164%	476.539%
Standard Deviation of Returns	3.702%	26.699%

	Weekly	Annual
Minimum Return	-28.967%	-1506.307%
Mean Return	0.153%	7.965%
Median Return	0.579%	30.106%
Maximum Return	8.890%	462.265%
Standard Deviation of Returns	3.511%	25.319%

Appendix V: Correlation Matrix

Basket #1 Assets – Correlation Matrix

Portfolio 1A

	SGX	SVX	RTY	SPEU	MSDUJN
SGX	1.000	0.829	0.751	0.665	0.195
SVX	0.829	1.000	0.747	0.737	0.243
RTY	0.751	0.747	1.000	0.644	0.227
SPEU	0.665	0.737	0.644	1.000	0.267
MSDUJN	0.195	0.243	0.227	0.267	1.000

Portfolio 1B

	SGX	SVX	RTY	SPEU	MSDUJN	GT2	GT10
SGX	1.000	0.829	0.751	0.665	0.195	-0.073	-0.016
SVX	0.829	1.000	0.747	0.737	0.243	-0.081	-0.036
RTY	0.751	0.747	1.000	0.644	0.227	-0.189	-0.135
SPEU	0.665	0.737	0.644	1.000	0.267	-0.234	-0.182
MSDUJN	0.195	0.243	0.227	0.267	1.000	-0.065	-0.086
GT2	-0.073	-0.081	-0.189	-0.234	-0.065	1.000	0.864
GT10	-0.016	-0.036	-0.135	-0.182	-0.086	0.864	1.000

Portfolio 1C

	SGXcc	SVXcc	RTYcc	SPEUcc	MSDUJNcc
SGXcc	1.000	0.820	0.777	0.639	0.199
SVXcc	0.820	1.000	0.755	0.724	0.253
RTYcc	0.777	0.755	1.000	0.621	0.233
SPEUcc	0.639	0.724	0.621	1.000	0.279
MSDUJNcc	0.199	0.253	0.233	0.279	1.000

Portfolio 2A

	SGX	SVX	RTY	SPEU	MSDUJN	MSDUP	MSDUCA	MSEUSBR	MSDUAS	MSEUSSA
SGX	1.000	0.829	0.751	0.665	0.195	0.256	0.695	0.353	0.380	0.358
SVX	0.829	1.000	0.747	0.737	0.243	0.319	0.660	0.424	0.455	0.407
RTY	0.751	0.747	1.000	0.644	0.227	0.294	0.651	0.399	0.432	0.428
SPEU	0.665	0.737	0.644	1.000	0.267	0.343	0.617	0.424	0.448	0.455
MSDUJN	0.195	0.243	0.227	0.267	1.000	0.982	0.282	0.190	0.350	0.267
MSDUP	0.256	0.319	0.294	0.343	0.982	1.000	0.351	0.243	0.472	0.341
MSDUCA	0.695	0.660	0.651	0.617	0.282	0.351	1.000	0.371	0.473	0.494
MSEUSBR	0.353	0.424	0.399	0.424	0.190	0.243	0.371	1.000	0.350	0.349
MSDUAS	0.380	0.455	0.432	0.448	0.350	0.472	0.473	0.350	1.000	0.483
MSEUSSA	0.358	0.407	0.428	0.455	0.267	0.341	0.494	0.349	0.483	1.000

Portfolio 2B

	SGX	SVX	RTY	SPEU	MSDUJN	MSDUP	MSDUCA	MSEUSBR	MSDUAS	MSEUSSA	GT2	GT10
SGX	1.000	0.829	0.751	0.665	0.195	0.256	0.695	0.353	0.380	0.358	-0.073	-0.016
SVX	0.829	1.000	0.747	0.737	0.243	0.319	0.660	0.424	0.455	0.407	-0.081	-0.036
RTY	0.751	0.747	1.000	0.644	0.227	0.294	0.651	0.399	0.432	0.428	-0.189	-0.135
SPEU	0.665	0.737	0.644	1.000	0.267	0.343	0.617	0.424	0.448	0.455	-0.234	-0.182
MSDUJN	0.195	0.243	0.227	0.267	1.000	0.982	0.282	0.190	0.350	0.267	-0.065	-0.086
MSDUP	0.256	0.319	0.294	0.343	0.982	1.000	0.351	0.243	0.472	0.341	-0.079	-0.102
MSDUCA	0.695	0.660	0.651	0.617	0.282	0.351	1.000	0.371	0.473	0.494	-0.112	-0.058
MSEUSBR	0.353	0.424	0.399	0.424	0.190	0.243	0.371	1.000	0.350	0.349	-0.068	-0.056
MSDUAS	0.380	0.455	0.432	0.448	0.350	0.472	0.473	0.350	1.000	0.483	-0.021	-0.028
MSEUSSA	0.358	0.407	0.428	0.455	0.267	0.341	0.494	0.349	0.483	1.000	-0.133	-0.131
GT2	-0.073	-0.081	-0.189	-0.234	-0.065	-0.079	-0.112	-0.068	-0.021	-0.133	1.000	0.864
GT10	-0.016	-0.036	-0.135	-0.182	-0.086	-0.102	-0.058	-0.056	-0.028	-0.131	0.864	1.000

Portfolio 2C

	SGXcc	SVXcc	RTYcc	SPEUcc	MSDUJNcc	MSDUPcc	MSDUCAcc	MSEUSBRcc	MSDUAScc	MSEUSSAcc
SGXcc	1.000	0.820	0.777	0.639	0.199	0.264	0.678	0.347	0.390	0.374
SVXcc	0.820	1.000	0.755	0.724	0.253	0.336	0.637	0.410	0.482	0.430
RTYcc	0.777	0.755	1.000	0.621	0.233	0.303	0.650	0.385	0.452	0.450
SPEUcc	0.639	0.724	0.621	1.000	0.279	0.358	0.601	0.439	0.458	0.475
MSDUJNcc	0.199	0.253	0.233	0.279	1.000	0.979	0.299	0.205	0.363	0.297
MSDUPcc	0.264	0.336	0.303	0.358	0.979	1.000	0.369	0.265	0.494	0.377
MSDUCAcc	0.678	0.637	0.650	0.601	0.299	0.369	1.000	0.387	0.482	0.510
MSEUSBRcc	0.347	0.410	0.385	0.439	0.205	0.265	0.387	1.000	0.364	0.396
MSDUAScc	0.390	0.482	0.452	0.458	0.363	0.494	0.482	0.364	1.000	0.519
MSEUSSAcc	0.374	0.430	0.450	0.475	0.297	0.377	0.510	0.396	0.519	1.000

Basket #2 Assets – Correlation Matrix

Portfolio 3A

	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN
DJUSBM	1.000	0.594	0.482	0.366	0.498
DJUSCY	0.594	1.000	0.647	0.545	0.393
DJUSNC	0.482	0.647	1.000	0.610	0.357
DJUSHC	0.366	0.545	0.610	1.000	0.424
DJUSEN	0.498	0.393	0.357	0.424	1.000

Portfolio 3B

	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	GT2	GT10
DJUSBM	1.000	0.594	0.482	0.366	0.498	-0.146	-0.149
DJUSCY	0.594	1.000	0.647	0.545	0.393	-0.117	-0.083
DJUSNC	0.482	0.647	1.000	0.610	0.357	0.009	0.070
DJUSHC	0.366	0.545	0.610	1.000	0.424	-0.038	0.017
DJUSEN	0.498	0.393	0.357	0.424	1.000	-0.082	-0.038
GT2	-0.146	-0.117	0.009	-0.038	-0.082	1.000	0.864
GT10	-0.149	-0.083	0.070	0.017	-0.038	0.864	1.000

Portfolio 3C

	DJUSBMcc	DJUSCYcc	DJUSNCcc	DJUSHCcc	DJUSENcc
DJUSBMcc	1.000	0.634	0.530	0.412	0.536
DJUSCYcc	0.634	1.000	0.668	0.535	0.425
DJUSNCcc	0.530	0.668	1.000	0.604	0.401
DJUSHCcc	0.412	0.535	0.604	1.000	0.435
DJUSENcc	0.536	0.425	0.401	0.435	1.000

Portfolio 4A

	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	DJUSRE	TRAN	CXBT	GSM	GSO
DJUSBM	1.000	0.594	0.482	0.366	0.498	0.473	0.674	0.288	0.330	0.307
DJUSCY	0.594	1.000	0.647	0.545	0.393	0.516	0.758	0.478	0.585	0.639
DJUSNC	0.482	0.647	1.000	0.610	0.357	0.411	0.540	0.375	0.248	0.353
DJUSHC	0.366	0.545	0.610	1.000	0.424	0.363	0.443	0.532	0.245	0.362
DJUSEN	0.498	0.393	0.357	0.424	1.000	0.343	0.409	0.311	0.200	0.243
DJUSRE	0.473	0.516	0.411	0.363	0.343	1.000	0.512	0.288	0.224	0.246
TRAN	0.674	0.758	0.540	0.443	0.409	0.512	1.000	0.365	0.463	0.464
CXBT	0.288	0.478	0.375	0.532	0.311	0.288	0.365	1.000	0.526	0.634
GSM	0.330	0.585	0.248	0.245	0.200	0.224	0.463	0.526	1.000	0.799
GSO	0.307	0.639	0.353	0.362	0.243	0.246	0.464	0.634	0.799	1.000

Portfolio 4B

	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	DJUSRE	TRAN	CXBT	GSM	GSO	GT2	GT10
DJUSBM	1.000	0.594	0.482	0.366	0.498	0.473	0.674	0.288	0.330	0.307	-0.146	-0.149
DJUSCY	0.594	1.000	0.647	0.545	0.393	0.516	0.758	0.478	0.585	0.639	-0.117	-0.083
DJUSNC	0.482	0.647	1.000	0.610	0.357	0.411	0.540	0.375	0.248	0.353	0.009	0.070
DJUSHC	0.366	0.545	0.610	1.000	0.424	0.363	0.443	0.532	0.245	0.362	-0.038	0.017
DJUSEN	0.498	0.393	0.357	0.424	1.000	0.343	0.409	0.311	0.200	0.243	-0.082	-0.038
DJUSRE	0.473	0.516	0.411	0.363	0.343	1.000	0.512	0.288	0.224	0.246	-0.061	-0.013
TRAN	0.674	0.758	0.540	0.443	0.409	0.512	1.000	0.365	0.463	0.464	-0.096	-0.060
CXBT	0.288	0.478	0.375	0.532	0.311	0.288	0.365	1.000	0.526	0.634	-0.095	-0.067
GSM	0.330	0.585	0.248	0.245	0.200	0.224	0.463	0.526	1.000	0.799	-0.192	-0.158
GSO	0.307	0.639	0.353	0.362	0.243	0.246	0.464	0.634	0.799	1.000	-0.162	-0.112
GT2	-0.146	-0.117	0.009	-0.038	-0.082	-0.061	-0.096	-0.095	-0.192	-0.162	1.000	0.864
GT10	-0.149	-0.083	0.070	0.017	-0.038	-0.013	-0.060	-0.067	-0.158	-0.112	0.864	1.000

Portfolio 4C

	DJUSBMcc	DJUSCYcc	DJUSNCcc	DJUSHCcc	DJUSENcc	DJUSREcc	TRANcc	CXBTcc	GSMcc	GSOcc
DJUSBMcc	1.000	0.634	0.530	0.412	0.536	0.467	0.695	0.334	0.364	0.341
DJUSCYcc	0.634	1.000	0.668	0.535	0.425	0.550	0.756	0.514	0.596	0.653
DJUSNCcc	0.530	0.668	1.000	0.604	0.401	0.436	0.560	0.431	0.287	0.387
DJUSHCcc	0.412	0.535	0.604	1.000	0.435	0.387	0.456	0.530	0.242	0.366
DJUSENcc	0.536	0.425	0.401	0.435	1.000	0.380	0.458	0.327	0.225	0.265
DJUSREcc	0.467	0.550	0.436	0.387	0.380	1.000	0.542	0.309	0.244	0.281
TRANcc	0.695	0.756	0.560	0.456	0.458	0.542	1.000	0.405	0.474	0.477
CXBTcc	0.334	0.514	0.431	0.530	0.327	0.309	0.405	1.000	0.546	0.661
GSMcc	0.364	0.596	0.287	0.242	0.225	0.244	0.474	0.546	1.000	0.805
GSOcc	0.341	0.653	0.387	0.366	0.265	0.281	0.477	0.661	0.805	1.000

Appendix VI: Portfolio Weights in Construction of Efficient Frontiers

Basket #1 – Portfolio Weights

Portfolio 1A

Returns	SGX	SVX	RTY	SPEU	MSDUJN
1.00%	-18.92%	45.63%	5.74%	4.58%	62.98%
2.00%	-13.01%	47.38%	6.72%	4.00%	54.92%
3.00%	-7.11%	49.13%	7.70%	3.42%	46.87%
4.00%	-1.20%	50.88%	8.68%	2.84%	38.81%
5.00%	4.70%	52.63%	9.66%	2.26%	30.75%
6.00%	10.61%	54.38%	10.64%	1.68%	22.69%
7.00%	16.52%	56.12%	11.62%	1.11%	14.63%
8.00%	22.42%	57.87%	12.60%	0.53%	6.58%
9.00%	28.33%	59.62%	13.58%	-0.05%	-1.48%
10.00%	34.24%	61.37%	14.56%	-0.63%	-9.54%
11.00%	40.14%	63.12%	15.54%	-1.21%	-17.60%
12.00%	46.05%	64.87%	16.52%	-1.78%	-25.66%
13.00%	51.95%	66.62%	17.50%	-2.36%	-33.71%
14.00%	57.86%	68.37%	18.48%	-2.94%	-41.77%
15.00%	63.77%	70.12%	19.46%	-3.52%	-49.83%
16.00%	69.67%	71.87%	20.44%	-4.10%	-57.89%
17.00%	75.58%	73.62%	21.42%	-4.67%	-65.94%
18.00%	81.49%	75.37%	22.40%	-5.25%	-74.00%
19.00%	87.39%	77.12%	23.38%	-5.83%	-82.06%
20.00%	93.30%	78.87%	24.36%	-6.41%	-90.12%
21.00%	99.20%	80.62%	25.34%	-6.99%	-98.18%
22.00%	105.11%	82.37%	26.32%	-7.56%	-106.23%
23.00%	111.02%	84.12%	27.30%	-8.14%	-114.29%
24.00%	116.92%	85.87%	28.28%	-8.72%	-122.35%
25.00%	122.83%	87.62%	29.26%	-9.30%	-130.41%
26.00%	128.74%	89.36%	30.24%	-9.88%	-138.47%
27.00%	134.64%	91.11%	31.22%	-10.45%	-146.52%
28.00%	140.55%	92.86%	32.20%	-11.03%	-154.58%
29.00%	146.45%	94.61%	33.18%	-11.61%	-162.64%
30.00%	152.36%	96.36%	34.16%	-12.19%	-170.70%

Portfolio 1B

Returns	SGX	SVX	RTY	SPEU	MSDUJN	GT2	GT10
1.00%	-0.52%	-1.08%	3.02%	3.26%	-0.30%	95.63%	0.00%
2.00%	4.21%	3.66%	4.17%	2.76%	-4.31%	89.50%	0.00%
3.00%	8.82%	8.30%	5.38%	2.35%	-8.24%	82.49%	0.90%
4.00%	12.34%	11.98%	7.07%	2.75%	-11.49%	67.09%	10.27%
5.00%	15.85%	15.65%	8.76%	3.14%	-14.74%	51.69%	19.64%
6.00%	19.37%	19.33%	10.45%	3.54%	-17.99%	36.30%	29.01%
7.00%	22.89%	23.00%	12.14%	3.93%	-21.24%	20.90%	38.38%
8.00%	26.41%	26.68%	13.83%	4.33%	-24.49%	5.50%	47.75%
9.00%	31.29%	30.73%	14.98%	3.88%	-29.27%	0.00%	48.39%
10.00%	36.94%	34.99%	15.84%	2.96%	-34.91%	0.00%	44.18%
11.00%	42.59%	39.26%	16.70%	2.04%	-40.55%	0.00%	39.97%
12.00%	48.24%	43.52%	17.56%	1.12%	-46.19%	0.00%	35.76%
13.00%	53.89%	47.78%	18.41%	0.20%	-51.83%	0.00%	31.55%
14.00%	59.54%	52.05%	19.27%	-0.72%	-57.47%	0.00%	27.34%
15.00%	65.18%	56.31%	20.13%	-1.64%	-63.11%	0.00%	23.13%
16.00%	70.83%	60.57%	20.99%	-2.56%	-68.75%	0.00%	18.92%
17.00%	76.48%	64.84%	21.85%	-3.48%	-74.39%	0.00%	14.71%
18.00%	82.13%	69.10%	22.70%	-4.40%	-80.03%	0.00%	10.50%
19.00%	87.78%	73.36%	23.56%	-5.32%	-85.67%	0.00%	6.29%
20.00%	93.43%	77.63%	24.42%	-6.24%	-91.31%	0.00%	2.08%
21.00%	99.20%	80.62%	25.34%	-6.99%	-98.18%	0.00%	0.00%
22.00%	105.11%	82.37%	26.32%	-7.56%	-106.23%	0.00%	0.00%
23.00%	111.02%	84.12%	27.30%	-8.14%	-114.29%	0.00%	0.00%
24.00%	116.92%	85.87%	28.28%	-8.72%	-122.35%	0.00%	0.00%
25.00%	122.83%	87.62%	29.26%	-9.30%	-130.41%	0.00%	0.00%
26.00%	128.74%	89.36%	30.24%	-9.88%	-138.47%	0.00%	0.00%
27.00%	134.64%	91.11%	31.22%	-10.45%	-146.52%	0.00%	0.00%
28.00%	140.55%	92.86%	32.20%	-11.03%	-154.58%	0.00%	0.00%
29.00%	146.45%	94.61%	33.18%	-11.61%	-162.64%	0.00%	0.00%
30.00%	152.36%	96.36%	34.16%	-12.19%	-170.70%	0.00%	0.00%

Portfolio 1C

Returns	SGXcc	SVXcc	RTYcc	SPEUcc	MSDUJNcc
1.00%	-30.01%	55.89%	-15.15%	10.06%	79.20%
2.00%	-17.91%	53.93%	-10.00%	8.28%	65.71%
3.00%	-5.81%	51.96%	-4.85%	6.50%	52.21%
4.00%	6.29%	49.99%	0.30%	4.72%	38.71%
5.00%	18.39%	48.02%	5.45%	2.94%	25.21%
6.00%	30.48%	46.06%	10.60%	1.15%	11.71%
7.00%	42.59%	44.09%	15.74%	-0.63%	-1.79%
8.00%	54.68%	42.12%	20.89%	-2.41%	-15.29%
9.00%	66.78%	40.15%	26.04%	-4.19%	-28.79%
10.00%	78.88%	38.19%	31.19%	-5.97%	-42.29%
11.00%	90.98%	36.22%	36.33%	-7.75%	-55.79%
12.00%	103.08%	34.25%	41.48%	-9.53%	-69.29%
13.00%	115.18%	32.28%	46.63%	-11.31%	-82.79%
14.00%	127.28%	30.32%	51.78%	-13.09%	-96.28%
15.00%	139.38%	28.35%	56.93%	-14.87%	-109.78%
16.00%	151.48%	26.38%	62.07%	-16.65%	-123.28%
17.00%	163.58%	24.41%	67.22%	-18.43%	-136.78%
18.00%	175.68%	22.45%	72.37%	-20.21%	-150.28%
19.00%	187.78%	20.48%	77.52%	-21.99%	-163.78%
20.00%	199.88%	18.51%	82.67%	-23.78%	-177.28%
21.00%	211.98%	16.55%	87.81%	-25.56%	-190.78%
22.00%	224.08%	14.58%	92.96%	-27.34%	-204.28%
23.00%	236.18%	12.61%	98.11%	-29.12%	-217.78%
24.00%	248.27%	10.64%	103.26%	-30.90%	-231.28%
25.00%	260.37%	8.68%	108.41%	-32.68%	-244.78%
26.00%	272.47%	6.71%	113.55%	-34.46%	-258.27%
27.00%	284.57%	4.74%	118.70%	-36.24%	-271.77%
28.00%	296.67%	2.77%	123.85%	-38.02%	-285.27%
29.00%	308.77%	0.81%	129.00%	-39.80%	-298.77%
30.00%	320.87%	-1.16%	134.15%	-41.58%	-312.27%

Portfolio 2A

Returns	SGX	SVX	RTY	SPEU	MSDUJN	MSDUP	MSDUCA	MSEUSBR	MSDUAS	MSEUSSA
1.00%	-1.80%	29.63%	2.40%	-3.85%	-85.53%	158.14%	-11.83%	-1.70%	11.98%	2.57%
2.00%	-0.10%	31.85%	2.71%	-3.47%	-61.97%	122.55%	-8.60%	-2.44%	17.19%	2.29%
3.00%	1.59%	34.08%	3.03%	-3.10%	-38.40%	86.95%	-5.37%	-3.18%	22.40%	2.01%
4.00%	3.29%	36.30%	3.34%	-2.72%	-14.77%	51.29%	-2.14%	-3.92%	27.62%	1.72%
5.00%	4.98%	38.52%	3.66%	-2.35%	8.77%	15.72%	1.10%	-4.66%	32.82%	1.44%
6.00%	6.69%	40.74%	3.97%	-1.98%	32.12%	-19.63%	4.34%	-5.41%	37.99%	1.16%
7.00%	8.49%	42.87%	4.32%	-1.65%	53.65%	-52.86%	7.66%	-6.17%	42.87%	0.82%
8.00%	10.76%	44.86%	4.75%	-1.67%	63.76%	-72.92%	11.47%	-7.05%	45.85%	0.20%
9.00%	12.18%	47.09%	5.05%	-1.05%	94.59%	-116.90%	14.40%	-7.72%	52.26%	0.10%
10.00%	13.67%	49.70%	5.25%	-0.72%	119.69%	-154.26%	17.57%	-8.45%	57.68%	-0.13%
11.00%	15.46%	51.83%	5.59%	-0.37%	141.75%	-188.12%	20.86%	-9.20%	62.65%	-0.46%
12.00%	17.15%	54.02%	5.90%	0.03%	165.96%	-224.46%	24.06%	-9.93%	67.97%	-0.71%
13.00%	18.78%	56.28%	6.22%	0.41%	191.14%	-261.90%	27.24%	-10.66%	73.45%	-0.97%
14.00%	20.24%	58.51%	6.49%	1.03%	220.98%	-304.75%	30.20%	-11.33%	79.71%	-1.09%
15.00%	21.94%	60.74%	6.80%	1.41%	244.53%	-340.33%	33.43%	-12.07%	84.92%	-1.37%
16.00%	23.94%	62.87%	7.19%	1.57%	261.53%	-368.33%	36.89%	-12.88%	89.03%	-1.81%
17.00%	25.56%	65.05%	7.53%	1.99%	285.98%	-404.95%	40.13%	-13.61%	94.39%	-2.07%
18.00%	27.37%	67.27%	7.88%	2.21%	305.83%	-436.24%	43.56%	-14.39%	98.98%	-2.46%
19.00%	29.35%	69.34%	8.31%	2.42%	324.11%	-465.73%	46.95%	-15.19%	103.31%	-2.87%
20.00%	30.77%	71.68%	8.52%	2.97%	353.09%	-507.57%	50.03%	-15.87%	109.42%	-3.03%
21.00%	32.66%	73.78%	8.91%	3.24%	373.17%	-539.15%	53.37%	-16.65%	114.06%	-3.39%
22.00%	34.46%	75.95%	9.26%	3.53%	393.95%	-571.52%	56.72%	-17.42%	118.80%	-3.74%
23.00%	36.33%	78.12%	9.49%	3.89%	414.28%	-603.38%	60.11%	-18.19%	123.46%	-4.10%
24.00%	37.69%	80.48%	9.80%	4.44%	445.13%	-647.38%	63.02%	-18.85%	129.88%	-4.20%
25.00%	39.47%	82.65%	10.19%	4.69%	466.80%	-680.77%	66.35%	-19.62%	134.79%	-4.54%
26.00%	41.26%	84.81%	10.53%	5.00%	488.12%	-713.78%	69.68%	-20.38%	139.63%	-4.88%
27.00%	43.05%	86.99%	10.82%	5.39%	509.17%	-746.48%	73.02%	-21.15%	144.40%	-5.22%
28.00%	44.91%	89.19%	11.09%	5.70%	530.13%	-779.06%	76.34%	-21.91%	149.17%	-5.56%
29.00%	46.66%	91.31%	11.57%	5.92%	551.08%	-811.63%	79.72%	-22.68%	153.96%	-5.92%
30.00%	48.46%	93.47%	11.93%	6.20%	572.05%	-844.21%	83.08%	-23.45%	158.75%	-6.28%

Portfolio 2B

Returns	SGX	SVX	RTY	SPEU	MSDUJN	MSDUP	MSDUCA	MSEUSBR	MSDUAS	MSEUSSA	GT2	GT10
1.00%	-1.07%	-1.03%	2.91%	3.29%	1.19%	-1.66%	0.67%	-0.42%	-0.07%	0.39%	95.81%	0.00%
2.00%	0.59%	2.79%	3.20%	3.29%	20.25%	-28.95%	3.25%	-1.23%	5.77%	0.22%	90.82%	0.00%
3.00%	2.25%	6.61%	3.48%	3.30%	39.32%	-56.25%	5.83%	-2.04%	11.60%	0.05%	85.84%	0.00%
4.00%	3.90%	10.42%	3.77%	3.30%	58.39%	-83.54%	8.42%	-2.84%	17.44%	-0.12%	80.85%	0.00%
5.00%	5.48%	14.14%	4.13%	3.39%	77.04%	-110.24%	10.95%	-3.65%	23.16%	-0.26%	74.80%	1.05%
6.00%	6.76%	17.45%	4.79%	3.83%	94.01%	-134.56%	13.29%	-4.43%	28.40%	-0.32%	64.46%	6.32%
7.00%	8.03%	20.76%	5.45%	4.26%	110.99%	-158.87%	15.62%	-5.22%	33.64%	-0.37%	54.12%	11.59%
8.00%	9.31%	24.07%	6.11%	4.70%	127.96%	-183.19%	17.96%	-6.01%	38.88%	-0.42%	43.77%	16.86%
9.00%	10.58%	27.38%	6.77%	5.13%	144.94%	-207.51%	20.29%	-6.80%	44.12%	-0.48%	33.43%	22.13%
10.00%	11.86%	30.70%	7.43%	5.56%	161.92%	-231.83%	22.63%	-7.59%	49.36%	-0.53%	23.09%	27.40%
11.00%	13.13%	34.01%	8.08%	6.00%	178.89%	-256.14%	24.97%	-8.38%	54.61%	-0.59%	12.75%	32.67%
12.00%	14.41%	37.32%	8.74%	6.43%	195.87%	-280.46%	27.30%	-9.17%	59.85%	-0.64%	2.41%	37.94%
13.00%	16.18%	40.93%	8.93%	6.40%	216.49%	-310.38%	30.09%	-9.97%	65.72%	-0.87%	0.00%	36.47%
14.00%	18.10%	44.64%	8.98%	6.22%	238.22%	-341.99%	33.02%	-10.77%	71.78%	-1.14%	0.00%	32.94%
15.00%	20.03%	48.35%	9.03%	6.04%	259.95%	-373.61%	35.95%	-11.57%	77.84%	-1.42%	0.00%	29.41%
16.00%	21.95%	52.06%	9.07%	5.86%	281.69%	-405.23%	38.88%	-12.37%	83.90%	-1.69%	0.00%	25.88%
17.00%	23.88%	55.77%	9.12%	5.68%	303.42%	-436.84%	41.81%	-13.17%	89.96%	-1.97%	0.00%	22.35%
18.00%	25.80%	59.47%	9.17%	5.50%	325.15%	-468.46%	44.74%	-13.97%	96.02%	-2.24%	0.00%	18.82%
19.00%	27.73%	63.18%	9.22%	5.32%	346.88%	-500.08%	47.67%	-14.77%	102.08%	-2.52%	0.00%	15.29%
20.00%	29.65%	66.89%	9.26%	5.14%	368.61%	-531.69%	50.60%	-15.57%	108.14%	-2.80%	0.00%	11.76%
21.00%	31.58%	70.60%	9.31%	4.96%	390.34%	-563.31%	53.53%	-16.37%	114.20%	-3.07%	0.00%	8.23%
22.00%	33.50%	74.31%	9.36%	4.78%	412.07%	-594.93%	56.46%	-17.17%	120.26%	-3.35%	0.00%	4.70%
23.00%	35.42%	78.02%	9.41%	4.60%	433.81%	-626.54%	59.39%	-17.97%	126.32%	-3.62%	0.00%	1.17%
24.00%	37.20%	80.73%	9.63%	4.79%	456.77%	-660.83%	62.53%	-18.73%	131.82%	-3.90%	0.00%	0.00%
25.00%	38.89%	82.95%	9.95%	5.16%	480.35%	-696.44%	65.76%	-19.47%	137.03%	-4.18%	0.00%	0.00%
26.00%	40.59%	85.17%	10.26%	5.54%	503.93%	-732.05%	68.99%	-20.21%	142.24%	-4.46%	0.00%	0.00%
27.00%	42.28%	87.40%	10.57%	5.92%	527.51%	-767.65%	72.22%	-20.95%	147.45%	-4.75%	0.00%	0.00%
28.00%	43.98%	89.62%	10.89%	6.29%	551.09%	-803.26%	75.46%	-21.69%	152.66%	-5.03%	0.00%	0.00%
29.00%	45.67%	91.84%	11.20%	6.67%	574.67%	-838.87%	78.69%	-22.43%	157.87%	-5.31%	0.00%	0.00%
30.00%	47.37%	94.06%	11.52%	7.04%	598.25%	-874.48%	81.92%	-23.17%	163.08%	-5.59%	0.00%	0.00%

Portfolio 2C

Returns	SGXcc	SVXcc	RTYcc	SPEUcc	MSDUJNcc	MSDUPcc	MSDUCAcc	MSEUSBRcc	MSDUAScc	MSEUSSAcc
1.00%	-5.28%	33.89%	-8.55%	4.44%	-112.83%	209.65%	-17.52%	-4.82%	3.75%	-2.72%
2.00%	-0.42%	33.23%	-6.90%	2.96%	-85.74%	163.48%	-10.63%	-4.88%	11.41%	-2.51%
3.00%	3.71%	33.36%	-5.65%	2.02%	-45.24%	101.86%	-4.53%	-4.88%	21.42%	-2.06%
4.00%	7.83%	33.49%	-4.40%	1.07%	-4.77%	40.27%	1.58%	-4.89%	31.41%	-1.61%
5.00%	11.96%	33.62%	-3.14%	0.13%	35.71%	-21.32%	7.69%	-4.89%	41.41%	-1.17%
6.00%	16.98%	32.73%	-1.37%	-1.44%	59.62%	-63.83%	14.76%	-4.96%	48.53%	-1.02%
7.00%	20.57%	33.24%	-0.32%	-1.97%	109.82%	-136.58%	20.37%	-4.93%	60.22%	-0.43%
8.00%	24.34%	34.00%	0.61%	-2.70%	157.05%	-205.99%	26.01%	-4.90%	71.39%	0.17%
9.00%	28.57%	34.04%	1.92%	-3.74%	195.24%	-264.95%	32.26%	-4.91%	80.98%	0.58%
10.00%	32.60%	34.24%	3.13%	-4.59%	237.76%	-328.88%	38.24%	-4.90%	91.34%	1.06%
11.00%	36.71%	34.39%	4.37%	-5.51%	278.62%	-390.92%	44.33%	-4.91%	101.41%	1.52%
12.00%	40.84%	34.51%	5.62%	-6.46%	318.99%	-452.38%	50.44%	-4.91%	111.38%	1.96%
13.00%	45.34%	34.25%	7.09%	-7.69%	352.44%	-505.88%	56.96%	-4.94%	120.16%	2.29%
14.00%	49.68%	34.04%	8.58%	-8.86%	387.86%	-561.63%	63.38%	-4.98%	129.31%	2.62%
15.00%	53.94%	33.95%	9.97%	-9.95%	424.83%	-619.19%	69.72%	-4.99%	138.73%	3.00%
16.00%	57.65%	34.69%	10.82%	-10.46%	474.94%	-691.89%	75.22%	-4.95%	150.34%	3.65%
17.00%	61.87%	34.71%	12.13%	-11.48%	513.63%	-751.42%	81.43%	-4.95%	160.02%	4.06%
18.00%	66.11%	34.72%	13.44%	-12.50%	552.09%	-810.68%	87.65%	-4.97%	169.67%	4.47%
19.00%	70.36%	34.72%	14.75%	-13.53%	590.31%	-869.69%	93.89%	-4.98%	179.27%	4.89%
20.00%	74.62%	34.70%	16.08%	-14.58%	628.32%	-928.43%	100.14%	-4.99%	188.85%	5.28%
21.00%	78.90%	34.66%	17.42%	-15.63%	666.11%	-986.92%	106.41%	-5.00%	198.38%	5.68%
22.00%	84.34%	33.67%	19.19%	-17.62%	683.02%	-1021.37%	113.89%	-5.10%	204.25%	5.74%
23.00%	87.50%	34.70%	19.99%	-17.81%	740.91%	-1103.02%	118.98%	-5.03%	217.28%	6.49%
24.00%	91.82%	34.65%	21.34%	-18.90%	777.91%	-1160.60%	125.30%	-5.05%	226.66%	6.88%
25.00%	96.19%	34.55%	22.70%	-20.02%	814.03%	-1217.18%	131.66%	-5.07%	235.89%	7.25%
26.00%	100.53%	34.47%	24.05%	-21.14%	850.56%	-1274.23%	138.00%	-5.09%	245.22%	7.63%
27.00%	104.88%	34.40%	25.41%	-22.25%	887.07%	-1331.24%	144.33%	-5.11%	254.49%	8.01%
28.00%	109.27%	34.28%	26.78%	-23.39%	922.90%	-1387.47%	150.71%	-5.13%	263.67%	8.38%
29.00%	113.65%	34.18%	28.15%	-24.52%	958.84%	-1443.84%	157.09%	-5.16%	272.85%	8.75%
30.00%	118.06%	34.05%	29.54%	-25.65%	994.18%	-1499.50%	163.49%	-5.19%	281.91%	9.12%

Basket #2 – Portfolio Weights

Portfolio 3A

Returns	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN
1.00%	85.37%	34.45%	33.63%	-76.69%	23.24%
2.00%	76.45%	31.05%	36.37%	-67.20%	23.34%
3.00%	67.52%	27.64%	39.10%	-57.71%	23.45%
4.00%	58.59%	24.24%	41.83%	-48.21%	23.55%
5.00%	49.66%	20.84%	44.57%	-38.72%	23.66%
6.00%	40.73%	17.43%	47.30%	-29.23%	23.76%
7.00%	31.81%	14.03%	50.04%	-19.74%	23.87%
8.00%	22.88%	10.63%	52.77%	-10.25%	23.97%
9.00%	13.95%	7.22%	55.51%	-0.76%	24.07%
10.00%	5.02%	3.82%	58.24%	8.74%	24.18%
11.00%	-3.91%	0.42%	60.97%	18.23%	24.28%
12.00%	-12.83%	-2.98%	63.71%	27.72%	24.39%
13.00%	-21.76%	-6.39%	66.44%	37.21%	24.49%
14.00%	-30.69%	-9.79%	69.18%	46.70%	24.60%
15.00%	-39.62%	-13.19%	71.91%	56.19%	24.70%
16.00%	-48.54%	-16.60%	74.65%	65.69%	24.81%
17.00%	-57.47%	-20.00%	77.38%	75.18%	24.91%
18.00%	-66.40%	-23.40%	80.12%	84.67%	25.02%
19.00%	-75.33%	-26.81%	82.85%	94.16%	25.12%
20.00%	-84.26%	-30.21%	85.58%	103.65%	25.23%
21.00%	-93.18%	-33.61%	88.32%	113.14%	25.33%
22.00%	-102.11%	-37.02%	91.05%	122.64%	25.44%
23.00%	-111.04%	-40.42%	93.79%	132.13%	25.54%
24.00%	-119.97%	-43.82%	96.52%	141.62%	25.65%
25.00%	-128.90%	-47.22%	99.26%	151.11%	25.75%
26.00%	-137.82%	-50.63%	101.99%	160.60%	25.86%
27.00%	-146.75%	-54.03%	104.73%	170.09%	25.96%
28.00%	-155.68%	-57.43%	107.46%	179.59%	26.07%
29.00%	-164.61%	-60.84%	110.19%	189.08%	26.17%
30.00%	-173.54%	-64.24%	112.93%	198.57%	26.28%

Portfolio 3B

Returns	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	GT2	GT10
1.00%	0.91%	1.26%	0.00%	0.98%	1.01%	95.85%	0.00%
2.00%	-1.48%	0.43%	5.34%	4.45%	2.83%	88.42%	0.00%
3.00%	-3.86%	-0.40%	10.68%	7.93%	4.66%	81.00%	0.00%
4.00%	-6.24%	-1.23%	16.02%	11.40%	6.49%	73.57%	0.00%
5.00%	-8.62%	-2.07%	21.36%	14.87%	8.32%	66.14%	0.00%
6.00%	-10.99%	-2.89%	26.68%	18.34%	10.14%	58.57%	0.15%
7.00%	-12.97%	-3.50%	31.45%	21.65%	11.88%	46.99%	4.50%
8.00%	-14.95%	-4.10%	36.22%	24.96%	13.62%	35.42%	8.84%
9.00%	-16.93%	-4.71%	40.99%	28.28%	15.35%	23.84%	13.18%
10.00%	-18.92%	-5.31%	45.76%	31.59%	17.09%	12.26%	17.52%
11.00%	-20.90%	-5.92%	50.53%	34.90%	18.83%	0.69%	21.86%
12.00%	-24.49%	-7.33%	56.45%	39.19%	20.62%	0.00%	15.56%
13.00%	-28.19%	-8.78%	62.44%	43.54%	22.42%	0.00%	8.58%
14.00%	-31.89%	-10.24%	68.43%	47.89%	24.21%	0.00%	1.61%
15.00%	-39.62%	-13.19%	71.91%	56.19%	24.70%	0.00%	0.00%
16.00%	-48.54%	-16.60%	74.65%	65.69%	24.81%	0.00%	0.00%
17.00%	-57.47%	-20.00%	77.38%	75.18%	24.91%	0.00%	0.00%
18.00%	-66.40%	-23.40%	80.12%	84.67%	25.02%	0.00%	0.00%
19.00%	-75.33%	-26.81%	82.85%	94.16%	25.12%	0.00%	0.00%
20.00%	-84.26%	-30.21%	85.58%	103.65%	25.23%	0.00%	0.00%
21.00%	-93.18%	-33.61%	88.32%	113.14%	25.33%	0.00%	0.00%
22.00%	-102.11%	-37.01%	91.05%	122.64%	25.44%	0.00%	0.00%
23.00%	-111.04%	-40.42%	93.79%	132.13%	25.54%	0.00%	0.00%
24.00%	-119.97%	-43.82%	96.52%	141.62%	25.65%	0.00%	0.00%
25.00%	-128.90%	-47.22%	99.26%	151.11%	25.75%	0.00%	0.00%
26.00%	-137.82%	-50.63%	101.99%	160.60%	25.86%	0.00%	0.00%
27.00%	-146.75%	-54.03%	104.73%	170.09%	25.96%	0.00%	0.00%
28.00%	-155.68%	-57.43%	107.46%	179.59%	26.07%	0.00%	0.00%
29.00%	-164.61%	-60.84%	110.19%	189.08%	26.17%	0.00%	0.00%
30.00%	-173.54%	-64.24%	112.93%	198.57%	26.28%	0.00%	0.00%

Portfolio 3C

Returns	DJUSBMcc	DJUSCYcc	DJUSNCcc	DJUSHCcc	DJUSENcc
1.00%	113.57%	22.70%	52.04%	-108.79%	20.48%
2.00%	97.24%	19.40%	53.53%	-90.91%	20.74%
3.00%	80.91%	16.10%	55.02%	-73.02%	21.00%
4.00%	64.57%	12.79%	56.51%	-55.14%	21.26%
5.00%	48.24%	9.49%	57.99%	-37.25%	21.52%
6.00%	31.91%	6.19%	59.48%	-19.37%	21.79%
7.00%	15.58%	2.88%	60.98%	-1.49%	22.05%
8.00%	-0.75%	-0.42%	62.46%	16.40%	22.31%
9.00%	-17.08%	-3.72%	63.95%	34.28%	22.57%
10.00%	-33.41%	-7.02%	65.44%	52.17%	22.83%
11.00%	-49.75%	-10.33%	66.93%	70.05%	23.10%
12.00%	-66.08%	-13.63%	68.42%	87.93%	23.36%
13.00%	-82.41%	-16.93%	69.90%	105.82%	23.62%
14.00%	-98.74%	-20.23%	71.39%	123.70%	23.88%
15.00%	-115.07%	-23.54%	72.88%	141.58%	24.14%
16.00%	-131.40%	-26.84%	74.37%	159.47%	24.40%
17.00%	-147.73%	-30.14%	75.86%	177.35%	24.67%
18.00%	-164.06%	-33.45%	77.35%	195.24%	24.93%
19.00%	-180.40%	-36.75%	78.84%	213.12%	25.19%
20.00%	-196.73%	-40.06%	80.32%	231.01%	25.45%
21.00%	-213.06%	-43.36%	81.81%	248.89%	25.71%
22.00%	-229.39%	-46.66%	83.30%	266.77%	25.98%
23.00%	-245.72%	-49.96%	84.79%	284.66%	26.24%
24.00%	-262.05%	-53.26%	86.28%	302.54%	26.50%
25.00%	-278.38%	-56.57%	87.77%	320.42%	26.76%
26.00%	-294.72%	-59.87%	89.25%	338.31%	27.02%
27.00%	-311.05%	-63.17%	90.74%	356.19%	27.29%
28.00%	-327.38%	-66.48%	92.23%	374.07%	27.55%
29.00%	-343.71%	-69.78%	93.72%	391.96%	27.81%
30.00%	-360.04%	-73.08%	95.21%	409.84%	28.07%

Portfolio 4A

Returns	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	DJUSRE	TRAN	CXBT	GSM	GSO
1.00%	28.12%	12.26%	1.82%	-34.74%	5.72%	103.71%	-0.77%	-11.32%	-13.12%	8.32%
2.00%	23.94%	9.44%	6.75%	-29.07%	6.77%	98.05%	-2.12%	-10.16%	-10.68%	7.07%
3.00%	19.76%	6.63%	11.68%	-23.40%	7.82%	92.39%	-3.47%	-9.00%	-8.24%	5.82%
4.00%	15.58%	3.82%	16.60%	-17.73%	8.88%	86.73%	-4.82%	-7.84%	-5.80%	4.57%
5.00%	11.40%	1.01%	21.53%	-12.05%	9.93%	81.07%	-6.17%	-6.68%	-3.36%	3.32%
6.00%	7.22%	-1.80%	26.46%	-6.38%	10.98%	75.41%	-7.52%	-5.52%	-0.92%	2.07%
7.00%	3.04%	-4.61%	31.39%	-0.71%	12.03%	69.75%	-8.87%	-4.36%	1.52%	0.82%
8.00%	-1.14%	-7.41%	36.31%	4.97%	13.08%	64.09%	-10.22%	-3.20%	3.95%	-0.43%
9.00%	-5.32%	-10.23%	41.24%	10.63%	14.14%	58.43%	-11.57%	-2.04%	6.40%	-1.68%
10.00%	-9.50%	-13.04%	46.17%	16.30%	15.19%	52.77%	-12.92%	-0.88%	8.84%	-2.93%
11.00%	-13.68%	-15.85%	51.10%	21.98%	16.24%	47.11%	-14.28%	0.28%	11.28%	-4.18%
12.00%	-17.86%	-18.66%	56.03%	27.65%	17.29%	41.45%	-15.63%	1.44%	13.72%	-5.43%
13.00%	-22.04%	-21.47%	60.96%	33.32%	18.34%	35.79%	-16.98%	2.60%	16.16%	-6.68%
14.00%	-26.22%	-24.28%	65.88%	38.99%	19.39%	30.13%	-18.33%	3.76%	18.60%	-7.93%
15.00%	-30.40%	-27.09%	70.81%	44.66%	20.45%	24.47%	-19.68%	4.92%	21.03%	-9.18%
16.00%	-34.58%	-29.90%	75.74%	50.33%	21.50%	18.81%	-21.03%	6.09%	23.47%	-10.43%
17.00%	-38.76%	-32.71%	80.67%	56.00%	22.55%	13.15%	-22.38%	7.25%	25.91%	-11.68%
18.00%	-42.94%	-35.52%	85.60%	61.68%	23.60%	7.49%	-23.73%	8.41%	28.35%	-12.93%
19.00%	-47.12%	-38.33%	90.53%	67.35%	24.65%	1.83%	-25.08%	9.57%	30.79%	-14.18%
20.00%	-51.30%	-41.14%	95.45%	73.02%	25.71%	-3.83%	-26.43%	10.73%	33.23%	-15.43%
21.00%	-55.48%	-43.95%	100.38%	78.69%	26.76%	-9.49%	-27.78%	11.89%	35.67%	-16.68%
22.00%	-59.66%	-46.76%	105.31%	84.36%	27.81%	-15.14%	-29.14%	13.05%	38.11%	-17.93%
23.00%	-63.84%	-49.57%	110.24%	90.03%	28.86%	-20.80%	-30.49%	14.21%	40.55%	-19.18%
24.00%	-68.02%	-52.39%	115.17%	95.70%	29.91%	-26.46%	-31.84%	15.37%	42.99%	-20.43%
25.00%	-72.20%	-55.19%	120.09%	101.38%	30.97%	-32.12%	-33.19%	16.53%	45.43%	-21.69%
26.00%	-76.38%	-58.00%	125.02%	107.05%	32.02%	-37.78%	-34.54%	17.69%	47.87%	-22.94%
27.00%	-80.56%	-60.82%	129.95%	112.72%	33.07%	-43.44%	-35.89%	18.85%	50.31%	-24.19%
28.00%	-84.74%	-63.63%	134.88%	118.39%	34.12%	-49.10%	-37.24%	20.01%	52.75%	-25.44%
29.00%	-88.92%	-66.44%	139.81%	124.06%	35.17%	-54.76%	-38.59%	21.17%	55.19%	-26.69%
30.00%	-93.10%	-69.25%	144.73%	129.73%	36.23%	-60.42%	-39.94%	22.33%	57.63%	-27.94%

Portfolio 4B

Returns	DJUSBM	DJUSCY	DJUSNC	DJUSHC	DJUSEN	DJUSRE	TRAN	CXBT	GSM	GSO	GT2	GT10
1.00%	1.24%	-0.15%	-0.16%	0.63%	0.71%	2.22%	-0.97%	-0.19%	0.85%	0.26%	95.55%	0.00%
2.00%	-0.77%	-1.96%	4.93%	3.46%	2.17%	4.72%	-2.30%	0.08%	2.17%	-0.34%	87.86%	0.00%
3.00%	-2.79%	-3.78%	10.02%	6.28%	3.62%	7.23%	-3.64%	0.34%	3.48%	-0.94%	80.18%	0.00%
4.00%	-4.81%	-5.59%	15.10%	9.11%	5.08%	9.73%	-4.97%	0.61%	4.80%	-1.55%	72.49%	0.00%
5.00%	-6.83%	-7.40%	20.19%	11.94%	6.53%	12.23%	-6.31%	0.87%	6.11%	-2.15%	64.81%	0.00%
6.00%	-8.76%	-9.17%	25.17%	14.73%	7.97%	14.73%	-7.66%	1.14%	7.43%	-2.75%	56.34%	0.83%
7.00%	-10.32%	-10.72%	29.68%	17.39%	9.33%	17.17%	-9.08%	1.41%	8.75%	-3.33%	44.42%	5.29%
8.00%	-11.88%	-12.27%	34.19%	20.05%	10.70%	19.62%	-10.50%	1.67%	10.08%	-3.91%	32.51%	9.76%
9.00%	-13.44%	-13.82%	38.69%	22.70%	12.06%	22.07%	-11.93%	1.94%	11.40%	-4.50%	20.59%	14.23%
10.00%	-15.00%	-15.38%	43.20%	25.36%	13.43%	24.51%	-13.35%	2.21%	12.72%	-5.08%	8.68%	18.70%
11.00%	-16.99%	-17.16%	48.08%	28.29%	14.83%	26.53%	-14.71%	2.52%	14.10%	-5.71%	0.00%	20.21%
12.00%	-20.12%	-19.55%	53.97%	31.96%	16.33%	27.41%	-15.93%	2.97%	15.65%	-6.47%	0.00%	13.79%
13.00%	-23.25%	-21.95%	59.86%	35.62%	17.83%	28.29%	-17.14%	3.42%	17.19%	-7.24%	0.00%	7.37%
14.00%	-26.38%	-24.34%	65.74%	39.29%	19.33%	29.16%	-18.35%	3.87%	18.73%	-8.00%	0.00%	0.95%
15.00%	-30.40%	-27.09%	70.81%	44.66%	20.45%	24.47%	-19.68%	4.92%	21.03%	-9.18%	0.00%	0.00%
16.00%	-34.58%	-29.90%	75.74%	50.33%	21.50%	18.81%	-21.03%	6.09%	23.47%	-10.43%	0.00%	0.00%
17.00%	-38.76%	-32.71%	80.67%	56.00%	22.55%	13.15%	-22.38%	7.25%	25.91%	-11.68%	0.00%	0.00%
18.00%	-42.94%	-35.52%	85.60%	61.68%	23.60%	7.49%	-23.73%	8.41%	28.35%	-12.93%	0.00%	0.00%
19.00%	-47.12%	-38.33%	90.53%	67.35%	24.65%	1.83%	-25.08%	9.57%	30.79%	-14.18%	0.00%	0.00%
20.00%	-51.30%	-41.14%	95.45%	73.02%	25.71%	-3.83%	-26.43%	10.73%	33.23%	-15.43%	0.00%	0.00%
21.00%	-55.48%	-43.95%	100.38%	78.69%	26.76%	-9.49%	-27.78%	11.89%	35.67%	-16.68%	0.00%	0.00%
22.00%	-59.66%	-46.77%	105.31%	84.36%	27.81%	-15.14%	-29.13%	13.05%	38.11%	-17.93%	0.00%	0.00%
23.00%	-63.84%	-49.58%	110.24%	90.03%	28.86%	-20.80%	-30.49%	14.21%	40.55%	-19.18%	0.00%	0.00%
24.00%	-68.02%	-52.39%	115.17%	95.70%	29.91%	-26.46%	-31.84%	15.37%	42.99%	-20.43%	0.00%	0.00%
25.00%	-72.20%	-55.20%	120.09%	101.38%	30.97%	-32.12%	-33.19%	16.53%	45.43%	-21.68%	0.00%	0.00%
26.00%	-76.38%	-58.01%	125.02%	107.05%	32.02%	-37.78%	-34.54%	17.69%	47.87%	-22.93%	0.00%	0.00%
27.00%	-80.56%	-60.82%	129.95%	112.72%	33.07%	-43.44%	-35.89%	18.85%	50.31%	-24.19%	0.00%	0.00%
28.00%	-84.74%	-63.63%	134.88%	118.39%	34.12%	-49.10%	-37.24%	20.01%	52.75%	-25.44%	0.00%	0.00%
29.00%	-88.92%	-66.44%	139.81%	124.06%	35.17%	-54.76%	-38.59%	21.17%	55.19%	-26.69%	0.00%	0.00%
30.00%	-93.10%	-69.25%	144.73%	129.73%	36.23%	-60.42%	-39.94%	22.33%	57.63%	-27.94%	0.00%	0.00%

Portfolio 4C

Returns	DJUSBMcc	DJUSCYcc	DJUSNCcc	DJUSHCcc	DJUSENcc	DJUSREcc	TRANcc	CXBTcc	GSMcc	GSOcc
1.00%	36.05%	-2.35%	13.04%	-31.12%	2.26%	110.29%	-11.67%	-18.76%	-24.60%	26.86%
2.00%	29.63%	-3.57%	17.59%	-24.15%	3.72%	102.39%	-11.98%	-15.92%	-18.92%	21.22%
3.00%	23.21%	-4.80%	22.14%	-17.18%	5.18%	94.48%	-12.29%	-13.09%	-13.23%	15.58%
4.00%	16.79%	-6.02%	26.70%	-10.21%	6.64%	86.58%	-12.61%	-10.25%	-7.55%	9.93%
5.00%	10.37%	-7.25%	31.25%	-3.24%	8.10%	78.68%	-12.92%	-7.41%	-1.86%	4.29%
6.00%	3.95%	-8.47%	35.80%	3.72%	9.57%	70.78%	-13.24%	-4.57%	3.82%	-1.35%
7.00%	-2.48%	-9.68%	40.36%	10.69%	11.02%	62.87%	-13.54%	-1.74%	9.50%	-7.00%
8.00%	-8.89%	-10.92%	44.91%	17.66%	12.48%	54.97%	-13.85%	1.09%	15.19%	-12.64%
9.00%	-15.31%	-12.15%	49.46%	24.63%	13.94%	47.06%	-14.16%	3.93%	20.87%	-18.28%
10.00%	-21.73%	-13.37%	54.02%	31.60%	15.40%	39.16%	-14.47%	6.76%	26.56%	-23.93%
11.00%	-28.15%	-14.60%	58.57%	38.57%	16.86%	31.26%	-14.79%	9.60%	32.24%	-29.57%
12.00%	-34.57%	-15.82%	63.13%	45.54%	18.32%	23.35%	-15.10%	12.44%	37.93%	-35.21%
13.00%	-40.99%	-17.05%	67.68%	52.50%	19.78%	15.45%	-15.41%	15.27%	43.61%	-40.85%
14.00%	-47.41%	-18.27%	72.23%	59.47%	21.24%	7.55%	-15.72%	18.11%	49.30%	-46.50%
15.00%	-53.83%	-19.50%	76.79%	66.44%	22.70%	-0.36%	-16.03%	20.94%	54.98%	-52.14%
16.00%	-60.25%	-20.72%	81.34%	73.41%	24.16%	-8.26%	-16.34%	23.78%	60.66%	-57.78%
17.00%	-66.67%	-21.95%	85.89%	80.38%	25.62%	-16.16%	-16.65%	26.61%	66.35%	-63.43%
18.00%	-73.09%	-23.17%	90.45%	87.35%	27.08%	-24.07%	-16.97%	29.45%	72.03%	-69.07%
19.00%	-79.50%	-24.40%	95.00%	94.32%	28.55%	-31.97%	-17.28%	32.29%	77.72%	-74.71%
20.00%	-85.92%	-25.62%	99.55%	101.28%	30.01%	-39.87%	-17.59%	35.12%	83.40%	-80.36%
21.00%	-92.34%	-26.85%	104.11%	108.25%	31.47%	-47.78%	-17.90%	37.96%	89.08%	-86.00%
22.00%	-98.76%	-28.07%	108.66%	115.22%	32.93%	-55.68%	-18.21%	40.79%	94.77%	-91.64%
23.00%	-105.18%	-29.30%	113.21%	122.19%	34.39%	-63.58%	-18.52%	43.63%	100.45%	-97.29%
24.00%	-111.60%	-30.52%	117.77%	129.16%	35.85%	-71.49%	-18.83%	46.47%	106.14%	-102.93%
25.00%	-118.02%	-31.75%	122.32%	136.13%	37.31%	-79.39%	-19.14%	49.30%	111.82%	-108.57%
26.00%	-124.44%	-32.97%	126.87%	143.10%	38.77%	-87.29%	-19.46%	52.14%	117.51%	-114.22%
27.00%	-130.86%	-34.20%	131.43%	150.06%	40.23%	-95.20%	-19.77%	54.97%	123.19%	-119.86%
28.00%	-137.28%	-35.42%	135.98%	157.03%	41.69%	-103.10%	-20.08%	57.81%	128.87%	-125.50%
29.00%	-143.70%	-36.65%	140.53%	164.00%	43.15%	-111.00%	-20.39%	60.64%	134.56%	-131.14%
30.00%	-150.12%	-37.87%	145.09%	170.97%	44.61%	-118.91%	-20.70%	63.48%	140.24%	-136.79%

Appendix VII: Data for Risk Aversion Coefficient vs. Diversification Benefit

Risk Aversion Coefficient	R^{CE} 1A	R^{CE} 2A	Benefits
0.5	27.86%	43.11%	15.24%
1.0	15.68%	23.56%	7.89%
1.5	11.25%	16.74%	5.49%
2.0	8.77%	13.10%	4.33%
2.5	7.07%	10.73%	3.67%
3.0	5.75%	9.00%	3.25%
3.5	4.65%	7.63%	2.98%
4.0	3.70%	6.49%	2.80%
4.5	2.83%	5.50%	2.67%
5.0	2.03%	4.62%	2.59%

Risk Aversion Coefficient	R^{CE} 1B	R^{CE} 2B	Benefits
0.5	37.12%	53.63%	16.52%
1.0	18.80%	27.07%	8.27%
1.5	12.69%	18.21%	5.51%
2.0	9.64%	13.78%	4.14%
2.5	7.80%	11.12%	3.31%
3.0	6.58%	9.34%	2.76%
3.5	5.70%	8.08%	2.37%
4.0	5.05%	7.12%	2.08%
4.5	4.53%	6.38%	1.85%
5.0	4.12%	5.79%	1.66%

Risk Aversion Coefficient	R^{CE} 1C	R^{CE} 2C	Benefits
0.5	25.80%	42.05%	16.25%
1.0	14.79%	23.09%	8.30%
1.5	10.99%	16.66%	5.67%
2.0	8.99%	13.36%	4.37%
2.5	7.71%	11.31%	3.60%
3.0	6.79%	9.90%	3.10%
3.5	6.08%	8.84%	2.75%
4.0	5.50%	8.00%	2.50%
4.5	5.00%	7.31%	2.31%
5.0	4.57%	6.73%	2.16%

Risk Aversion Coefficient	R^{CE} 3A	R^{CE} 4A	Benefits
0.5	27.83%	38.23%	10.40%
1.0	17.96%	22.49%	4.53%
1.5	14.33%	17.03%	2.70%
2.0	12.26%	14.14%	1.88%
2.5	10.81%	12.27%	1.46%
3.0	9.67%	10.92%	1.25%
3.5	8.71%	9.86%	1.15%
4.0	7.86%	8.98%	1.12%
4.5	7.09%	8.23%	1.14%
5.0	6.37%	7.56%	1.19%

Risk Aversion Coefficient	R^{CE} 3B	R^{CE} 4B	Benefits
0.5	60.82%	72.56%	11.74%
1.0	30.67%	36.57%	5.89%
1.5	20.62%	24.57%	3.95%
2.0	15.59%	18.57%	2.97%
2.5	12.58%	14.96%	2.39%
3.0	10.56%	12.56%	2.00%
3.5	9.12%	10.84%	1.72%
4.0	8.04%	9.55%	1.51%
4.5	7.20%	8.55%	1.35%
5.0	6.53%	7.75%	1.22%

Risk Aversion Coefficient	R^{CE} 3C	R^{CE} 4C	Benefits
0.5	24.98%	53.20%	28.22%
1.0	15.98%	29.56%	13.58%
1.5	12.86%	21.60%	8.74%
2.0	11.20%	17.57%	6.37%
2.5	10.13%	15.11%	4.97%
3.0	9.36%	13.42%	4.07%
3.5	8.75%	12.19%	3.44%
4.0	8.25%	11.24%	2.99%
4.5	7.81%	10.47%	2.66%
5.0	7.43%	9.83%	2.41%

Appendix VIII: Regressions for Base Portfolio Aggressiveness vs. Diversification Benefit

Portfolio 1A & Portfolio 2A

Regression: ben		
	constant	agr
coefficient	-0.0111722	0.13661482
std error of coef	0.00074907	0.00132296
t-ratio	-14.9148	103.2642
p-value	0.0000%	0.0000%
beta-weight		0.9989
standard error of regression		0.00162949
R-squared		99.78%
adjusted R-squared		99.77%
number of observations		26
residual degrees of freedom		24
t-statistic for computing 95%-confidence intervals		2.0639

Portfolio 3A & Portfolio 4A

Regression: ben		
	constant	agr
coefficient	-0.009166	0.06774888
std error of coef	0.00299885	0.0040556
t-ratio	-3.0565	16.7050
p-value	0.6795%	0.0000%
beta-weight		0.9692
standard error of regression		0.00515598
R-squared		93.94%
adjusted R-squared		93.60%
number of observations		20
residual degrees of freedom		18
t-statistic for computing 95%-confidence intervals		2.1009

Portfolio 1B & Portfolio 2B

Regression: ben		
	constant	agr
coefficient	-0.0265567	0.2204847
std error of coef	0.00130447	0.00330694
t-ratio	-20.3582	66.6733
p-value	0.0000%	0.0000%
beta-weight		0.9969
standard error of regression		0.00287217
R-squared		99.37%
adjusted R-squared		99.35%
number of observations		30
residual degrees of freedom		28
t-statistic for computing 95%-confidence intervals		2.0484

Portfolio 3B & Portfolio 4B

Regression: ben		
	constant	agr
coefficient	-0.0130356	0.10123349
std error of coef	0.00095169	0.00213597
t-ratio	-13.6973	47.3945
p-value	0.0000%	0.0000%
beta-weight		0.9938
standard error of regression		0.00254747
R-squared		98.77%
adjusted R-squared		98.72%
number of observations		30
residual degrees of freedom		28
t-statistic for computing 95%-confidence intervals		2.0484

Portfolio 1C & Portfolio 2C

Regression: ben		
	constant	agr
coefficient	-0.0045183	0.08755161
std error of coef	0.00053044	0.00054782
t-ratio	-8.5182	159.8197
p-value	0.0000%	0.0000%
beta-weight		0.9995
standard error of regression		0.00125405
R-squared		99.91%
adjusted R-squared		99.90%
number of observations		26
residual degrees of freedom		24
t-statistic for computing 95%-confidence intervals		2.0639

Portfolio 3C & Portfolio 4C

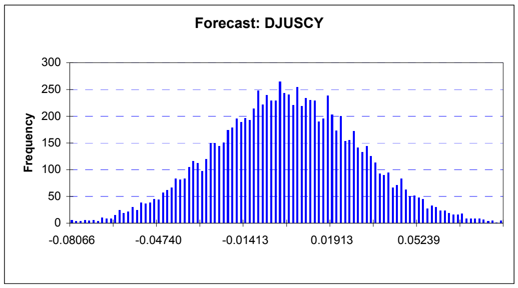
Regression: ben		
	constant	agr
coefficient	-0.0225809	0.10713177
std error of coef	0.00240556	0.00202351
t-ratio	-9.3869	52.9436
p-value	0.0000%	0.0000%
beta-weight		0.9963
standard error of regression		0.00505557
R-squared		99.26%
adjusted R-squared		99.22%
number of observations		23
residual degrees of freedom		21
t-statistic for computing 95%-confidence intervals		2.0796

Appendix IX: Monte Carlo Simulation Forecasts

Forecast: DJUSCY

Summary:
Display Range is from -0.08149 to 0.08482
Entire Range is from -0.09707 to 0.11229
After 10,000 Trials, the Std. Error of the Mean is 0.00028

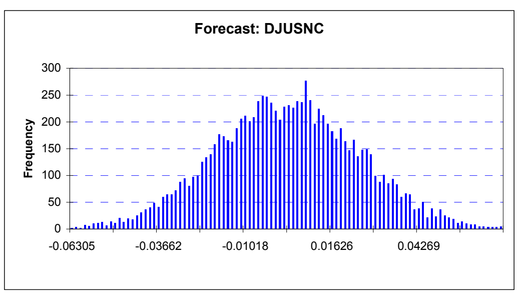
Statistics:	Value
Trials	10000
Mean	0.00167
Median	0.00158
Mode	---
Standard Deviation	0.02772
Variance	0.00077
Skewness	-0.01107
Kurtosis	2.94534
Coeff. of Variability	16.64544
Range Minimum	-0.09707
Range Maximum	0.11229
Range Width	0.20936
Mean Std. Error	0.00028



Forecast: DJUSNC

Summary:
Display Range is from -0.06371 to 0.06847
Entire Range is from -0.07652 to 0.08605
After 10,000 Trials, the Std. Error of the Mean is 0.00022

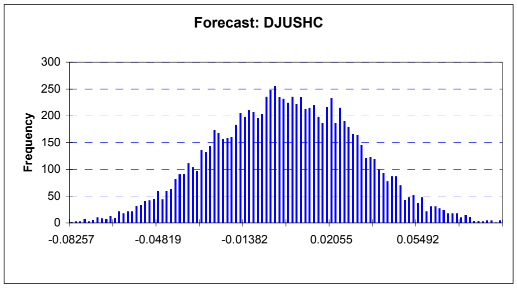
Statistics:	Value
Trials	10000
Mean	0.00238
Median	0.00242
Mode	---
Standard Deviation	0.02203
Variance	0.00049
Skewness	-0.00511
Kurtosis	2.95195
Coeff. of Variability	9.26398
Range Minimum	-0.07652
Range Maximum	0.08605
Range Width	0.16256
Mean Std. Error	0.00022



Forecast: DJUSHC

Summary:
Display Range is from -0.08343 to 0.08843
Entire Range is from -0.11026 to 0.10930
After 10,000 Trials, the Std. Error of the Mean is 0.00029

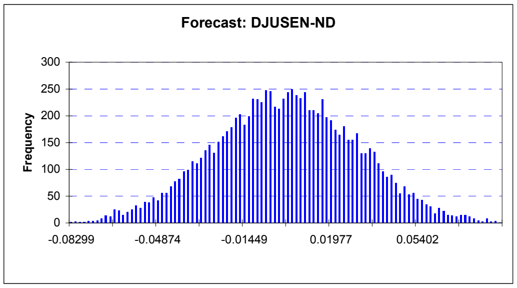
Statistics:	Value
Trials	10000
Mean	0.00250
Median	0.00253
Mode	---
Standard Deviation	0.02864
Variance	0.00082
Skewness	-0.00618
Kurtosis	2.92887
Coeff. of Variability	11.44091
Range Minimum	-0.11026
Range Maximum	0.10930
Range Width	0.21956
Mean Std. Error	0.00029



Forecast: DJUSEN-ND

Summary:
Display Range is from -0.08385 to 0.08742
Entire Range is from -0.12030 to 0.11847
After 10,000 Trials, the Std. Error of the Mean is 0.00029

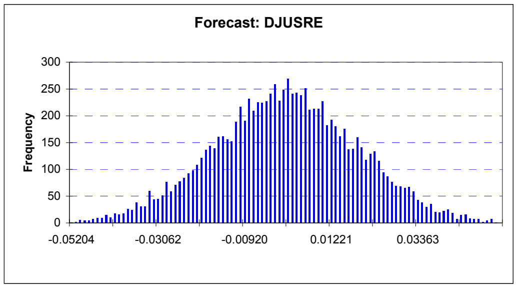
Statistics:	Value
Trials	10000
Mean	0.00178
Median	0.00189
Mode	---
Standard Deviation	0.02854
Variance	0.00081
Skewness	-0.01717
Kurtosis	2.96341
Coeff. of Variability	15.99861
Range Minimum	-0.12030
Range Maximum	0.11847
Range Width	0.23877
Mean Std. Error	0.00029



Forecast: DJUSRE

Summary:
Display Range is from -0.05257 to 0.05451
Entire Range is from -0.06154 to 0.07359
After 10,000 Trials, the Std. Error of the Mean is 0.00018

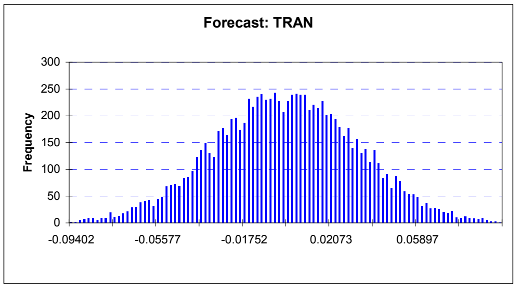
Statistics:	Value
Trials	10000
Mean	0.00097
Median	0.00096
Mode	---
Standard Deviation	0.01785
Variance	0.00032
Skewness	0.02126
Kurtosis	3.00400
Coeff. of Variability	18.43427
Range Minimum	-0.06154
Range Maximum	0.07359
Range Width	0.13513
Mean Std. Error	0.00018



Forecast: TRAN

Summary:
Display Range is from -0.09497 to 0.09626
Entire Range is from -0.12460 to 0.15347
After 10,000 Trials, the Std. Error of the Mean is 0.00032

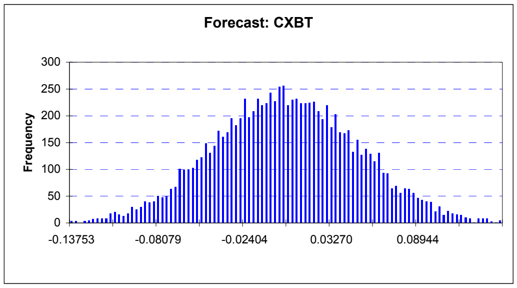
Statistics:	Value
Trials	10000
Mean	0.00065
Median	0.00077
Mode	---
Standard Deviation	0.03187
Variance	0.00102
Skewness	-0.00120
Kurtosis	3.01248
Coeff. of Variability	49.33658
Range Minimum	-0.12460
Range Maximum	0.15347
Range Width	0.27807
Mean Std. Error	0.00032



Forecast: CXBT

Summary:
Display Range is from -0.13895 to 0.14477
Entire Range is from -0.19201 to 0.18363
After 10,000 Trials, the Std. Error of the Mean is 0.00047

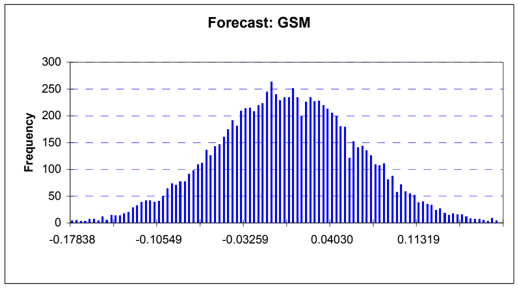
Statistics:	Value
Trials	10000
Mean	0.00291
Median	0.00223
Mode	---
Standard Deviation	0.04729
Variance	0.00224
Skewness	0.01582
Kurtosis	2.95645
Coeff. of Variability	16.25712
Range Minimum	-0.19201
Range Maximum	0.18363
Range Width	0.37564
Mean Std. Error	0.00047



Forecast: GSM

Summary:
Display Range is from -0.18020 to 0.18427
Entire Range is from -0.25404 to 0.22688
After 10,000 Trials, the Std. Error of the Mean is 0.00061

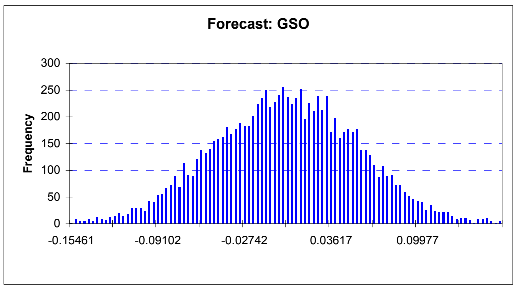
Statistics:	Value
Trials	10000
Mean	0.00203
Median	0.00199
Mode	---
Standard Deviation	0.06074
Variance	0.00369
Skewness	-0.02445
Kurtosis	3.06097
Coeff. of Variability	29.89369
Range Minimum	-0.25404
Range Maximum	0.22688
Range Width	0.48092
Mean Std. Error	0.00061



Forecast: GSO

Summary:
Display Range is from -0.15620 to 0.16178
Entire Range is from -0.20135 to 0.19476
After 10,000 Trials, the Std. Error of the Mean is 0.00053

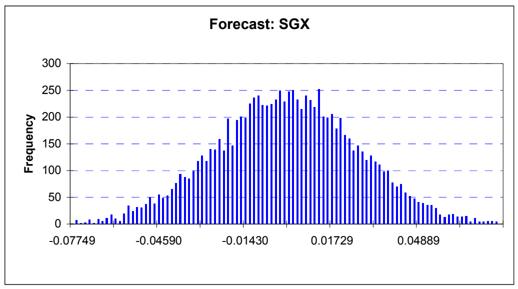
Statistics:	Value
Trials	10000
Mean	0.00279
Median	0.00302
Mode	---
Standard Deviation	0.05300
Variance	0.00281
Skewness	-0.02208
Kurtosis	2.96260
Coeff. of Variability	19.01908
Range Minimum	-0.20135
Range Maximum	0.19476
Range Width	0.39611
Mean Std. Error	0.00053



Forecast: SGX

Summary:
Display Range is from -0.07828 to 0.07969
Entire Range is from -0.10109 to 0.09908
After 10,000 Trials, the Std. Error of the Mean is 0.00026

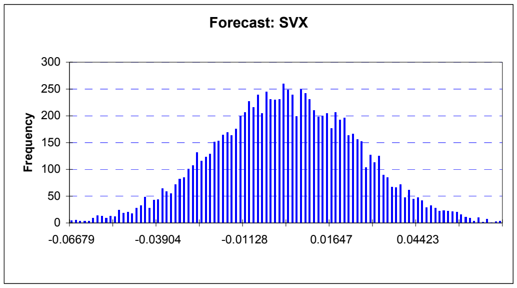
Statistics:	Value
Trials	10000
Mean	0.00071
Median	0.00076
Mode	---
Standard Deviation	0.02633
Variance	0.00069
Skewness	-0.00665
Kurtosis	3.07388
Coeff. of Variability	37.30098
Range Minimum	-0.10109
Range Maximum	0.09908
Range Width	0.20017
Mean Std. Error	0.00026



Forecast: SVX

Summary:
Display Range is from -0.06749 to 0.07129
Entire Range is from -0.09333 to 0.09075
After 10,000 Trials, the Std. Error of the Mean is 0.00023

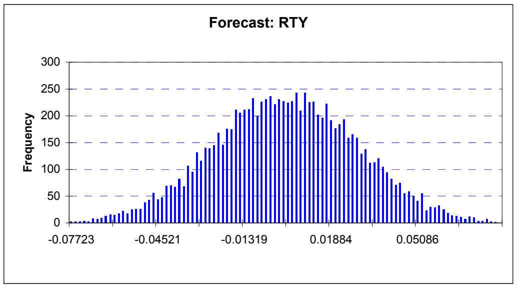
Statistics:	Value
Trials	10000
Mean	0.00190
Median	0.00189
Mode	---
Standard Deviation	0.02313
Variance	0.00054
Skewness	-0.02571
Kurtosis	3.00052
Coeff. of Variability	12.15658
Range Minimum	-0.09333
Range Maximum	0.09075
Range Width	0.18409
Mean Std. Error	0.00023



Forecast: RTY

Summary:
Display Range is from -0.07803 to 0.08208
Entire Range is from -0.10757 to 0.09846
After 10,000 Trials, the Std. Error of the Mean is 0.00027

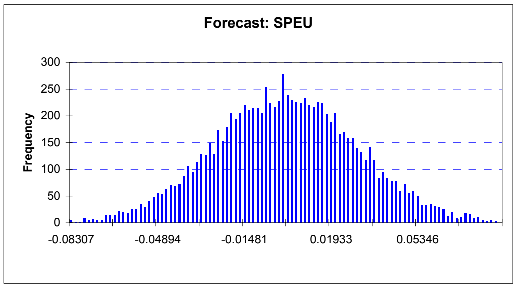
Statistics:	Value
Trials	10000
Mean	0.00202
Median	0.00201
Mode	---
Standard Deviation	0.02669
Variance	0.00071
Skewness	-0.03084
Kurtosis	2.92620
Coeff. of Variability	13.18312
Range Minimum	-0.10757
Range Maximum	0.09846
Range Width	0.20603
Mean Std. Error	0.00027



Forecast: SPEU

Summary:
Display Range is from -0.08393 to 0.08674
Entire Range is from -0.10014 to 0.12854
After 10,000 Trials, the Std. Error of the Mean is 0.00028

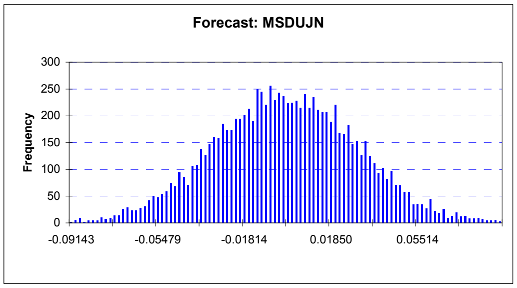
Statistics:	Value
Trials	10000
Mean	0.00141
Median	0.00119
Mode	---
Standard Deviation	0.02845
Variance	0.00081
Skewness	0.03730
Kurtosis	2.96032
Coeff. of Variability	20.21588
Range Minimum	-0.10014
Range Maximum	0.12854
Range Width	0.22867
Mean Std. Error	0.00028



Forecast: MSDUJN

Summary:
Display Range is from -0.09235 to 0.09087
Entire Range is from -0.13997 to 0.14139
After 10,000 Trials, the Std. Error of the Mean is 0.00031

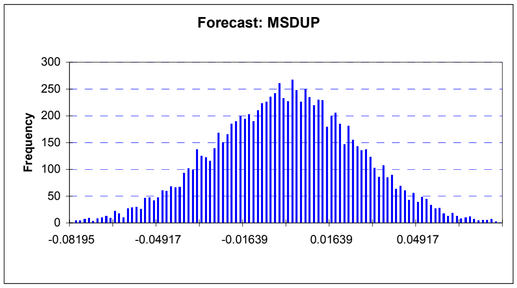
Statistics:	Value
Trials	10000
Mean	-0.00074
Median	-0.00116
Mode	---
Standard Deviation	0.03054
Variance	0.00093
Skewness	0.04119
Kurtosis	3.10249
Coeff. of Variability	-41.42433
Range Minimum	-0.13997
Range Maximum	0.14139
Range Width	0.28136
Mean Std. Error	0.00031



Forecast: MSDUP

Summary:
Display Range is from -0.08277 to 0.08113
Entire Range is from -0.10997 to 0.09956
After 10,000 Trials, the Std. Error of the Mean is 0.00027

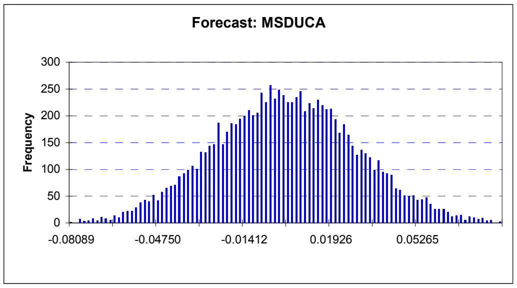
Statistics:	Value
Trials	10000
Mean	-0.00082
Median	-0.00052
Mode	---
Standard Deviation	0.02732
Variance	0.00075
Skewness	-0.02837
Kurtosis	3.00843
Coeff. of Variability	-33.31781
Range Minimum	-0.10997
Range Maximum	0.09956
Range Width	0.20953
Mean Std. Error	0.00027



Forecast: MSDUCA

Summary:
Display Range is from -0.08172 to 0.08519
Entire Range is from -0.09935 to 0.10830
After 10,000 Trials, the Std. Error of the Mean is 0.00028

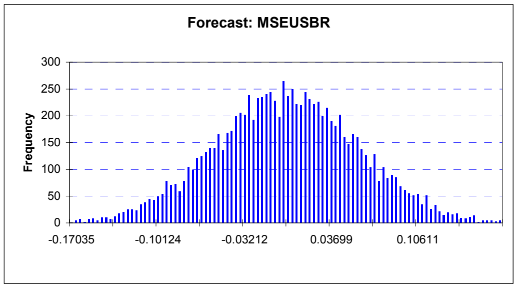
Statistics:	Value
Trials	10000
Mean	0.00174
Median	0.00151
Mode	---
Standard Deviation	0.02782
Variance	0.00077
Skewness	0.04313
Kurtosis	2.96423
Coeff. of Variability	16.01250
Range Minimum	-0.09935
Range Maximum	0.10830
Range Width	0.20765
Mean Std. Error	0.00028



Forecast: MSEUSBR

Summary:
Display Range is from -0.17208 to 0.17349
Entire Range is from -0.25710 to 0.20810
After 10,000 Trials, the Std. Error of the Mean is 0.00058

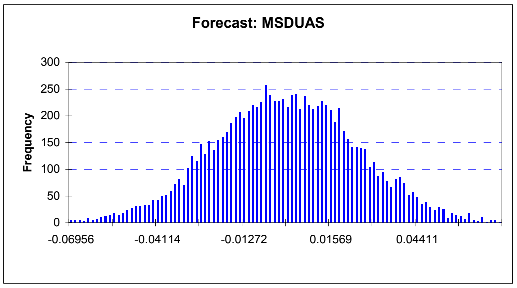
Statistics:	Value
Trials	10000
Mean	0.00071
Median	0.00062
Mode	---
Standard Deviation	0.05760
Variance	0.00332
Skewness	-0.01740
Kurtosis	2.99716
Coeff. of Variability	81.36911
Range Minimum	-0.25710
Range Maximum	0.20810
Range Width	0.46520
Mean Std. Error	0.00058



Forecast: MSDUAS

Summary:
Display Range is from -0.07027 to 0.07182
Entire Range is from -0.09128 to 0.08227
After 10,000 Trials, the Std. Error of the Mean is 0.00024

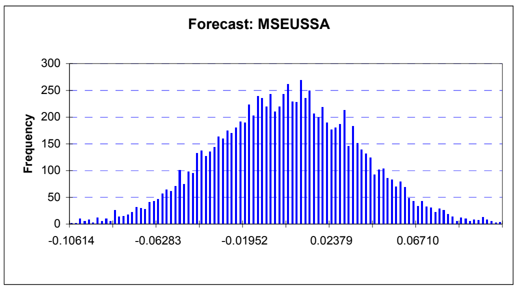
Statistics:	Value
Trials	10000
Mean	0.00078
Median	0.00070
Mode	---
Standard Deviation	0.02368
Variance	0.00056
Skewness	-0.00799
Kurtosis	3.01558
Coeff. of Variability	30.52634
Range Minimum	-0.09128
Range Maximum	0.08227
Range Width	0.17355
Mean Std. Error	0.00024



Forecast: MSEUSSA

Summary:
Display Range is from -0.10722 to 0.10933
Entire Range is from -0.15711 to 0.14054
After 10,000 Trials, the Std. Error of the Mean is 0.00036

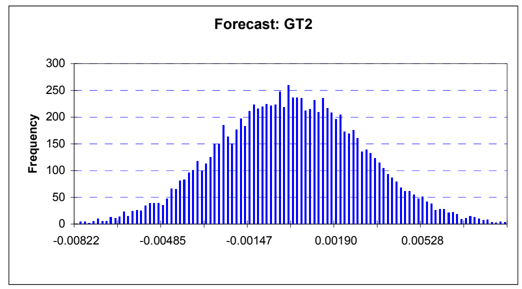
Statistics:	Value
Trials	10000
Mean	0.00105
Median	0.00143
Mode	---
Standard Deviation	0.03609
Variance	0.00130
Skewness	-0.01945
Kurtosis	3.04367
Coeff. of Variability	34.25854
Range Minimum	-0.15711
Range Maximum	0.14054
Range Width	0.29764
Mean Std. Error	0.00036



Forecast: GT2

Summary:
Display Range is from -0.00831 to 0.00857
Entire Range is from -0.00998 to 0.00999
After 10,000 Trials, the Std. Error of the Mean is 0.00003

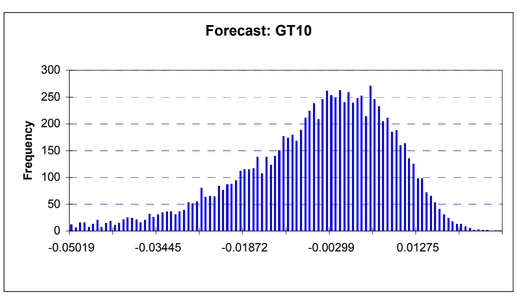
Statistics:	Value
Trials	10000
Mean	0.00013
Median	0.00013
Mode	---
Standard Deviation	0.00281
Variance	0.00001
Skewness	-0.00789
Kurtosis	2.94819
Coeff. of Variability	21.38304
Range Minimum	-0.00998
Range Maximum	0.00999
Range Width	0.01997
Mean Std. Error	0.00003



Forecast: GT10

Summary:
Display Range is from -0.05058 to 0.02809
Entire Range is from -0.10484 to 0.02809
After 10,000 Trials, the Std. Error of the Mean is 0.00015

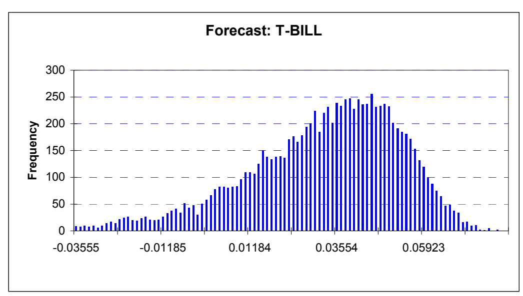
Statistics:	Value
Trials	10000
Mean	-0.00624
Median	-0.00369
Mode	---
Standard Deviation	0.01478
Variance	0.00022
Skewness	-1.17025
Kurtosis	5.37587
Coeff. of Variability	-2.36942
Range Minimum	-0.10484
Range Maximum	0.02809
Range Width	0.13293
Mean Std. Error	0.00015



Forecast: T-BILL

Summary:
Display Range is from -0.03614 to 0.08234
Entire Range is from -0.10515 to 0.08234
After 10,000 Trials, the Std. Error of the Mean is 0.00022

Statistics:	Value
Trials	10000
Mean	0.03125
Median	0.03497
Mode	---
Standard Deviation	0.02246
Variance	0.00050
Skewness	-1.09488
Kurtosis	5.13591
Coeff. of Variability	0.71883
Range Minimum	-0.10515
Range Maximum	0.08234
Range Width	0.18749
Mean Std. Error	0.00022



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