Incomplete Contracts and Strategic Ambiguity

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Why are observed contracts so often incomplete in the sense that they leave contracting parties' obligations vague or unspecified? Traditional answers to this question invoke transaction costs or bounded rationality. In contrast, we argue that such incompleteness is often an essential feature of a well-designed contract. Specifically, once some aspects of performance are unverifiable, it is often optimal to leave other verifiable aspects of performance unspecified. We explore the conditions under which this occurs, and investigate the structure of optimal contracts when these conditions are satisfied. (*JEL* D23, D82, L14)

Economic agents rarely write contracts that are complete in the Arrow-Debreu sense. Few regard this as puzzling, because many relevant events and actions are not verifiable. Nevertheless, one also frequently observes contracts that seem "excessively" incomplete. This is true in two different senses. First, contracts sometimes make actions less sensitive to verifiable events than would appear optimal. For example, a manager may receive a fixed wage, even when it is possible to condition pay on verifiable measures of performance. Second, and more striking, contracts often fail even to specify verifiable obligations of the parties. For example, a managerial employment contract may contain no explicit provision regarding hours of work.

There are two common explanations for these observations. The first holds that parties write simple (i.e., noncontingent) contracts and fail to specify obligations in unlikely states of the world to economize on transaction costs (see, for example, Ronald H. Coase [1937] and Oliver Williamson [1975, 1985]).[1] The second argues that boundedly rational parties may not distinguish certain contingencies or recognize the need to specify some dimension of contractual performance (see, for example, Herbert A. Simon, 1981).

Though appealing, these theories do not explain the apparent propensity for even sophisticated parties to leave contracts incomplete in circumstances in which transaction costs are low. To take a familiar example, the typical employment contract for an academic economist rarely ties salary explicitly to any observable output measure such as teaching ratings or the placement of Ph.D. students, and it usually fails to specify many easily described aspects of the university's obligations (e.g., secretarial support, office space, and—most notably—future wages). More generally, business executives often seem to prefer contracts that provide parties with considerable discretion (Stewart Macauley, 1963).

A number of recent papers offer explanations for the first aspect of contractual incompleteness mentioned above: the failure to write contracts that make outcomes sensitive to observable events. Bengt Holmstrom and Paul R. Milgrom (1992), for example, show how

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[1] In the extreme, transaction costs can render complete contracts infeasible: even if certain events are verifiable, they may be too costly to describe in a contract.

contracts that tie an agent's compensation to verifiable measures of performance can divert effort and attention from other more important, but less easily measured, aspects of performance.[2] Alternatively, Franklin Allen and Douglas Gale (1992) and Kathryn E. Spier (1992) describe circumstances in which the act of proposing a contingent contract can lead other parties to draw negative inferences.[3]

In contrast to these papers, we focus on the second aspect of contractual incompleteness: why contracts often contain obvious "gaps" that provide parties with so much discretion. We show that if *some* aspects of performance are noncontractible, then it may be optimal to leave *other* verifiable aspects of performance unspecified. In other words, if contracts must be somewhat incomplete (due to transaction costs or limits on verifiability) then it is often optimal for parties to write contracts that are even more incomplete.

Our analysis proceeds on the basis of two assumptions: first, certain actions that are observable to the contracting parties are not publicly verifiable (and hence cannot be specified contractually); second, strategic play governs all discretionary choices permitted to the agents under their contract. The first assumption implies that the parties necessarily have discretionary choices regardless of the contract they sign. Under the second assumption, all discretionary choices (both nonverifiable actions and any verifiable actions not specified in the contract) are determined by strategic play, and the equilibrium of the associated game forms an implicit or self-enforcing agreement that complements the explicit agreement embodied in the contract.

Within this framework, the design of the explicit agreement affects outcomes *directly* by specifying particular choices for some verifiable actions. This is the effect typically considered in arguments for the optimality of "complete" contracts, but it is not the only effect. Contract design also affects outcomes *indirectly* by altering the set of feasible self-enforcing implicit agreements covering those aspects of performance that are determined by strategic play.

The indirect effect of contract design has two components. First, without altering the set of actions that are tied down in the contract, changes in the contractually mandated values of particular actions may affect the set of equilibria governing actions that are left unspecified. Second, the implicit portion of an agreement may work better—and the entire relationship may be more beneficial to the parties—if the parties *expand* the set of discretionary choices to include actions that are verifiable. In particular, by leaving some verifiable actions unspecified, an incomplete contract may enable each player to influence the other's nonverifiable choices in a mutually beneficial way. Consequently, as long as some aspects of performance are nonverifiable, parties to a contract may prefer to leave some verifiable aspects of performance ambiguous, rather than specify them explicitly. We use the term "*strategic ambiguity*" to describe contractual incompleteness that arises from this consideration.

As an illustration of this point, consider again the contracting problem involving a faculty member and a university. In this relationship there is one very important aspect of performance that is nonverifiable but reasonably observable: the level of the faculty member's effort (interpreted broadly to include his/her cooperation in providing services that are beneficial to the university). By contrast, it would be fairly easy to write a contract specifying most aspects of the university's obligations to the faculty member (e.g., secretarial support, office location, current and future salary). The theory we present here suggests that the extensive degree of incompleteness in the specification of the university's obligations to

[2] For example, a faculty member who is compensated based on teaching ratings might be wary of challenging his students or making exams difficult.

[3] Both papers combine asymmetric information with some other cost of contingent contracts. In Spier (1992) there is an exogenous cost of writing a contingent contract: she shows that asymmetric information expands the set of cases in which a noncontingent contract is selected. In contrast, Allen and Gale (1992) assume that one of the contracting parties is able to distort the realization of an accounting signal at some cost. Such distortion occurs, however, only under a contingent contract. This causes the standard single-crossing property to fail at the noncontingent contract, and leads to a (refined) pooling equilibrium at this contract.

the faculty member arises from the need to encourage the faculty member's effort. With a complete contract, the faculty member would have recourse to the courts if the university reneged on its contractual obligations, but the university would have no recourse if the faculty member shirked. Giving the Dean discretion helps to establish appropriate incentives, since it provides the university with recourse. In particular, the Dean can adjust the terms of future employment should the faculty member shirk (e.g., the Dean can put the faculty member in the basement or not give him/her a raise). If the Dean uses this discretion opportunistically and renegs on a nonconcontractual promise, the faculty member also has recourse (shirking). Shirking is thereby reduced and the overall relationship works better (joint surplus is increased).

This principle is widely applicable in practice. According to Macauley (1963 p. 64):

> Not only are contract and contract law not needed in many situations, their use may have, or may be thought to have, undesirable consequences... . Even where agreement can be reached at the negotiation stage, carefully planned arrangements may create undesirable exchange relationships between business units. Some businessmen object that in such a carefully worked out relationship, one gets performance only to the letter of the contract.

This theme—that the specification of too many aspects of verifiable performance in a contract may worsen noncontractible aspects of performance—is precisely the key to the theory of incomplete contracts that we present here.

The remainder of the paper is organized as follows. In Section I we consider static contracting problems. Subject to some qualifications, we establish that there is never an affirmative reason to write an incomplete contract in a static setting. Thus, in this limited context, the commonly assumed desirability of complete contracts is affirmed. This result leads us to focus in the rest of the paper on dynamic models.

Section II provides a preliminary discussion of contractual incompleteness in dynamic settings. We clarify the relevant differences between static and dynamic settings, and we explain why the results of Section I do not carry over once dynamics are introduced.

The remainder of the paper explores the role of contractual incompleteness in more specific dynamic settings. In Section III, we consider models with ''structural'' intertemporal linkages (current choices affect subsequent incentives directly through payoff functions). We investigate the conditions under which parties can improve outcomes by permitting flexible responses. Much of this section focuses on the characteristics of optimal contracts when actions are either strategic complements or strategic substitutes (Jeremy Bulow et al., 1985). We show that incompleteness is optimal in the former instances, but not in the latter.

In Section IV, we examine dynamic settings in which structural intertemporal linkages are absent and dynamic linkages are purely strategic. We investigate the conditions under which contractual incompleteness helps parties sustain cooperation by allowing them to create more severe history-dependent punishments. For the sake of tractability (particularly regarding the treatment of renegotiation), we restrict our attention to settings with finitely repeated interactions. As in Section III, we find that the desirability of contractual incompleteness depends critically on whether actions are strategic complements or strategic substitutes.

Finally, in Section V we offer concluding remarks and discuss some related papers.

I. Static Contracting Problems

In this section, we study the nature of optimal contracts for situations in which two individuals can contract prior to playing a static (i.e., simultaneous-move) game. We will see that this setting provides one environment in which the often-presumed optimality of complete contracts is affirmed.

Before introducing a formal model, we begin with a simple illustration. Imagine that two players ($i = 1, 2$) seek to write a contract prior to playing the simultaneous-move game depicted in Figure 1. Each player i has three possible actions, $\{a_{i1}, a_{i2}, a_{i3}\}$. We will assume that a court cannot distinguish a_{i1} from a_{i2};

however, it can tell whether player i has chosen a_{i3}. Hence, the most a contract can do is require player i to choose one of its first two actions, or require it to choose its third action. In this contracting problem, there are four possible "complete contracts" which specify each player's action choice as fully as is possible given the limits of verifiability (both players could be required to choose their third action, player 1 could be required to choose one of his first two actions and player 2 could be required to choose his third action, etc.). There are also five possible incomplete contracts: the fully incomplete "contract" specifying nothing, and four partially incomplete contracts specifying one of the players' choices as tightly as possible (given the limits imposed by verifiability), and specifying nothing for the other player. Under the fully incomplete contract, equilibrium play results in action choices (a_{13}, a_{23}) and payoffs of (2, 2). One can easily check that no other incomplete contract does better for either player. Note, however, that the players can do as well with a complete contract requiring each player i to choose a_{i3}. In fact, they can do even better: with the complete contract specifying that each player choose one of its first two actions, equilibrium play results in action choices (a_{11}, a_{21}) and a payoff of (3, 3). Hence, a complete contract is optimal.

We now extend this point formally to a general class of simultaneous-move contracting problems. We suppose that the two players will play a simultanous-move game in which each player $i = 1, 2$ chooses an action a_i from a finite set of possible actions A_i. Player i's payoff function is $u_i : A_1 \times A_2 \to \mathbb{R}$. Note that an element $a_i \in A_i$ may be a vector comprising many activities, including perhaps a monetary transfer from player i to player $j \neq i$. In what follows, we let Q_i denote the power set of A_i (that is, the set whose elements are all possible subsets of A_i).

In anticipation of playing this game, the two individuals may choose to write a contract that restricts their actions. The scope of any such agreement is necessarily limited because courts may be incapable of verifying certain choices. Formally, we model the verifiability constraints as partitions $P_1 = \{p_{11}, ..., p_{1N_1}\}$ and $P_2 = \{p_{21}, ..., p_{2N_2}\}$ of the sets A_1 and A_2,

			Player 2: p_{21}		p_{22}
			a_{21}	a_{22}	a_{23}
Player 1	p_{11}	a_{11}	3,3	0,0	0,5
		a_{12}	0,0	-1,-1	1,1
	p_{12}	a_{13}	5,0	1,1	2,2

FIGURE 1. AN EXAMPLE FOR WHICH THE UNIQUE OPTIMAL CONTRACT IS COMPLETE

respectively. The interpretation of the partition P_i is that a court cannot distinguish two actions belonging to the same element of the partition, say a' and $a'' \in p_{in} \subseteq A_i$, but it can distinguish between actions belonging to two different elements of P_i, say $a' \in p_{in}$ and $a'' \in p_{im}$. In Figure 1, for example, each player's partition contains two elements: $p_{i1} = \{a_{i1}, a_{i2}\}$ and $p_{i2} = \{a_{i3}\}$. In what follows, it will be convenient to define $p_i(a)$ to be the element of P_i containing the action $a \in A_i$.

For most of this paper, we focus on contracts that stipulate a set of allowed actions $q_i \subseteq A_i$ for each player $i = 1, 2$ (an exception will be the mediated contracts discussed briefly later in this section). Such a contract is enforceable only if a court can determine whether each player i has in fact chosen an action in q_i. Formally, this amounts to the requirement that q_i be an element of the set

$$\bar{Q}_i \equiv \{q_i \in Q_i\text{: For } n = 1, ..., N_i \text{ either } p_{in} \subseteq q_i \text{ or } p_{in} \cap q_i = \varnothing\}.$$

We refer to such contracts as *simple contracts.*

Definition 1: A simple contract is a pair (q_1, q_2) with $q_i \in \bar{Q}_i$ for $i = 1, 2$.

For example, in Figure 1 the contract (p_{11}, p_{21}) (stipulating that each player must choose one of his first two actions) is a simple contract. Note that a simple contract induces a game in which player i's strategy set is q_i.

We shall say that the contract signed by the players is *complete* if it restricts the players' actions to the maximal extent possible given the limits on verifiability.[4,5] Formally:

Definition 2: The simple contract (q_1, q_2) is complete if $q_i \in P_i$ for $i = 1, 2$.

In the example of Figure 1, the four complete contracts are (p_{11}, p_{21}), (p_{12}, p_{21}), (p_{11}, p_{22}), and (p_{12}, p_{22}). Note that even with a complete contract, if a player's actions are not fully verifiable (i.e., if p_{in} contains more that one element), then this player may have some remaining discretion when it comes time to choose an action.

To assess the desirability of complete contracts, we need to make more specific assumptions about the players' behavior in the game induced by a contract. Throughout most of this paper, we assume that the outcome is a pure strategy Nash equilibrium (or, in dynamic games, a pure strategy subgame-perfect Nash equilibrium), that the players only sign contracts for which such an equilibrium exists, and that the players are able to coordinate on any desired pure strategy equilibrium when more than one exists.[6]

Given these behavioral assumptions, the following simple result tells us that the players never have a reason to sign an incomplete contract in a static contracting problem.

PROPOSITION 1: *Consider any simple contract* (q_1, q_2) *and associated pure strategy Nash equilibrium* $(a_1, a_2) \in q_1 \times q_2$. *Then there is a complete simple contract* $(\hat{q}_1, \hat{q}_2)$ *for which* (a_1, a_2) *is a Nash equilibrium outcome.*

The logic behind Proposition 1 is straightforward: setting $\hat{q}_i = p_i(a_i) \subseteq q_i$ (for $i = 1, 2$) creates a complete contract that reduces the set of possible deviations for each player i from a_i while retaining a_i as a feasible choice. Consequently, if (a_1, a_2) is a Nash equilibrium under contract (q_1, q_2), it is still a Nash equilibrium under contract $(\hat{q}_1, \hat{q}_2)$.

Proposition 1 merely establishes that the players suffer no loss by restricting themselves to complete contracts. It does not imply that one can *always* Pareto improve upon an incomplete contract; however, this is often the case (as in Figure 1).

While Proposition 1 endorses contractual completeness in static environments, it does depend on the assumptions that players confine themselves to pure strategies, and that they can coordinate on any Nash equilibrium. To see the importance of the coordination assumption, consider the contracting problem depicted in Figure 2, where $(\varepsilon, \delta) \geq 0$. The

[4] Some would argue that one should allow the players to supplement their strategy spaces contractually with publicly verifiable, payoff-irrelevant "messages," and to condition contractual terms on these messages. In this context, the appropriate definition of a "complete contract" may be controversial. One possible definition is that a complete contract ties down payoff-relevant actions as narrowly as possible, contingent on each possible profile of messages. In this view, a complete contract need not tie down players' choices of payoff-irrelevant messages. Note, however, that with this definition, there is always a complete contract that does as well as any incomplete contract. Specifically, take any outcome that is sustained by an incomplete contract. Supplement the strategy space so that each player is required to say what it is going to do immediately before acting. Impose the same restrictions on these messages that the original incomplete contract imposed on the players' actions, and restrict each party, as narrowly as possible, to make an action choice that is consistent with its message. This "complete" contract will achieve the same outcome as the original incomplete contract. Our view, however, is that this contract should *not* be regarded as complete. To the extent that messages are publicly verifiable, a complete contract would also have to specify the contracting parties' choices of messages as narrowly as possible.

[5] Our treatment of contract enforcement implicitly assumes that, subject to any restrictions agreed to in the contract, a player retains all of the discretion that it would have in the absence of a contract. Thus, in the absence of any stipulation regarding a particular dimension of performance, the player may choose any level of performance that would be available to him in the absence of a contract. In reality, however, courts sometimes "fill" gaps in contracts through default rules (see Ian Ayres and Robert H. Gertner [1989, 1992] for an interesting analysis of such default rules). In such a case, our analysis should be interpreted as applying to the economic consequences of the judicially amended contract.

[6] In our discussion of the example in Figure 1, we implicitly made the first assumption, while the second assumption was irrelevant because there was a unique Nash equilibrium for every simple contract.

		Player 2		
		p_{21}		p_{22}
		a_{21}	a_{22}	a_{23}
Player 1, p_{11}	a_{11}	3,3	$-\delta, 3-\varepsilon$	0,0
Player 1, p_{11}	a_{12}	$3-\varepsilon, -\delta$	1,1	-20,2
Player 1, p_{12}	a_{13}	0,0	2,-20	-2,-2

FIGURE 2. AN EXAMPLE IN WHICH THE COORDINATION ASSUMPTION PLAYS A CRITICAL ROLE

		Player 2			
		p_{21}		p_{22}	
		a_{21}	a_{22}	a_{23}	a_{24}
Player 1, p_{11}	a_{11}	2,2	0,4	1,1	4,0
Player 1, p_{11}	a_{12}	4,0	1,1	0,4	2,2
Player 1, p_{12}	a_{13}	1,1	4,0	2,2	0,4
Player 1, p_{12}	a_{14}	0,4	2,2	4,0	1,1

FIGURE 3. AN EXAMPLE IN WHICH THE PURE STRATEGY ASSUMPTION PLAYS A CRITICAL ROLE

Pareto-optimal payoff, (3, 3), is associated with the action pair (a_{11}, a_{21}). If players can coordinate on any Nash equilibrium, then this outcome is achievable with the complete contract (p_{11}, p_{21}). Note, however, that this contract also gives rise to a second pure strategy equilibrium, (a_{12}, a_{22}), with payoffs (1, 1). Moreover, the players might actually end up at this inefficient equilibrium. If, for example, ε is small while δ is large, then player i risks a large loss $(-\delta)$ by playing a_{i1}, in return for a small gain of at most ε. Consequently, cautious players might gravitate toward (a_{12}, a_{22}).[7] In contrast, for any $(\varepsilon, \delta) \geq 0$, (a_{11}, a_{21}) is the *unique* pure strategy Nash equilibrium for the totally incomplete contract (A_1, A_2). Once the players are allowed to choose their third actions (a_{13} and a_{23}, respectively), (a_{12}, a_{22}) is no longer a Nash equilibrium. Thus, if players cannot coordinate on the Nash equilibrium of their choosing, contractual incompleteness may be optimal even in static contexts because it can be used to eliminate undesirable equilibria.

Consider next the role of our assumption that the outcome of a contract is a pure strategy equilibrium. In Figure 3 we depict a contracting problem in which every complete contract has a unique Nash equilibrium, and all of these equilibria yield payoffs of (1, 1). But the fully incomplete contract (A_1, A_2) has a mixed strategy equilibrium that involves each party playing every action with probability $1/4$, and that generates payoffs of $(7/4, 7/4)$. Thus, when mixed strategy equilibria are allowed, it may be possible to Pareto improve upon all complete contracts through contractual incompleteness.

It may at first glance be surprising that one cannot generalize the logic of Proposition 1 to mixed strategy Nash equilibria. At a formal level, the problem is that the support of a particular mixed strategy for player i may not be contained in any element of P_i. Consequently, if we start with an incomplete contract (q_1, q_2) and an associated mixed strategy equilibrium (μ_1, μ_2), there may not be a complete contract (q_1', q_2') (with $q_i' \in P_i$ for $i = 1, 2$) for which μ_i is still feasible (for example, in Figure 3 there is no complete contract for which the equal $1/4$ probability randomization strategy is feasible). However, it *is* possible to extend our result to mixed strategy equilibria, and even to correlated equilibria, if we allow contracts to include a mediator who privately announces an allowable action set to each player based on the outcome of private exogenous randomization. In effect, by introducing a mediator we can convert any mixed strategy (or correlated) equilibrium to an equivalent pure strategy equilibrium, in which case the logic of Proposition 1 is applicable. We establish this result formally in Appendix A.

[7] To push this point further, consider the limiting case in which $\varepsilon = 0$ (for any value of δ). In this case, under contract (p_{11}, p_{21}) strategy a_{i1} is actually *weakly dominated* for each player i by a_{i2}.

In the remainder of the paper, we shall maintain our assumptions of pure strategy equilibrium and player coordination, in part because they strike us as natural, but more importantly because doing so allows us to identify the particular aspects of dynamic environments that lead to the optimality of contractual incompleteness.

II. Dynamic Contracting Problems: Preliminaries

The lesson of the preceding section was that, subject to some qualifications, contractual incompleteness is not required to achieve efficiency in (deterministic) static settings. One might imagine that this result would carry over directly to dynamic settings. A natural conjecture is that one can apply the same logic as in Proposition 1 when analyzing potential contractual restrictions on the normal form of a dynamic game, and reach the same conclusions. Though apparently intuitive, this conjecture is not correct.

We illustrate this point through a simple example. Consider the following two-stage, two-player game. First, player 1 selects an element of the set $\{A, B\}$. This choice is observable, but not verifiable. Second, player 2 selects an element of the set $\{a, b\}$. This choice is fully verifiable. Player 2's strategy set is then $\{(a, a), (a, b), (b, a), (b, b)\}$, where the first action in each pair indicates player 2's choice in the event player 1 selects A, and where the second action in each pair indicates player 2's choice in the event player 1 selects B. Because it is only possible to verify actions and not strategies, verifiability no longer induces a partition of player 2's strategy set. In particular, it is possible to restrict player 2 to the strategy (a, a), or to the strategy (b, b), but it is not possible to force a choice of either (a, b) or (b, a). Indeed, one cannot even restrict player 2's choice to the set $\{(a, b), (b, a)\}$, even though one can restrict player 2's choice to any element of the complement of this set. In short, the feasible restrictions on player 2's strategies are: the entire strategy set $\{(a, a), (a, b), (b, a), (b, b)\}$, $\{(a, a)\}$, or $\{(b, b)\}$. The logic of Proposition 1 continues to apply, but with much different implications. If the optimal contract involves play of a pure strategy equilibrium in which player 2's strategy is either (a, b) or (b, a), then, as in Proposition 1, one can achieve an equivalent outcome by imposing the narrowest restriction on player 2's choice that does not rule out (a, b). However, in this dynamic setting, the narrowest restriction on player 2's choice that does not rule out (a, b) is no restriction at all.[8]

At an economic level, this point is a significant one because player 2's response to player 1's action choice may have important effects on player 1's choice. In particular, if it is possible to achieve a response function for player 2 that induces player 1 to behave efficiency only by leaving player 2 unrestricted, then an incomplete contract may be optimal.

In the following two sections, we explore the role of contractual incompleteness in more focused dynamic settings. In Section III, we assume that choices are sequential rather than simultaneous, and that current choices affect subsequent incentives directly through payoff functions. We refer to the intertemporal linkages that characterize these settings as "structural." In Section IV, we assume that agents play a sequence of unrelated static games with no structural intertemporal linkages through payoff functions; however, play is linked

[8] The issue here is very similar to the considerations that arose with mixed strategy equilibria, and that were discussed in Section I and Appendix A. To complete the analogy, we modify the preceding example so that players 1 and 2 make their choices simultaneously. When mixed strategies are permitted, only three contractual restrictions are feasible: requiring 2 to play a with certainty [which is analogous to (a, a) in the dynamic setting], requiring 2 to play b with certainty [which is analogous to (b, b) in the dynamic setting], and the absence of a restriction [again analogous to the dynamic setting]. This similarity suggests that one might be able to use mediation to achieve a result parallel to Proposition A1. But this is not the case. One can think of a mixed strategy as a dynamic choice where, first, a player generates a random number, and second, the player makes a choice contingent on this realization. Proposition A1 holds because it is possible to delegate the generation of the random number to a mediator, who can then select an appropriate restriction. In contrast, in a dynamic setting, the choices of players may be contingent on the choices of other players, rather than on randomly generated numbers. Clearly, it is not possible to delegate the choice of another player to a mediator, unless the choice is fully verifiable. Consequently, dynamic games provide natural settings in which the need for strategic ambiguity gives rise to contractual incompleteness.

across periods through history-dependent strategies. These settings permit us to study the manner in which incomplete contracts affect cooperation in long-term relationships by altering feasible punishments.[9]

III. Dynamic Contracting Problems with Structural Intertemporal Linkages

In this section we study contracting problems in which players' actions are taken sequentially, rather than simultaneously as in Section I. We show that the introduction of dynamics can render contractual incompleteness optimal and, for a class of simple two-stage contracting problems, we characterize the circumstances in which optimal contracts are either complete or incomplete.

The central insight in this section is that when choices are sequential, the incentives of players' taking early, nonverifiable actions are determined in part by the subsequent reactions of other players. An optimal contract seeks to induce best-response functions for later-moving players that create desirable incentives for early-moving players. However, because contractual provisions restrict actions and not strategies, many best-response functions may be achievable only through the use of an incomplete contract that leaves the later-moving player with discretion. To illustrate, we begin with an example.

Example 1: A buyer and a seller contemplate the exchange of a good. At time 0 the buyer and seller may sign a contract constraining any subsequent verifiable action. At time 1, the procurement game begins, with the seller choosing an investment in quality, denoted by $r \geq 0$. This investment is observable but not verifiable. At time 2, the buyer makes a take-it-or-leave-it offer of a quantity $x \geq 0$ and total payment $T \in \mathbb{R}$ to the seller. This offer is verifiable. At time 3, the seller may accept or reject this offer. Again this is verifiable. Finally, at time 4 the good and money may be transferred between the players. This too is verifiable. The buyer's utility from consuming quantity x of quality r and making a payment of T is $u(x, r) - T$, where $u(0, r) = 0$ for all $r \geq 0$ and $u_{xr}(x, r) > 0$ for all (x, r) (subscripts denote partial derivatives). The seller's payoff under these circumstances is $T - cx - r$, where $c > 0$. Note that, in this example, the seller's investment in quality directly affects the payoffs of both parties.[10]

In the setting described above (in which lump-sum transfers are feasible), the first-best outcome is defined by $(x^*, r^*) \in \text{Argmax}_{x,r}\, u(x, r) - cx - r$. This outcome, if interior, must satisfy the conditions $u_x(x^*, r^*) = c$ and $u_r(x^*, r^*) = 1$. In what follows, we assume that $(x^*, r^*) >> 0$.

Since a complete contract necessarily fixes quantity (x) and payments (T), it leads the seller to set $r = 0$. This is plainly inefficient. Likewise, in the absence of a contract, once the seller chooses r, the buyer sets $T = cx$ (so that the seller is just willing to accept); consequently, the seller again sets $r = 0$.

What about partially incomplete contracts? Consider first a contract that sets a price per unit of $p > c$ but leaves quantity unspecified (i.e., it requires that the buyer's offer satisfy $T = px$). Given r, the buyer's quantity offer will solve $\text{Max}_x\, u(x, r) - px$, which implies $u_x(x, r) = p$. This leads to a demand function $x(r)$ that is strictly increasing in r. The seller therefore chooses r to solve $\text{Max}_r (p - c)x(r) - r$. Although such a contract may improve on the outcome with either a complete contract or a totally incomplete contract, it cannot achieve the first best. In particular, if we have an outcome (x, r) with $r > 0$, then we must have $p > c$. But this implies $u_x(x, r) > c$, and so the quantity consumed is inefficient given the quality level r.

It is, in fact, possible to achieve the first best through a partially incomplete *option*

[9] Obviously, structural intertemporal linkages are not incompatible with long-run cooperative relationships, but for simplicity we follow much of the existing game-theoretic literature by treating these issues separately.

[10] There is a large literature focusing on such "hold-up" problems. Following Oliver Hart and John Moore (1988), however, nearly all of this literature has studied the case in which the seller invests in cost reduction (these papers typically also have random valuations for the buyer and seller). Two exceptions are W. Bentley MacLeod and James M. Malcomsen (1993) and Yeon-Koo Che and Donald B. Hausch (1999), which we discuss below.

contract: The buyer is given the right, exercisable at time 3, to take delivery of quantity x^* at price $T^* = u(x^*, r^*)$ (the surplus can be divided in any way desired by including a noncontingent lump-sum monetary transfer). With this contract, if the seller chooses $r < r^*$, then the buyer will elect not to exercise the purchase option and the seller will earn $-r$. On the other hand, for any $r > r^*$, the buyer will opt to purchase and the seller will earn $T^* - cx^* - r$. Thus, the seller optimizes by setting $r = r^*$, and the buyer consumes quantity x^*. We therefore conclude that an optimal contract is necessarily partially incomplete whenever $(x^*, r^*) >> 0$.[11]

Suppose that we now alter the example by assuming that the seller invests not in quality enhancement, but rather in cost reduction. That is, let the buyer's utility be $u(x)$ and the seller's cost be $c(r)$ where $c'(r) < 0$. The first best (x^*, r^*) then solves $\text{Max}_{x,r}\ u(x) - c(r)x - r$. As is well known, a complete contract is optimal in this case: By setting the quantity x^* and a lump-sum transfer T in the contract, the seller will choose r to solve $\text{Min}_r\ c(r)x^* + 1$, leading to the choice of r^*.[12]

The preceding example illustrates two points: First, when players take their actions sequentially, contractual incompleteness may be optimal. Second, the optimal degree of contractual incompleteness may depend in interesting ways on the nature of the underlying contracting problem.

It is natural to wonder whether one can say anything general about the situations in which incompleteness is optimal. The buyer-seller example suggests an interesting possibility. Notice that when the noncontractible investment determines quality, an increase in the quality chosen by the seller enhances the buyer's willingness to purchase any given quantity at any given price. Thus, higher investment by the seller can lead the buyer to react in a way that is desirable from the seller's standpoint. In this setting, incompleteness was optimal. By way of contrast, when the seller's investment determines production costs, more investment by the seller encourages the buyer to reduce its price offer, a reaction that is disadvantageous to the seller. In this case, completeness was optimal.

In the remainder of this section we examine a class of simple two-stage contracting problems and investigate a conjecture suggested by the preceding observations: that optimal contracts are incomplete when actions are strategic complements, and complete when actions are strategic substitutes (see Bulow et al., 1985).

In the two-stage contracting problems considered here, player 1 moves first, choosing an observable but nonverifiable action $a_1 \in A_1 = [L_1, U_1] \subset \mathbb{R}$. After player 1 has chosen a_1, player 2 chooses a fully verifiable action $a_2 \in A_2 = [L_2, U_2] \subset \mathbb{R}$.[13] Each player i's payoff is given by the twice continuously differentiable

[11] MacLeod and Malcomsen (1993) show that in any case of one-sided investments (including seller investment in the buyer's valuation), the first best can be achieved if all bargaining power can be allocated to the investing party. In the text, we have taken the view that this is not possible (we take the order and nature of bargaining moves as fixed). Clearly, if this were possible, then such a contract would implement the first best in our example as well. Note, however, that such a contract is *also* partially incomplete since the seller is given discretion over the offer made to the buyer. For a recent interesting analysis of the case of two-sided "external" investments in the hold-up model (i.e., investments benefiting the other contracting party), see Che and Hausch (1999). Finally, Joel S. Demski and David Sappington (1991) show how an option contract (a "buy-out clause") can achieve the first best in an agency model with double moral hazard when efforts are observable but not verifiable. In contrast to the hold-up model studied here, in their model the decision to exercise the option (sell the firm) results in a firm's revenue stream being transferred from one agent to the other. There is, however, a close relationship between the two models; for more on this see Aaron S. Edlin et al. (1997).

[12] George Noldeke and Klaus M. Schmidt (1995) show how an option contract can achieve the first best in a hold-up model with 0–1 trade and uncertain valuations when the seller invests in cost reduction and the buyer invests in value enhancement. An important role of the option contract in their analysis, however, is to implement a level of expected trade intermediate between 0 and 1 as the default outcome under the contract, an issue which is not relevant with continuous quantities. (A second role of the option contract in their model is to shift all of the bargaining power during renegotiation to one agent in the event that the default outcome is inefficient.)

[13] If instead player 1's action choice were fully verifiable and player 2's were nonverifiable, then it is easy to see that a complete contract would always be optimal.

function $u_i(a_i, a_j)$. We assume that $\partial^2 u_i(a_i, a_j)/\partial a_i^2 < 0$ and $\partial u_i(a_i, a_j)/\partial a_j < b < 0$ at all $(a_i, a_j) \in A_1 \times A_2$; that is, higher action levels by a player are bad (to a degree bounded away from zero) for the player's rival.[14] We denote by $b_i(a_j)$ player i's (unique) best response in A_i to a_j. We also make the standard stability assumption that $b_1'(a_2)b_2'(a_1) > 1$ at all $(a_1, a_2) \in \text{int}(A_1 \times A_2)$. With this assumption, for the two cases studied below there is a unique Nash equilibrium of the static game in which the players choose actions simultaneously, denoted (a_1^N, a_2^N). In what follows, we assume that this equilibrium is interior; that is, $(a_1^N, a_2^N) \in \text{int}(A_1 \times A_2)$.

In this setting, the intuition for our conjecture is simple. With strategic complements, player 2 responds cooperatively to cooperative play by player 1. Hence, contracts that provide player 2 with flexibility encourage player 1 to behave more efficiently. By contrast, with strategic substitutes, player 2 accommodates player 1 in response to aggressive play. Hence, contracts that remove player 2's flexibility eliminate incentives for player 1 to behave inefficiently. As we will see, this intuition turns out to be correct, subject to some qualifications.

A. *Strategic Complements*

To begin, we assume that $\partial^2 u_i(a_i, a_j)/\partial a_i \partial a_j > 0$ so that the players' action choices are strategic complements. Under the present assumptions, we have $b_i'(a_j) > 0$ at any a_j such that $b_i(a_j) \in \text{int}(A_i)$. In this case, the players' best-response functions $b_1(\cdot)$ and $b_2(\cdot)$ have the relationship illustrated in Figure 4. In the absence of any contract, the equilibrium outcome would be $(a_1^s, b_2(a_1^s))$ ("s" stands for "Stackleberg," since player 1 is then in a position analogous to that of a Stackleberg leader).[15] The shaded area in Figure 4 is the set of individually rational outcomes, that is, those pairs (a_1, a_2) at which both players do at least as well as they would without a contract. The curve PP represents the set of Pareto-efficient outcomes, while CC represents the set of individually rational Pareto-efficient outcomes. Note that, as a general matter, the set CC lies within the set $\overline{BB} \equiv \{(a_1, a_2) \in A_1 \times A_2 : a_1 \leq b_1(a_2)$ and $a_2 \leq b_2(a_1)\}$, the upper boundary of which is formed by the two players' best-response functions.

What is the form of an optimal contract? When $a_1^s = L_1$, the no-contract outcome is Pareto efficient and nothing else is individually rational. However, when $a_1^s > L_1$, the no-contract outcome is not Pareto efficient.

At the other extreme, a complete contract specifying that player 2 choose, say, action a_2' results in the outcome $(b_1(a_2'), a_2')$. Thus, if an action pair (a_1^*, a_2^*) is implemented by a complete contract, it must be that $a_1^* = b_1(a_2^*)$.

In light of this observation, two properties of CC are worth emphasizing. First, there are always points $(a_1, a_2) \in CC$ such that $a_1 < b_1(a_2)$.[16] No complete contract can achieve these Pareto-efficient individually rational outcomes. Second, it is entirely possible that $a_1 < b_1(a_2)$ for *all* $(a_1, a_2) \in CC$ (refer again to Figure 4). In such cases, no Pareto-efficient individually rational outcome is implementable using a complete contract.

Although neither complete nor fully incomplete contracts can implement an individually rational Pareto-efficient outcome when $a_1^s > L_1$ and $a_1 < b_1(a_2)$ for all $(a_1, a_2) \in CC$, it turns out that we can *always* achieve efficiency with a partially incomplete contract as long as

[14] This is essentially an assumption of monotonicity and "consonance"—the sign of $\partial u_i(a_i, a_j)/\partial a_j$ is merely a normalization as long as it is the same for both players. Note, however, that in some contracting problems it may be natural for the sign of this derivative to differ across players (the effects may be "dissonant"). Although we can always redefine variables to make the effects consonant (say, by instead letting player 1's choice variable be $\bar{a}_1 = -a_1$), this has the effect of changing the sign of the relevant cross-partial derivative [that is, $\partial^2 u_i(\cdot)/\partial \bar{a}_1 \partial a_2 = -\partial^2 u_i(\cdot)/\partial a_1 \partial a_2$]. Hence, the results stated in the text for consonant variables that are strategic complements (resp. strategic substitutes), apply also to dissonant variables that are strategic substitutes (resp. strategic complements). (We thank a referee for pointing this out.)

[15] For simplicity, we will assume throughout that the Stackleberg point is unique, both for the case of strategic complements and for the case of strategic substitutes.

[16] The demonstration of this property appears in the proof of Proposition 2 in Appendix B.

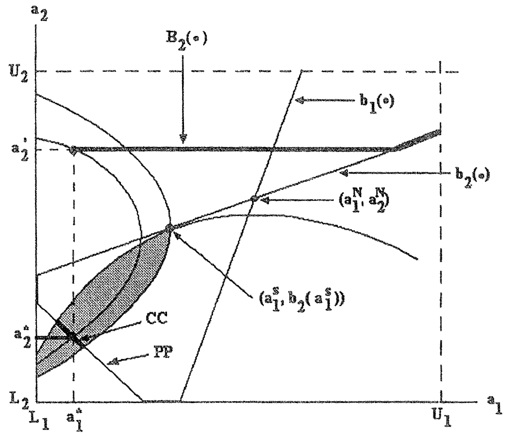

FIGURE 4. STRATEGIC COMPLEMENTS

player 2 can choose large enough levels of a_2. To illustrate this point, Figure 4 depicts an interval exclusion contract that sustains a particular individually rational Pareto-efficient outcome, (a_1^*, a_2^*), by precluding player 2 from choosing any element of the set (a_2^*, a_2'), where a_2' is defined by $u_2(a_1^*, a_2^*) = u_2(a_1^*, a_2')$. The figure indicates player 2's induced best response under this contract, denoted $B_2(a_1)$, by a heavy trace. This induced best-response function displays a discontinuity at a_1^*, where it jumps upward.[17] As is evident in the figure, given this induced best-response function for player 2, player 1's optimal choice is indeed a_1^*.

These observations lead to the following result.

PROPOSITION 2: *With strategic complements there exists a $\bar{U}$ such that if $U_2 \geq \bar{U}$, then every individually rational Pareto-efficient outcome can be achieved by means of a partially incomplete interval exclusion contract.*[18] *There are always individually rational Pareto-efficient outcomes $(a_1, a_2) \in CC$ that cannot be implemented with a complete contract. If $a_1^s > L_1$ and $a_1 < b_1(a_2)$ for all $(a_1, a_2) \in CC$, then no individually rational Pareto-efficient outcome is implementable with a complete contract or a totally incomplete contract.*

PROOF:

In Appendix B.

B. *Strategic Substitutes*

Suppose that $\partial^2 u_i(a_i, a_j)/\partial a_i \partial a_j < 0$, so that actions are strategic substitutes: $b_i'(a_j) < 0$ at any a_j such that $b_i(a_j) \in \text{int}(A_i)$. Figure 5 illustrates the best-response functions and the set of individually rational Pareto-efficient outcomes. Note that any individually rational outcome $(a_1, a_2) \in A_1 \times A_2$ with $a_1 > b_1(a_2)$ is Pareto dominated by the point $(b_1(a_2), a_2)$, which can be implemented by means of a complete contract specifying $q_2 = \{a_2\}$. Thus, we can confine our search for an optimal contract to agreements that implement individually rational outcomes in the set $\bar{B}_1 \equiv \{(a_1, a_2) \in A_1 \times A_2 : a_1 \leq b_1(a_2)\}$.

Next note that no contract (satisfying the condition that player 2's induced best response is well defined) can implement any outcome in the set $B_1 \equiv \{(a_1, a_2) \in A_1 \times A_2 : a_1 < b_1(a_2)\}$. To understand this result, suppose on the contrary that there is a contract implementing some $(\hat{a}_1, \hat{a}_2) \in B_1$. Holding a_2 fixed, a small increase in a_1 necessarily increases player 1's payoff [since $\hat{a}_1 < b_1(\hat{a}_1)$]. Of course, the increase in a_1 may lead player 2 to change a_2. However, with strategic substitutes, player 2's induced best-response correspondence is always nonincreasing in a_1.[19] To the extent that a_2 falls in response to the increase in a_1, this is an added bonus from player 1's perspective. Thus, player 1 will certainly deviate from $\hat{a}_1$.

As a result, the best the players can do is to implement an individually rational element of the set $\{(a_1, a_2) \in A_1 \times A_2 : a_1 = b_1(a_2)\}$ through a complete contract. Thus, we have:

[17] The fact that it jumps upward follows from the assumption of strategic complements.

[18] In the special case in which $a_1^s = L_1$, this contract is in fact fully incomplete.

[19] The formal demonstration of this property appears in the proof of Proposition 3 in Appendix B.

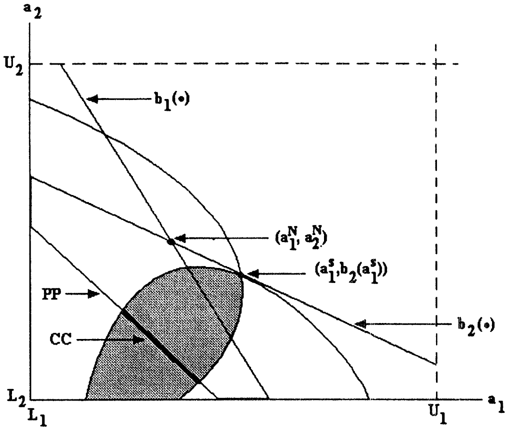

FIGURE 5. STRATEGIC SUBSTITUTES

PROPOSITION 3: *With strategic substitutes, every point in the Pareto frontier of the set of individually rational implementable outcomes can be implemented with a complete contract. With the possible exception of* $(b_1(L_2), L_2)$, *none of these points is Pareto efficient.*

PROOF:
In Appendix B.

Note that in contrast to the case of strategic complements, points in the Pareto frontier of the set of individually rational implementable outcomes are *not* generally Pareto efficient. The point $(b_1(L_2), L_2)$ is the only possible exception, and for L_2 sufficiently low it is not individually rational.

Together, Propositions 3 and 4 formalize the conjecture that situations with strategic complements call for incomplete contracts, while situations with strategic substitutes call for complete contracts. The following example provides an application of these results.

Example 2: Imagine a situation in which an entrepreneur (player 2) starts a business but needs capital from a venture capitalist (player 1). The venture requires \$$K$ as start-up capital at date 0, and can benefit from a future capital infusion at date 2. In the interim (at date 1), the entrepreneur can put in effort. The (verifiable) profits of the enterprise are given by $\Pi(e, m)$, where $e \in [0, \bar{e}]$ is the date 1 effort of the entrepreneur, and $m \in [0, \bar{m}]$ is the date 2 venture capital infusion ($\bar{e}$ and $\bar{m}$ are taken to be large). The capital infusions of the entrepreneur are verifiable, but the effort of the entrepreneur is not. We assume that any profit-sharing agreement takes a linear form, so that the entrepreneur's share is $s(\Pi) = \alpha + \beta\Pi$ with $\beta \in (0, 1)$. The venture capitalist seeks to maximize profits, while the entrepreneur's payoff from income I and effort is e is $I - g(e)$, with $g'(\cdot) \geq 0$, $g'(0) = 0$, and $g''(\cdot) > 0$.

Under what circumstances should a contract specify the level of the date 2 capital infusion in advance? For any given profit-sharing parameters (α, β), we can write $u_1(e, m) = \alpha + \beta\Pi(e, m) - g(e)$, and $u_2(e, m) = (1 - \beta)\Pi(e, m) - M - K$. It follows that the parties' actions are strategic complements if $\Pi_{em}(e, m) > 0$, and strategic substitutes if $\Pi_{em}(e, m) < 0$. From Propositions 3 and 4 we conclude that m should be fully specified when effort and capital are substitutes in the production of profits (i.e., when $\Pi_{em} < 0$), but should be left incompletely specified and at the partial discretion of the entrepreneur when effort and capital are complementary ($\Pi_{em}(e, m) > 0$). A first-best outcome is typically achievable only in the second case.

C. *Introducing Monetary Transfers*

In many situations, contracting parties are permitted to make side-payments. It is therefore natural to wonder whether the introduction of monetary transfers affects our results. We address this issue in the context of our two-stage contracting problem by permitting the players to make transfers after player 2 chooses a_2.[20] We assume that transfers are fully contractible, and that utility is quasilinear (linear in money). Although the ability to make transfers alters our results somewhat, a fundamental distinction remains between the cases of strategic complements and strategic substitutes.

[20] This restriction on the timing of transfers is for convenience only (note that we assume no discounting).

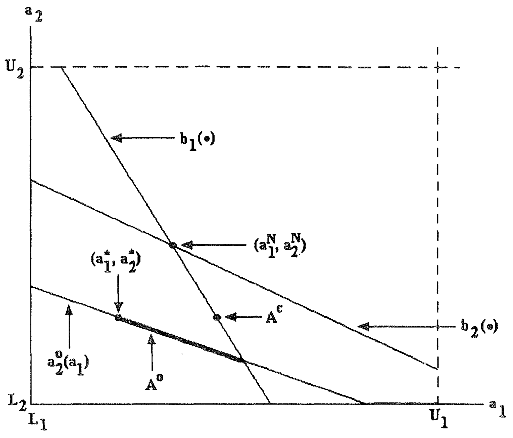

FIGURE 6. STRATEGIC SUBSTITUTES WITH MONETARY TRANSFERS

Suppose we define $t \geq 0$ to be player 2's transfer to player 1, and $s \geq 0$ to be player 1's transfer to player 2. In general, a simple contract in a dynamic setting can make the sets of allowed actions at some date τ contingent upon verifiable choices made earlier in the game. In the present situation, this means that the contract can make the sets of allowed transfers contingent upon player 2's action choice. Since players will always make the smallest allowed transfer, any such contract is equivalent to a contract that simply specifies a required net transfer from player 1 to player 2, $T = t - s$, for each possible action choice by player 2. Hence, we can view a simple contract as providing player 2 with a menu of action-transfer pairs, $\{(a_{21}, T_{21}), (a_{22}, T_{22}), \ldots\}$. In what follows we restrict attention to the case in which total profits, $\Sigma_i u_i(a_1, a_2)$, are strictly concave in (a_1, a_2).

Consider first the case of strategic complements. In contrast to the no-transfer case, all Pareto-efficient outcomes involve the same action pair $(a_1^*, a_2^*) = \text{Max}_{(a_1,a_2)\in A_1\times A_2} \Sigma_i u_i(a_1, a_2)$; points on the Pareto frontier are distinguished only by the level of transfers. If (a_1^*, a_2^*) is an element of the set of individually rational action pairs considered in Proposition 3, then the contract used there—which involved no transfers—also implements the efficient outcome in this context. Even when (a_1^*, a_2^*) is not an element of this set, it is still possible to implement any Pareto-efficient outcome through a partially incomplete contract. Formally:

PROPOSITION 4: *If actions are strategic complements and monetary transfers are possible, there exists a $\bar{U}$ such that if $U_2 \geq \bar{U}$ then every Pareto-efficient outcome can be achieved by means of a partially incomplete contract.*[21] *A complete contract can implement the Pareto-efficient action pair (a_1^*, a_2^*) only if $a_1^* = L_1 = b_1(a_2^*)$; hence, if $b_1(L_2) > L_1$, a complete contract cannot be optimal.*

PROOF:

In Appendix B.

When actions are strategic complements and transfers are feasible, contracting parties can harness two forces to achieve an efficient outcome. First, as in Section III, subsection A, player 2 naturally responds aggressively to an inefficiently aggressive choice by player 1. Second, the contract can require player 1 to pay a monetary penalty to player 2 if player 1's choice does induce player 2 to play aggressively (see the proof of Proposition 4 in Appendix B).

With strategic substitutes, the first force is reversed (as in Section III, subsection B): player 2 becomes less aggressive when player 1 makes an inefficiently aggressive choice. However, when transfers are feasible, contracting parties may still be able to improve on the best complete contract by harnessing the second force. To investigate this possibility, we define

$$a_2^\circ(a_1) = \underset{a_2' \in A_2}{\text{Argmax}} \sum_i u_i(a_1, a_2')$$

and

$$A^\circ = \{(a_1, a_2) \in A_1 \times A_2 : a_2 = a_2^\circ(a_1),\ a_1 \geq a_1^*, \text{ and } a_1 < b_1(a_2)\}.$$

[21] A fully incomplete contract is optimal only if $(a_1^*, a_2^*) = (a_1^s, b_2(a_1^s))$, which requires $b_2(a_1^s) = L_2$.

The function $a_2^\circ(a_1)$ identifies the level of a_2 that maximizes total payoffs given a_1. The set A° is the portion of the locus of points $(a_1, a_2^\circ(a_1))$ involving an action choice by player 1 of at least the efficient level a_1^* and lying in set $B_1 = \{(a_1, a_2) \in A_1 \times A_2 : a_1 < b_1(a_2)\}$. Figure 6 depicts both of these objects. One can show that A° is always nonempty.[22] We also define

$$A^c \equiv \underset{(a_1,a_2)\in A_1\times A_2}{\operatorname{Argmax}} \sum_i u_i(a_1, a_2)$$

$$\text{subject to } a_1 = b_1(a_2).$$

A^c is the set of action pairs on player 1's best-response function that maximize total payoffs; it is also depicted in Figure 6.

Our next result partially characterizes optimal implementable action pairs.

PROPOSITION 5: *If actions are strategic substitutes and monetary transfers are possible, the optimal contract implements an element of* $A^c \cup A^\circ$. *Any action pair* $(\bar{a}_1, \bar{a}_2) \in A^c$ *is implementable with a complete contract. An action pair* $(\bar{a}_1, \bar{a}_2) \in A^\circ$ *is implementable if and only if there is an* $\hat{a}_2 \in [L_2, \bar{a}_2]$ *such that*

$$(1) \quad u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2) \geq u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2).$$

No such action pair is implementable with a complete contract.

PROOF.

In Appendix B.

Proposition 5 tells us that in searching for an optimal (simple) contract, the players can first determine the most profitable implementable point in A°. This is the point in A° that has the lowest level of a_1 among all points in A° satisfying (1) [if any satisfy (1)]. The total payoff from this point is then compared to the payoff associated with the best complete contract (an element of A^c).

As noted above, with strategic substitutes, the prospects for improving upon the best complete contract depend on whether it is possible to construct a transfer scheme that penalizes player 1 sufficiently for aggressive behavior. In general, this may or may not be possible. We illustrate this point through the following example.

Example 3: Suppose that $u_i(a_1, a_2) = f(a_i + \beta a_j) - \alpha a_j$ for $i = 1, 2$ and $j \neq i$, where $\alpha > 0$, $\beta \in (0, 1)$, $\alpha > \beta c$, and $f(\cdot)$ is a strictly concave function with $f(0) = f'(0) = 0$, $f''(\cdot) < 0$, $\lim_{z\to-\infty} f'(z) = c$, and $\lim_{z\to+\infty} f'(z) = -c$. The functions $u_i(\cdot)$ satisfy all of our assumptions.[23] For simplicity, we assume that action sets are symmetric as well: $A_1 = A_2 = [L, U]$. In this setting, we have $b_i(a_j) = \text{Max}\{L, \text{Min}\{U, -\beta a_j\}\}$. For the (simultaneous-move) Nash equilibrium point (a_1^N, a_2^N) to be interior, we must have $L < 0 < U$ for $i = 1, 2$: we then have $(a_1^N, a_2^N) = (0, 0)$. The Pareto-efficient action pair (a_1^*, a_2^*) is the symmetric point that solves

$$\text{Max}_{a\in[L,U]} \; f(a(1 + \beta)) - \alpha a.$$

The solution is characterized by the condition that

$$(2) \qquad f'(a(1 + \beta)) \leq \frac{\alpha}{1 + \beta},$$

where (2) holds with equality if $a > L$. A necessary condition for an interior optimum is that $c > \alpha/(1 + \beta)$, which we shall assume in what follows. With this condition, there is an $\bar{L}$ such that for all $L \leq \bar{L}$, the efficient action pair (a^*, a^*) involves the (unique) level of a that satisfies (2) with strict equality.

Can the players support an outcome that dominates the best complete contract?

[22] This follows from an argument similar to the ones given at the ends of the proofs of Propositions 3 and 4 in Appendix B.

[23] The condition $\alpha > \beta c$ insures that $(\partial u_i / \partial a_j) < 0$.

For an arbitrary $(\bar{a}_1, \bar{a}_2) \in A^\circ$, define the function:

$$\xi(\hat{a}_2) = [u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)] - [u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2)] = [u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)] - f(0) + \alpha\hat{a}_2 - f(\hat{a}_2 + \beta\bar{a}_1) + \alpha\bar{a}_1.$$

By Proposition 5, we know that to implement $(\bar{a}_1, \bar{a}_2)$ we must find an $\hat{a}_2 < \bar{a}_2$ such that $\xi(\hat{a}_2) \geq 0$. Note that $\xi(\bar{a}_2) < 0$. Now consider the derivatives of $\xi(\cdot)$:

$$\xi'(\hat{a}_2) = \alpha - f'(\hat{a}_2 + \beta\bar{a}_1)$$

and

$$\xi''(\hat{a}_2) = -f''(\hat{a}_2 + \beta\bar{a}_1).$$

Note that if $\xi'(\hat{a}_2) \geq 0$ at all $\hat{a}_2$, then $\xi(\hat{a}_2) < 0$ at all $\hat{a}_2 < \bar{a}_2$. A sufficient condition for this is that $\alpha \geq c$ [and so we have $(1 + \beta)c > \alpha \geq c > \beta c$]. In this case, no $(\bar{a}_1, \bar{a}_2) \in A^\circ$ is implementable regardless of the values of L and U: the optimal contract is the best complete contract.

On the other hand, if $c > \alpha$, then for small enough $\hat{a}_2$ we have $\xi'(\hat{a}_2) < 0$. Whether there is an $\hat{a}_2$ at which $\xi(\hat{a}_2) > 0$ depends on the point $(\bar{a}_1, \bar{a}_2)$ that we are trying to implement, and on the value of L. As L falls, more elements of A° become supportable; indeed, there exists some $\hat{L} < 0$ such that the efficient action pair (a_1^*, a_2^*) is implementable if $L < \hat{L}$.

Hence, for $L < \hat{L}$, a complete contract is optimal if and only if $\alpha \geq c$.

In summary, even when monetary transfers are permitted, a fundamental distinction remains between the cases of strategic complements and strategic substitutes. With strategic complements, an incomplete contract is always optimal and the first best is attainable; with strategic substitutes, the parties are sometimes unable to improve on a second-best complete contract.

D. *Renegotiation*

Up to this point, we have assumed that the contract between players 1 and 2, once written, is never renegotiated. Since this contract leads to inefficient levels of a_2 following some choices of a_1 by player 1, it is natural to wonder how renegotiation would affect our results. We now introduce renegotiation of a particular form: player 2 can make a take-it-or-leave-it offer to player 1 following player 1's choice of a_1.[24]

Consider first the case without monetary transfers. Suppose that in the absence of renegotiation player 2 would choose a_2' in response to player 1 choosing a_1'. Then prior to this occurring, player 2 will propose to player 1 that they revise their agreement to specify the level a_2 that maximizes $u_2(a_1', a_2)$ on the set $\{a_2 \in A_2 : u_1(a_1', a_2) \geq u_1(a_1', a_2')\}$. When $a_2' > b_2(a_1')$, player 2 will offer to set his action at level $b_2(a_1')$ [recall that $u_1(\cdot)$ is strictly decreasing in a_2]; when $a_2' \leq b_2(a_1')$, no renegotiation is possible since any a_2 that raises player 2's payoff above $u_2(a_1', a_2')$ gives player 1 a payoff below $u_1(a_1', a_2')$. Letting $B_2^*(a_1)$ denote player 2's *post-renegotiation* response function (i.e., it gives the level of a_2 that arises after player 1 chooses a_1 and renegotiation has occurred), we have $B_2^*(a_1) = \text{Min}\{b_2(a_1), B_2(a_1)\}$; player 1 chooses a_1 to maximize $u_1(a_1, B_2^*(a_1))$.

For the case of strategic complements, it is easily verified that the interval exclusion contract in Figure 4 still implements (a_1^*, a_2^*). Indeed, renegotiation of the form considered here does not alter Proposition 2. For the case of strategic substitutes, since $b_2(a_1)$ and $B_2(a_1)$ are both nonincreasing functions, so is $B_2^*(a_1)$. This immediately implies that it is still impossible to implement any $(\bar{a}_1, \bar{a}_2)$ in set B_1. Moreover, it is easily verified that the best complete contract implements the same outcome regardless of whether one permits renegotiation of the form considered here. Hence Proposition 3 emerges intact.

Consider next the case with monetary transfers. Since player 2 makes a take-it-or-leave-

[24] In performing this exercise we assume that the recontracting process is not verifiable to third parties.

it offer, player 1's post-renegotiation payoff is always exactly the same as in the absence of renegotiation. Consequently, the possibility of renegotiation does not alter the particular values of a_1 that are implementable. From this observation, it follows immediately that the contract constructed in the proof of Proposition 4 (for the case of strategic complements) continues to implement (a_1^*, a_2^*) when renegotiation is permitted. Indeed, it does so without the need for any renegotiation on the equilibrium path. With strategic substitutes, however, this same observation tells us that if (a_1^*, a_2^*) is not implementable in the absence of renegotiation, then it cannot be implemented directly (i.e., without renegotiation occurring on the equilibrium path) when renegotiation is permitted. However, if U_2 is large enough it *is* possible to implement (a_1^*, a_2^*) *indirectly* (i.e., as the post-renegotiation outcome) via a complete contract. This contract sets $a_2 = b_1^{-1}(a_1^*)$; player 1 chooses a_1^* and player 2 then offers to set $a_2 = a_2^*$ in return for a transfer. Thus, in contrast to the case without renegotiation, the first-best outcome is implementable. However, a complete contract is still optimal.[25]

IV. Dynamic Contracting Problems Without Structural Intertemporal Linkages

An important feature of ongoing relationships is the development of cooperation through repeated interaction. It is well known that players may be able to enforce cooperation through strategies that involve punishment in response to deviations from cooperative play (e.g., Dilip Abreu, 1988). With the usual requirement of subgame perfection, cooperation is achieved by varying the selection of a continuation equilibrium in a manner that depends on past play.

In this section, we examine the desirability of contractual completeness for settings in which players seek to cooperate over both contractible and noncontractible aspects of their relationships. We argue that the imposition of contractual restrictions on future actions can alter the scope for punishment (and thus for cooperation) by changing the sets of equilibria for continuation games. In general, an increase in the level of contractual incompleteness can either expand or contract the set of continuation equilibria. A more incomplete contract makes more continuation paths feasible, but also expands the possibilities for deviations from any given path. An optimal contract must balance these two concerns.

Although cooperation is most commonly studied in the context of infinitely repeated games, we focus here on games with finite horizons. We make this choice to simplify the task of modeling and analyzing the process of renegotiation. As is well known, the analytics of renegotiation in infinitely repeated games are extremely complex, and many key issues remain unresolved (see e.g., Bernheim and Debraj Ray, 1989; Joseph Farrell and Eric Maskin, 1989; or Abreu et al., 1993). Moreover, in the present context, we need to consider not only renegotiation of equilibrium strategies, but also renegotiation of explicit contracts. In contrast, the theory of renegotiation in finitely repeated games is reasonably well settled (see e.g., Bernheim et al. [1987] or Drew Fudenberg and Jean Tirole [1991]).[26]

We will simplify our task by restricting attention to the following class of two-player models. Play unfolds in two stages. In the first, players take a collection of noncontractible

[25] The nature of the optimal contract with renegotiation does appear to be sensitive to the distribution of bargaining power. Assume, for example, that player 1 has all the bargaining power. In the case of strategic complements without transfers, no interior Pareto-efficient outcome can be implemented. With strategic substitutes, the same outcome is implemented as without renegotiation, but in some cases this requires a partially incomplete contract. We do not have general results on the structure of optimal contracts with other distributions of bargaining power.

[26] MacLeod and Malcomson (1989) and George Baker et al. (1994) study the interaction between explicit and implicit contracts in an infinitely repeated setting. MacLeod and Malcomson consider the possibility of renegotiation of agreements using the renegotiation-proof notion adopted by Abreu et al. (1993). Baker et al. do not allow for renegotiation of implicit agreements but do allow for some renegotiation of explicit contracts (in particular, they assume that following a breakdown in cooperation the contracting parties adopt the best explicit contract). We discuss these papers further in Section V.

actions, leading to some observable but nonverifiable outcome $h \in H$. For reasons that will become apparent shortly, the particular structure of stage 1 is of little concern to us; it could, for example, involve highly complex dynamic interaction. In the second stage, each player selects a contractible action $a_i \in A_i$. To rule out structural intertemporal linkages between stages 1 and 2, we assume that A_i is independent of h, and that each player's utility is additively separable across stages: $U_i(h, a_1, a_2) = v_i(h) + u_i(a_1, a_2)$.

Thus, stages 1 and 2 are each self-contained games. Given our assumptions about contractibility, it is possible to obtain any efficient outcome for stage 2 (considered in isolation) through a complete contract. We will assume, however, that all equilibria are inefficient when the (noncontractible) game of stage 1 is played in isolation. The existence of stage 2 provides the players with an opportunity to improve stage 1 outcomes by constructing punishments. Although the players cannot enter into a formal agreement that directly imposes rewards and punishments contingent on stage 1 outcomes, it is conceivable that the removal of contractual restrictions for stage 2 may create the flexibility to structure implicit agreements that accomplish this objective. Our object in this section is to isolate conditions under which this flexibility is actually of use, and conditions under which it is not.

In considering this problem, we will concern ourselves with two distinct modes of renegotiation: renegotiation of *explicit* contracts, and renegotiation of *implicit* contracts (i.e., the selection of a continuation equilibrium). Here, one can make four different sets of assumptions about the feasibility of renegotiation: no renegotiation of any kind, renegotiation of implicit contracts only, renegotiation of explicit contracts only, and renegotiation of both implicit and explicit contracts. For the class of games currently under consideration, we obtain the cleanest characterization under the assumption that all kinds of renegotiation are feasible. Moreover, once this case is understood, the other three are straightforward. We therefore focus our attention on the assumption of full renegotiation, returning to the alternative cases at the end of the section.

Formally, we model the renegotiation of implicit agreements by restricting attention to *renegotiation-proof* equilibria (given the contract).[27] We model the renegotiation of explicit agreements by allowing for bargaining and recontracting between stages 1 and 2. For simplicity, we treat this bargaining process as a "black box" with two properties: (i) it always leads to a complete contract that selects a continuation outcome on the efficient frontier, and (ii) the particular outcome selected depends only on the players' threat points (the payoffs for the continuation equilibrium that would prevail if the explicit contract was not renegotiated)—specifically, an increase in a player's threat point leads to an increase in his/her renegotiated payoff, and a decrease in the renegotiated payoff of his/her opponent. One could justify these assumptions more formally by inserting a completely specified noncooperative bargaining model between stages 1 and 2, but this would complicate the analysis considerably without contributing further insight.

We now turn our attention to the following question: Is it possible to design contracts such that the associated equilibria give rise to different stage 2 outcomes (i.e., rewards and punishments) following different stage 1 outcomes (elements of H)?

A. *Basic Results*

In Section III, we linked the desirability of incomplete contracts to the existence of strategic complements or strategic substitutes. Here we consider how conditions of strategic complementarity and substitutability for the stage 2 game affect the desirability of contractual incompleteness. In this subsection we show that, under the assumption of full (explicit and implicit) renegotiation, the implications of strategic complementarity and substitutability are reversed from the main conclusions of Section III: incomplete contracts cannot improve on complete contracts in the presence of strategic complements, while contractual incompleteness is in general desir-

[27] The notion of renegotiation-proofness used here is also known as *Pareto perfection* (Fudenberg and Tirole, 1991), and is originally due to Bernheim et al. (1987).

able with strategic substitutes. (Formally, in the discussion of strategic complements and substitutes that follows we make the same assumptions on strategy spaces and payoff functions as in Section III, subsections A and B.)

With strategic complementarities, for any initial contract the (pure strategy) equilibrium set in stage 2 is Pareto ranked, irrespective of the restrictions that the contract imposes on the players' choice sets (see Paul R. Milgrom and John Roberts, 1990). Thus, fixing the initial contract, renegotiation-proofness selects the same continuation equilibrium for all stage 1 realizations h. This implies that, even if players renegotiate the original explicit contract, this process leads to the same stage 2 outcome following all stage 1 realizations (since the threat points are identical across all such realizations). Thus, we have the following.

PROPOSITION 6: *Assume that a_1 and a_2 are strategic complements. Then it is impossible to write a contract that makes stage 2 outcomes contingent on stage 1 outcomes. Consequently, any implementable outcome is achievable through a complete contract.*

PROOF:

Follows from discussion in the text.

Now consider the case of strategic substitutes. We have assumed that, in the absence of a contract, the Nash equilibrium for the stage 2 game is unique and interior. This implies that the parties can create multiple, Pareto-unranked equilibria for stage 2 by signing a partially incomplete contract (see the proof of Proposition 7 in Appendix B).[28] Consequently, it is possible to arrange contingent punishments for the two-stage game by prescribing different continuation equilibria following different stage 1 outcomes. Although the players may renegotiate the explicit contract between stages 1 and 2, the assumed monotonicity of the bargaining process (with respect to the threat points) guarantees that stage 2 outcomes will be contingent on stage 1 results (since the threat points are contingent on stage 1 results). Thus, a properly designed incomplete contract can create the scope for selective punishment and reward. Formally, we have the following.

PROPOSITION 7: *Assume that a_1 and a_2 are strategic substitutes. Then it is generally possible to write an incomplete contract that makes stage 2 outcomes contingent on stage 1 outcomes.*

PROOF:

In Appendix B.

Since inefficient contracts are always renegotiated in this setting, strategic flexibility is achieved without sacrificing efficiency. Players might nevertheless prefer to write complete contracts if incompleteness can only deliver second-period outcomes on some undesired portion of the Pareto frontier. However, if players can make contractible monetary transfers and if utility is quasi-linear in money, then it is always possible to achieve the desired distribution of the efficient aggregate payoff on the equilibrium continuation path through appropriate up-front payments. In such cases, appropriately designed incomplete contracts typically improve upon complete contracts (and always do so if the stage 1 game is sufficiently "smooth"), since they create some scope for punishing undesirable stage 1 choices without sacrificing efficiency or distorting distribution.

We present a detailed example illustrating these results in Appendix C.

B. *Alternative Assumptions*

Throughout this section, we have assumed that players can renegotiate both implicit and explicit agreements. In this subsection, we briefly describe the manner in which our results would change under three alternative assumptions.

Renegotiation of explicit agreements only.—For strategic substitutes, stage 2 continuation equilibria are always Pareto unranked. Since players are therefore unable to renegotiate implicit agreements even if such renegotiation is permitted, Proposition 7 is unchanged.

[28] Note that if there were multiple equilibria in the unrestricted stage 2 game (i.e., if we did not maintain the stability assumption), then we could create Pareto-unranked multiplicity with a fully incomplete contract.

In contrast, an inability to renegotiate implicit agreements does change Proposition 6 (the case of strategic complements). To understand this point, imagine that the unrestricted stage 2 continuation game gives rise to two Pareto-ranked equilibria. Suppose that the Pareto-superior equilibrium payoff vector favors player 1, while the Pareto-inferior equilibrium payoff vector favors player 2. When renegotiation of implicit agreements is permitted, the players know that the inferior equilibrium cannot be used in any continuation game; hence, it is impossible to create contingent punishments. However, if renegotiation of implicit agreements is not permitted, then the players could well renegotiate the explicit contract to obtain an efficient outcome that favors player 1 when the superior continuation equilibrium is specified, and an efficient outcome that favors player 2 when the inferior continuation equilibrium is specified. In that case, there would be scope for punishment.

Note, however, that punishment is not always possible with strategic complements. If, for example, the game is completely symmetric, then the equilibria will also be symmetric (otherwise there would be Pareto-unranked equilibrium pairs). With symmetric bargaining power, any set of symmetric threat points will lead to the same outcome when the explicit agreement is renegotiated.

Renegotiation of implicit agreements only.—Under this alternative assumption, the argument for strategic complements is unchanged from that in Section IV, subsection A, and Proposition 6 continues to hold. Punishment remains possible with strategic substitutes. However, if the continuation equilibria for the unrestricted stage 2 game are inefficient (as in our example), then there may be a trade-off between efficiency in stage 1 and efficiency in stage 2. That is, one can improve efficiency in stage 1 by creating stage 2 punishments, but this may reduce the efficiency of the stage 2 outcome.[29] Thus, the efficient contract may or may not be incomplete.

No renegotiation of any kind.—In the absence of any renegotiation, it is generally possible to establish punishments by using partially incomplete contracts to create continuation games with multiple equilibria. This is true regardless of whether actions are strategic complements or strategic substitutes. With strategic substitutes, there may be a trade-off between efficiency in stage 1 and efficiency in stage 2 for the same reason as above. Thus, the efficient contract may or may not involve partial incompleteness. With strategic complements, it is usually possible to create multiplicity wherein the Pareto-superior equilibrium is on the efficient frontier.[30] This implies that with strategic complementarities in stage 2 the efficient contract will typically be incomplete.

As is evident from this discussion, the precise conditions under which incomplete contracts are optimal depends on the extent to which renegotiation of both implicit and explicit contracts is possible. When no renegotiation is possible, incomplete contracts are optimal with strategic complements (as was true in Section III) and may or may not be optimal with strategic substitutes. However, renegotiation of implicit contracts destroys the usefulness of contractual incompleteness in the case of strategic complements, because the Pareto ranking of the equilibrium set precludes a multiplicity of equilibrium outcomes following renegotiation. In contrast, renegotiation of explicit contracts tends to make incompleteness more attractive in the case of strategic substitutes, because it allows the players to create multiple Pareto-unranked equilibria without sacrificing efficiency.

V. Concluding Remarks

In this paper, we have argued that, when some aspects of behavior are observable but not verifiable, it may be optimal to write a contract that leaves other potentially contractible

[29] In some instances, it is possible to create partially incomplete contracts that provide for an efficient continuation equilibrium if no one deviates in stage 1, as well as punishments when stage 1 deviations occur. However, there are also cases in which this is impossible.

[30] For a symmetric game, let (e, e) denote the first-best symmetric outcome. Define the action d implicitly as follows: $u_1(d, e) = u_1(e, e)$. It is easy to check that (d, d) and (e, e) are, in general, both equilibria.

aspects of the relationship unspecified. We have also explored the relation between the underlying structure of the economic environment and the optimal degree of contractual incompleteness.

While these themes have not been explored systematically in the preexisting literature, several recent papers have made related points. Perhaps closest to our work is MacLeod and Malcomson's (1989) study of implicit and explicit contracting in long-run (infinitely repeated) employment relationships. Although MacLeod and Malcomson do not explicitly set out to investigate the optimal degree of incompleteness for employment contracts, the optimal contract for their model includes a discretionary bonus structure, and is therefore incomplete in the sense defined here. Contractual incompleteness serves the same role in their paper as in the current analysis: a discretionary bonus allows the firm to discipline shirking more effectively.

Similar comments apply to Baker et al.'s (1994) analysis of subjective performance measures. Like MacLeod and Malcomson, Baker et al. study an infinitely repeated employment relationship, and exhibit optimal contracts in which the employer makes discretionary wage payments conditional on good behavior. The model of Baker et al. differs from that of MacLeod and Malcomson in that the firm observes only measures of the worker's stochastic output, rather than effort. Discretionary pay allows the firm to condition compensation on a superior but nonverifiable performance measure.[31]

Another closely related paper by Demski and Sappington (1991) examines the role of buy-out clauses in resolving agency problems with double moral hazard when effort is observable but not verifiable (see footnote 11 for a further discussion). Their analysis demonstrates that it is possible to improve contractual performance by giving one party discretion in the form of an option. This result is analogous to our Example 1, and reflects the general principles developed herein.[32]

Our study is also related to a paper by Farrell and Shapiro (1989) which investigates whether an optimal long-term contract can outperform an optimal short-term contract (one that is incomplete in the sense that it does not specify future terms of trade) in the presence of customer-switching costs when some dimension of product quality is noncontractible. They establish a "Principle of Negative Protection" (see their Proposition 1), stating that a buyer may be worse off specifying some contractual terms when other choices (quality) are noncontractible. Despite the apparent similarities, this principle is actually quite distinct from our argument: it establishes only that a *poorly chosen* complete contract may be suboptimal. Indeed, in their model there is always a complete contract that achieves the optimum (see again their Proposition 1). In contrast, our theory explains why an incomplete contract may be *strictly better* than any complete contract.

Finally, Arnoud W. A. Boot et al. (1993) propose an alternative (complementary) theory to explain why contracts often leave obligations incompletely specified. In their model, the terms of trade should ideally vary with an underlying but nonverifiable realization of the state of the world. If obligations are left vague, parties can adjust the terms of trade to reflect this realization.

APPENDIX A

In this Appendix, we discuss how introduction of a mediator allows us to extend the optimality of complete contracts not only to mixed strategy equilibria, but also to correlated equilibria. We now imagine that a

[31] Our analysis is also related to an older literature on reputation and product quality. For example, Benjamin Klein and Keith B. Leffler (1981) describe an equilibrium with short-term contracts where the terms of future trade are left unspecified. A complete contract (one that specified all future terms of trade) would be suboptimal in their model, as it would vitiate all possibilities for enforcing observable but nonverifiable dimensions of contractual performance.

[32] The papers by Gur Huberman and Charles M. Kahn (1988) and Benjamin E. Hermalin and Michael L. Katz (1991) on contract renegotiation can also be interpreted as containing a similar message: in these analyses, the process of contract renegotiation—in which parties' offers and counteroffers may depend on previously chosen observable but nonverifiable actions—can help improve the equilibrium choices of these nonverifiable actions.

contract can designate a mediator who makes a private observation of a random variable θ from a set Θ, where the distribution of the observation is governed by the probability measure μ. (It is sufficient for our purposes to take $\Theta = [0, 1]$ and μ to be the uniform distribution.) The contract provides the mediator with a finite message space for each player, M_1 and M_2. As a function of the realization of θ, the mediator privately gives each player i an allowed set of actions $q_i(\theta)$ and a message $m_i(\theta)$. In summary, a mediated contract is described by a collection

$$(q_1(\cdot), q_2(\cdot), m_1(\cdot), m_2(\cdot), M_1, M_2, \Theta, \mu)$$

where $q_i : \Theta \to \bar{Q}_i$ and $m_i : \Theta \to M_i$.

Note that the simple contract (q_1, q_2) is the special case of a mediated contract in which, for all $\theta \in \Theta$ and $i = 1, 2$, $q_i(\theta) = q_i$ and $m_i(\theta) = m_i$ for some $m_i \in M_i$. In parallel to our previous discussion, we say that a mediated contract is complete if $q_i(\theta) \in P_i$ for all θ and $i = 1, 2$.

Given a mediated contract, a correlated equilibrium (for which mixed and pure strategy Nash equilibria are special cases) is a collection $(\phi_1(\cdot), \phi_2(\cdot), \Omega_1, \Omega_2, \eta)$, where Ω_i is a set of possible signals that may be observed by player i (with elements $\omega_i \in \Omega_i$), η is a probability measure on $\Omega_1 \times \Omega_2$, and $\phi_i : \bar{Q}_i \times M_i \times \Omega_i \to A_i$ satisfies $\phi_i(q_i, m_i, \omega_i) \in q_i$ for all (q_i, m_i, ω_i). Formally:

Definition A1: The collection $(\phi_1(\cdot), \phi_2(\cdot), \Omega_1, \Omega_2, \eta)$ with $\phi_i(q_i, m_i, \omega_i) \in q_i$ for all $(q_i, m_i, \omega_i) \in \bar{Q}_i \times M_i \times \Omega_i$ is a correlated equilibrium of mediated contract $C = (q_1(\cdot), q_2(\cdot), m_1(\cdot), m_2(\cdot), M_1, M_2, \Theta, \mu)$ if, for $i = 1, 2$,

$$E_{\theta,\omega}\{u_i[\phi_i(q_i(\theta), m_i(\theta), \omega_i), \phi_{-i}(q_{-i}(\theta), m_{-i}(\theta), \omega_{-i})]\}$$
$$\geq E_{\theta,\omega}\{u_i[b_i(q_i(\theta), m_i(\theta), \omega_i), \phi_{-i}(q_{-i}(\theta), m_{-i}(\theta), \omega_{-i})]\}$$

for any function $b_i : \bar{Q}_i \times M_i \times A_i$ satisfying $b_i(q_i, m_i, \omega_i) \in q_i$ for all $(q_i, m_i, \omega_i) \in \bar{Q}_i \times M_i \times \Omega_i$, where the expectations are taken with respect to the measures μ and η on Θ and $\Omega_1 \times \Omega_2$, respectively.

Two equilibria are said to be *outcome equivalent* if they give rise to the same probability distribution over $A_1 \times A_2$.

Given these definitions, we have the following result.

PROPOSITION A1: *For any mediated contract* $C = (q_1(\cdot), q_2(\cdot), m_1(\cdot), m_2(\cdot), M_1, M_2, \Theta, \mu)$ and associated correlated equilibrium $(\phi_1(\cdot), \phi_2(\cdot), \Omega_1, \Omega_2, \eta)$ there is a complete mediated contract C^* and associated pure strategy equilibrium that leads to the same probability distribution over $A_1 \times A_2$.

Proposition A1 follows as a consequence of the following two lemmas.

LEMMA A1: *Consider any mediated contract* $C = (q_1(\cdot), q_2(\cdot), m_1(\cdot), m_2(\cdot), M_1, M_2, \Theta, \mu)$ *and an associated correlated equilibrium* $(\phi_1(\cdot), \phi_2(\cdot), \Omega_1, \Omega_2, \eta)$. *There exists a mediated contract that gives rise to an outcome equivalent pure strategy Nash equilibrium.*

PROOF:

Consider the contract $C^\circ = (q_1^\circ(\cdot), q_2^\circ(\cdot), m_1^\circ(\cdot), m_2^\circ(\cdot), M_1^\circ, M_2^\circ, \Theta^\circ, \mu^\circ)$ such that

$$\Theta^\circ = \Theta \times \Omega_1 \times \Omega_2$$

$$\mu^\circ = \mu \times \eta$$

$$M_i^\circ = M_i \times \Omega_i$$

$$q_i^\circ(\theta, \omega) = q_i(\theta)$$

$$m_i^\circ(\theta, \omega) = (m_i(\theta), \omega_i).$$

A pure strategy Nash equilibrium of this contract is a pair of strategies $s_i : \bar{Q}_i \times M_i^\circ \to A_i$ with $s_i(q_i, m_i) \in q_i$ for all (q_i, m_i). For each q_i and $m_i^\circ = (m_i, \omega_i)$, define the pure strategy

$$s_i^\circ(q_i, m_i^\circ) = \phi_i(q_i, m_i, \omega_i).$$

We argue that (s_1°, s_2°) is a pure strategy Nash equilibrium of contract C°. To see this, suppose otherwise. Then for some i there is a $b_i^\circ : \bar{Q}_i \times M_i^\circ \to A_i$ satisfying $b_i^\circ(q_i, m_i^\circ) \in q_i$ for all (q_i, m_i°) and for which

$$\text{(A1)}\quad E_{\theta,\omega}\{u_i[b_i^\circ(q_i^\circ(\theta,\omega), m_i^\circ(\theta,\omega)), s_{-i}^\circ(q_{-i}^\circ(\theta,\omega), m_{-i}^\circ(\theta,\omega))]\}$$
$$> E_{\theta,\omega}\{u_i[s_i^\circ(q_i^\circ(\theta,\omega), m_i^\circ(\theta,\omega)), s_{-i}^\circ(q_{-i}^\circ(\theta,\omega), m_{-i}^\circ(\theta,\omega))]\}$$
$$= E_{\theta,\omega}\{\phi_i(q_i(\theta), m_i(\theta), \omega_i), \phi_{-i}(q_{-i}(\theta), m_{-i}(\theta), \omega_{-i})]\}.$$

Now, consider the deviation $b_i : \bar{Q}_i \times M_i \times \Omega_i \to A_i$ for player i from the correlated equilibrium $(\phi_1(\cdot), \phi_2(\cdot), \Omega_1, \Omega_2, \eta)$ under contract C in which $b_i(q_i, m_i, \omega_i) = b_i^\circ(q_i, (m_i, \omega_i))$ for all (q_i, m_i, ω_i). Then since

$$E_{\theta,\omega}\{u_i[b_i^\circ(q_i^\circ(\theta,\omega), m_i^\circ(\theta,\omega)), s_{-i}^\circ(q_{-i}^\circ(\theta,\omega), m_{-i}^\circ(\theta,\omega))]\}$$
$$= E_{\theta,\omega}\{u_i[b_i(q_i(\theta), m_i(\theta), \omega_i), \phi_{-i}(q_{-i}(\theta), m_{-i}(\theta), \omega_{-i})]\},$$

(A1) implies that the deviation $b_i(\cdot)$ is profitable for player i under contract C—a contradiction.

Lemma A1 tells us that we can always reduce any correlated equilibrium to a pure strategy equilibrium by appropriately enriching the random signals upon which the mediator conditions his messages and the players' allowed strategy sets.

LEMMA A2: *Consider any mediated contract $C = (q_1(\cdot), q_2(\cdot), m_1(\cdot), m_2(\cdot), M_1, M_2, \Theta, \mu)$ and an associated pure strategy Nash equilibrium (s_1, s_2). There exists a complete mediated contract C^* that gives rise to an outcome equivalent pure strategy Nash equilibrium.*

PROOF:

Define the complete contract $C^* = (q_1^*(\cdot), q_2^*(\cdot), m_1^*(\cdot), m_2^*(\cdot), M_1^*, M_2^*, \Theta, \mu)$ such that

$$M_i^* = \bar{Q}_i \times M_i$$

$$m_i^*(\theta) = (q_i(\theta), m_i(\theta))$$

$$q_i^*(\theta) = p_i(s_i(q_i(\theta), m_i(\theta))).$$

Now consider the pair of pure strategies $s_i^* : \bar{Q}_i \times M_i^* \to A_i$ for $i = 1, 2$ such that

$$s_i^*(q_i', (q_i, m_i)) = s_i(q_i, m_i)$$

whenever $s_i(q_i, m_i) \in q_i'$ [$s_i^*(q_i', (q_i, m_i))$ can be anything otherwise]. Note that with this strategy, the action taken by player i for each realization under contract C^* is identical to the action he takes for realization θ under contract C; in particular, since $s_i(q_i(\theta), m_i(\theta)) \in p_i(s_i(q_i(\theta), m_i(\theta)))$ for all $\theta \in \Theta$, we have

$$\text{(A2)}\quad s_i^*(q_i^*(\theta), m_i^*(\theta)) = s_i^*(p_i(s_i(q_i(\theta), m_i(\theta))), (q_i(\theta), m_i(\theta))) = s_i(q_i(\theta), m_i(\theta))$$

for all $\theta \in \Theta$ and $i = 1, 2$. This immediately implies that if the pair of pure strategies (s_1^*, s_2^*) is an equilibrium for contract C^*, then it is outcome equivalent to the equilibrium (s_1, s_2) arising under contract C. We conclude by arguing that (s_1^*, s_2^*) is indeed an equilibrium under C^*. To see this, suppose otherwise. Then some player i has a feasible deviation strategy $b_i^* : \bar{Q}_i \times M_i^* \to A_i$ under contract C^* that raises his expected payoff relative to strategy s_i^* when his opponent plays s_{-i}^*. But note that under contract C the strategy $b_i : \bar{Q}_i \times M_i \to A_i$ such that

$$b_i(q_i, m_i) = b_i^*(p_i(s_i(q_i, m_i)), (q_i, m_i))$$

is feasible (i.e., $b_i(q_i, m_i) \in q_i$) and chooses for each $\theta \in \Theta$ the same action as does $b_i^*(\cdot)$ under contract C^*. Given (A2), this must mean that switching from $s_i(\cdot)$ to $b_i(\cdot)$ is a profitable deviation for i under contract C when his opponent plays $s_{-i}(\cdot)$—a contradiction to (s_1, s_2) being an equilibrium under contract C.

The complication in Lemma A2 is that in the incomplete contract C the announced allowable set of actions $q_i \in \bar{Q}_i$ may signal something to player i about the state θ, much like a message. So in establishing the result it is important to incorporate this information into the new contract C^* as an explicit message.

Proposition A1 tells us that, as long as a mediated contract is feasible, the optimality of complete contracts extends to correlated (and thus mixed) equilibria. In Figure A1 we depict a symmetric contracting problem in which attaining the best symmetric payoff (6, 6) requires a complete and mediated contract. In this problem, complete simple contracts can achieve only the payoffs (1, 1), (10, 2), and (2, 10). No incomplete simple contract can achieve (6, 6) either: To achieve (6, 6) requires a correlated equilibrium with $\text{Prob}(a_4, b_1) = \text{Prob}(a_1, b_4) = 1/2$. But with this distribution of strategies, player 1 would deviate to a_2 when (a_4, b_1) is called for. The payoff (6, 6) can be achieved with a mediated contract that announces the restrictions $(q_1 = p_{12}, q_2 = p_{21})$ with probability 1/2, and $(q_1 = p_{11}, q_2 = p_{22})$ with probability 1/2. But no incomplete mediated contract can achieve (6, 6). In general, however, Proposition A1 also tells us that an optimal complete contract may need to be quite complicated.

		Player 2			
		p_{21}		p_{22}	
Player 1		a_{21}	a_{22}	a_{23}	a_{24}
p_{11}	a_{11}	1,1	-20,20	1,1	2,10
	a_{12}	20,-20	1,1	0,0	1,1
p_{12}	a_{13}	1,1	0,0	1,1	20,-20
	a_{14}	10,2	1,1	-20,20	1,1

Convex Hull of Payoff Set

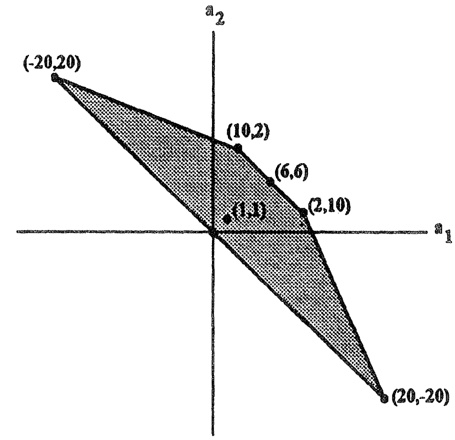

Figure A1. An Example for Which a Complete Mediated Contract is Required to Achieve the Efficient Symmetric Outcome

Appendix B

PROOF OF PROPOSITION 2:

That no individually rational Pareto-efficient outcome is implementable using either a complete contract or a totally incomplete contract when $a_1^s > L_1$ and $a_1 < b_1(a_2)$ for all $(a_1, a_2) \in CC$ follows from the reasoning in the text.

To demonstrate that there are always individually rational Pareto-efficient outcomes that cannot be implemented with a complete contract, we need only show that there are $(a_1, a_2) \in CC$ such that $a_1 \neq b_1(a_2)$ [note that any such point has $a_1 < b_1(a_2)$]. Consider any element $(\tilde{a}_1, \tilde{a}_2)$ of the set

$$\underset{(a_1,a_2)\in A_1\times A_2}{\text{Argmax}} \; u_2(a_1, a_2)$$

$$\text{subject to (i) } a_1 \leq a_1^s$$

$$\text{(ii) } u_1(a_1, a_2) = u_1(a_1^s, b_2(a_1^s)).$$

Clearly, this set is nonempty and $(\tilde{a}_1, \tilde{a}_2) \in CC$. If $\tilde{a}_1 = a_1^s$, then it is obvious that $\tilde{a}_1 \neq$

$b_1(\tilde{a}_2)$ (otherwise the Stackleberg point would coincide with the Nash point, which cannot occur with strategic complements when the Nash point lies in the interior of $A_1 \times A_2$). If $\tilde{a}_1 < a_1^s$, then $b_2(\tilde{a}_1) \leq b_2(a_1^s)$. Furthermore, by the definition of a_1^s we have $\tilde{a}_2 < b_2(\tilde{a}_1)$ and so $\tilde{a}_2 < b_2(a_1^s)$. Thus,

$$u_1(a_1^s, \tilde{a}_2) > u_1(a_1^s, b_2(a_1^s)) = u_1(\tilde{a}_1, \tilde{a}_2).$$

But this implies that $\tilde{a}_1 \neq b_1(\tilde{a}_2)$, as desired.

We now establish the first part of the result. Define $\bar{U}$ such that $u_2(L_1, \bar{U}) \leq \text{Min}_{(a_1,a_2)\in BB}\, u_2(a_1, a_2)$ (note that under our assumptions such a $\bar{U}$ exists). Assume, henceforth, that $U_2 \geq \bar{U}$.

Now consider an arbitrary individually rational Pareto-efficient contract (a_1^*, a_2^*). Let a_2' be defined by $u_2(a_1^*, a_2') = u_2(a_1^*, a_2^*)$ (such a value of a_2 exists by the assumption that $U_2 \geq \bar{U}$); note that since $u_2(\cdot)$ is strictly concave in a_2 and $a_2^* < b_2(a_1^*)$, we have $a_2' > b_2(a_1^*) > a_2^*$. Now consider the partially incomplete interval exclusion contract in which $q_2 = [L_2, a_2^*] \cup [a_2', U_2]$. We argue that there is a subgame-perfect Nash equilibrium under this contract that results in outcome (a_1^*, a_2^*).

Because $u_2(\cdot)$ is strictly concave in a_2, player 2's induced best response under this contract, $B_2(\cdot)$, has the property that $B_2(a_1) = b_2(a_1)$ if $b_2(a_1) \in q_2$, and $B_2(a_1) \in \{a_2^*, a_2'\}$ otherwise. Moreover, $\partial^2 u_i(a_i, a_j)/\partial a_i \partial a_j > 0$ implies that if $u_2(a_1^*, a_2^*) = u_2(a_1^*, a_2')$ then $u_2(a_1, a_2^*) > u_2(a_1, a_2')$ if $a_1 < a_1^*$, and $u_2(a_1, a_2^*) < u_2(a_1, a_2')$ if $a_1 > a_1^*$. This implies that

$$B_2(a_1) = \begin{Bmatrix} \text{Min}\{a_2^*, b_2(a_1)\} & \text{if } a_1 < a_1^* \\ \{a_2^*, a_2'\} & \text{if } a_1 = a_1^* \\ \text{Max}\{a_2', b_2(a_1)\} & \text{if } a_1 > a_1^* \end{Bmatrix}.$$

We suppose that if player 1 chooses a_1^*, player 2 resolves his indifference by choosing a_2^*.

Consider first whether any $\hat{a}_1 < a_1^*$ can give player 1 a higher payoff than $u_1(a_1^*, a_2^*)$ under this contract. If $B_2(\hat{a}_1) = b_2(\hat{a}_1)$ then we would have

$$u_1(\hat{a}_1, B_2(\hat{a}_1)) = u_1(\hat{a}_1, b_2(\hat{a}_1)) \leq \text{Max}_a\, u_1(a, b_2(a)) \leq u_1(a_1^*, a_2^*),$$

where the last inequality follows from individual rationality. If instead $B_2(\hat{a}_1) = a_2^*$, then since $a_1^* \leq b_1(a_2^*)$ and $u_1(\cdot)$ is strictly concave in a_1, we have $u_1(\hat{a}_1, a_2^*) < u_1(a_1^*, a_2^*)$ for any $\hat{a}_1 < a_1^*$. We conclude that no $\hat{a}_1 < a_1^*$ can be a better choice for player 1 than a_1^*.

Now consider whether choosing any $\hat{a}_1 > a_1^*$ can raise player 1's payoff. For such a choice we have

$$u_1(\hat{a}_1, B_2(\hat{a}_1)) = u_1(\hat{a}_1, \text{Max}\{a_2', b_2(\hat{a}_1)\}) \leq u_1(\hat{a}_1, b_2(\hat{a}_1)) \leq \text{Max}_a\, u_1(a, b_2(a)) \leq u_1(a_1^*, a_2^*),$$

where the last inequality again follows from individual rationality. Thus, a_1^* is at least as good a choice for player 1 as any $\hat{a}_1 > a_1^*$ under this contract.

We conclude that a_1^* is an optimal choice for player 1. Hence, there is a subgame-perfect Nash equilibrium with outcome (a_1^*, a_2^*) under the interval exclusion contract $q_2 = [L_2, a_2^*] \cup [a_2', U_2]$. Finally, recall that in the special case in which $a_1^s = L_1$, the no-contract outcome $(a_1^s, b_2(a_1^s))$ is the unique Pareto-efficient individually rational outcome. When $(a_1^*, a_2^*) = (a_1^s, b_2(a_1^s))$, the interval exclusion contract derived above has $a_2' = a_2^*$, and so the contract is in fact fully incomplete.

PROOF OF PROPOSITION 3:

The fact that a complete contract can implement any point in the set $\{(a_1, a_2) \in A_1 \times A_2 : a_1 = b_1(a_2)\}$ is immediate, as is the fact that any point in the set $\{(a_1, a_2) \in A_1 \times A_2 :$

$a_1 > b_1(a_2)\}$ is Pareto dominated by a point in set $\{(a_1, a_2) \in A_1 \times A_2 : a_1 = b_1(a_2)\}$. To establish the result, we therefore need only show formally that no (a_1, a_2) with $a_1 < b_1(a_2)$ is implementable.

To see this, suppose that the contract in which player 2's allowed set of actions is $q \subseteq A_2$ implements an outcome $(\bar{a}_1, \bar{a}_2)$ with $\bar{a}_1 < b_1(\bar{a}_2)$. Define $B_2(\cdot)$ as player 2's induced best response under this contract. Clearly, $\bar{a}_2 \in B_2(\bar{a}_1)$. Moreover, under the condition of strategic substitutes, $B_2(\cdot)$ must be weakly decreasing in the sense that for any $a_1' < a_1''$, if $a_2' \in B_2(a_1')$ and $a_2'' \in B_2(a_1'')$, then $a_2'' \leq a_2'$. To see this, assume to the contrary that $a_2'' > a_2'$. Then

$$0 \leq u_2(a_1'', a_2'') - u_2(a_1'', a_2')$$
$$< u_2(a_1', a_2'') - u_2(a_1', a_2')$$

(the strict inequality follows from $\partial^2 u_2(\cdot)/\partial a_1 \partial a_2 < 0$), which contradicts $a_2' \in B_2(a_1')$.

Now, since $(\bar{a}_1, \bar{a}_2)$ satisfies $\bar{a}_1 < b_1(\bar{a}_2)$, by choosing $\hat{a}_1 = b_1(\bar{a}_2)$ player 1 can earn at least

$$\text{Min}_{a_2 \in B_2(b_1(\bar{a}_2))} u_1(b_1(\bar{a}_2), a_2)$$
$$\geq u_1(b_1(\bar{a}_2), \bar{a}_2) > u_1(\bar{a}_1, \bar{a}_2).$$

Since $\bar{a}_2$ is player 2's response to player 1 choosing $\bar{a}_1$ [under the assumption that $(\bar{a}_1, \bar{a}_2)$ is implemented by this contract], this contradicts $\bar{a}_1$ being player 1's optimal choice.

Finally, we argue that points of the form $(b_1(a_2), a_2)$ are Pareto inefficient when $a_2 > L_2$. For $a_2 > a_2^N$, a small reduction in a_2 (holding a_1 fixed) increases the payoff to both play ers. For $L_2 < a_2 < a_2^N$, $b_1(a_2) > a_1^N > L_1$. Choose some

$$\alpha > -\frac{\left(\dfrac{\partial u_2(b_1(a_2), a_2)}{\partial a_2}\right)}{\left(\dfrac{\partial u_2(b_1(a_2), a_2)}{\partial a_1}\right)}.$$

Then we have

$$\left[\frac{du_2(b_1(a_2) - \alpha\varepsilon, a_2 - \varepsilon)}{d\varepsilon}\right]_{\varepsilon=0} > 0$$

(by construction), and

$$\left[\frac{du_1(b_1(a_2) - \alpha\varepsilon, a_2 - \varepsilon)}{d\varepsilon}\right]_{\varepsilon=0}$$
$$= -\frac{\partial u_1(b_1(a_2), a_2)}{\partial a_2} > 0.$$

This Pareto-improving change is, of course, infeasible when $a_2 = L_2$.

PROOF OF PROPOSITION 4:

To establish the first part of the proposition, we exhibit a contract that implements the action pair (a_1^*, a_2^*). Define

$$A_2' = \{a_2 \in A_2 : a_2 \geq a_2^* \text{ and } u_2(a_1^*, a_2^*)$$
$$\geq u_2(a_1^*, a_2)\}$$

and

$$A_2'' = \{a_2 \in A_2 : u_1(a_1^*, a_2^*)$$
$$\geq u_1(b_1(a_2), a_2)\}.$$

Under our assumptions, we can find a $\bar{U}$ such that, for all $U_2 \geq \bar{U}$, A_1' and A_2'' are both closed and nonempty and have a nonempty closed intersection. Thus, we can define

$$\hat{a}_2 = \text{Min}\, A_2' \cap A_2''.$$

Now consider the contract in which player 2's action set q_2 gives him a choice between the following two action-transfer pairs (recall that the transfer is a transfer from player 1 to player 2):

$$q_2 = \{(a_2^*, 0), (\hat{a}_2, \hat{T})\},$$

where $\hat{T} = u_2(a_1^*, a_2^*) - u_2(a_1^*, \hat{a}_2)$. Note that $\hat{T} \geq 0$ since $\hat{a}_2 \in A_2'$. By strategic complements, we have

$$u_2(a_1, a_2^*) - u_2(a_1, \hat{a}_2) \gtreqless \hat{T} \text{ if and only if } a_1 \lesseqgtr a_1^*.$$

Thus, player 2 optimally chooses $(a_2^*, 0)$ if $a_1 \leq a_1^*$, and $(\hat{a}_2, \hat{T})$ if $a_1 > a_1^*$.

Now consider player 1. For $a_1 \leq a_1^*$, player 1 earns $u_1(a_1, a_2^*)$. Since $a_1^* \leq b_1(a_2^*)$ [recall that $(a_1^*, a_2^*) \in BB$], the best such choice is $a_1 = a_1^*$. For $a_1 > a_1^*$, player 1 earns

$$u_1(a_1, \hat{a}_2) - \hat{T} \leq u_1(b_1(\hat{a}_2), \hat{a}_2) - \hat{T} \leq u_1(a_1^*, a_2^*),$$

where the last inequality follows because $\hat{a}_2 \in A_2''$ and $\hat{T} \geq 0$. Hence, player 1 finds it optimal to choose $a_1 = a_1^*$.

By combining this contract with an appropriately chosen up-front transfer, we can implement any Pareto-efficient outcome.

For the second part of the proposition, note first that a complete contract can implement the Pareto-efficient action pair (a_1^*, a_2^*) only if $a_1^* = b_1(a_2^*)$ (this follows from the same logic as in Proposition 2). But

$$\left[\frac{d(\sum_i u_i(b_1(a_2^*) - \varepsilon, a_2^*))}{d\varepsilon}\right]_{\varepsilon=0} = -\frac{\partial u_2(b_1(a_2^*), a_2^*)}{\partial a_1} > 0.$$

Thus, no such point can be Pareto efficient unless $a_1^* = b_1(a_2^*) = L_1$. Since $b_1(a_2^*) \geq b_1(L_2)$, this is clearly impossible if $b_1(L_2) > L_1$.

PROOF OF PROPOSITION 5:

That any action pair $(\bar{a}_1, \bar{a}_2) \in A^c$ is implementable with a complete contract is immediate. Note next that only action pairs lying in set $B_1 = \{(a_1, a_2) \in A_1 \times A_2 : a_1 < b_1(a_2)\}$ can yield a larger total payoff than the best complete contract.

Consider an arbitrary simple contract. Suppose that there is some equilibrium under this contract that implements an action pair $(\bar{a}_1, \bar{a}_2) \in B_1$. Let $\phi_2(a_1)$ denote player 2's equilibrium strategy and let $T(a_1)$ denote the associated (net) transfer made by player 1 to player 2. Note that, by the Revelation Principle, there must be an incentive compatible (i.e., truth-inducing) direct revelation contract in which player 2 is asked to announce player 1's action choice (after it has been taken), where player 2's announcements are mapped into choices of a_2 and transfers, and where the resulting action-transfer outcome is $[\phi_2(a_1), T(a_1)]$ for each $a_1 \in A_1$. Going in the other direction, if $[\phi_2(\cdot), T(\cdot)]$ is an incentive-compatible direct revelation mechanism, then there is a simple contract that implements outcome $[\phi_2(a_1), T(a_1)]$ for each $a_1 \in A_1$: this contract simply faces player 2 with the choice from the set of action-transfer pairs $\{(a_2, T) : \phi_2(a_1) = a_2 \text{ and } T(a_1) = T \text{ for some } a_1 \in A_1\}$.

For any direct revelation mechanism $[\phi_2(\cdot), T(\cdot)]$, let $U_i(a_1)$ denote player i's payoff when player 1 chooses a_1 and player 2 truthfully announces this fact. Standard results tell us that $[\phi_2(\cdot), T(\cdot)]$ is incentive compatible if and only if $\phi_2(\cdot)$ is nonincreasing and $T(\cdot)$ is such that[33]

$$U_2(a_1) = U_2(\bar{a}_1) + \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \phi_2(s))}{\partial a_1} ds \quad \text{for all } a_1.$$

Since we can always take $T_2(\bar{a}_1) = 0$, this is equivalent to the condition that

$$U_2(a_1) = u_2(\bar{a}_1, \bar{a}_2) + \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \phi_2(s))}{\partial a_1} ds \quad \text{for all } a_1.$$

[33] See Andreu Mas-Collell et al. (1995 Sec. 23.D).

This implies that player 1's payoff from choosing a_1 is

$$(\text{B1}) \quad U_1(a_1) = u_1(a_1, \phi_2(a_1)) + u_2(a_1, \phi_2(a_1)) - u_2(\bar{a}_1, \bar{a}_2) - \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \phi_2(s))}{\partial a_1} ds.$$

Hence, we see that action pair $(\bar{a}_1, \bar{a}_2) \in B_1$ is implementable if and only if there exists a nonincreasing function $\phi_2(\cdot)$ with $\phi_2(\bar{a}_1) = \bar{a}_2$ such that, for all $a_1 \in A_1$, $U_1(a_1) \leq u_1(\bar{a}_1, \bar{a}_2)$, or equivalently,

$$(\text{B2}) \quad u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2) \geq u_1(a_1, \phi_2(a_1)) + u_2(a_1, \phi_2(a_1)) - \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \phi_2(s))}{\partial a_1} ds \text{ for all } a_1.$$

We next argue that if $(\bar{a}_1, \bar{a}_2)$ is implementable by means of some implementable function $\phi_2(\cdot)$, then it is also implementable by means of the function

$$\tilde{\phi}_2(\cdot) = \begin{Bmatrix} \bar{a}_2 & \text{if } a_1 \leq \bar{a}_1 \\ \hat{a}_2 & \text{if } a_1 > \bar{a}_1 \end{Bmatrix}$$

where we define $\hat{a}_2$ as the smallest element of A_2 satisfying $\hat{a}_2 = \phi_2(b_1(\hat{a}_2))$ and we let $\hat{a}_1 = b_1(\hat{a}_2)$.[34] Note that $\hat{a}_2 \leq \bar{a}_2$. To establish the claim we need to verify that (B2) is satisfied with function $\tilde{\phi}_2(\cdot)$. Consider first $a_1 < \bar{a}_1$. For such levels of a_1 the right-hand side of (B2) becomes

$$u_1(a_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)$$

which is strictly lower than the left-hand side of (B2) given that $a_1 < \bar{a}_1 < b_1(\bar{a}_2)$. Now consider $a_1 > \bar{a}_1$. Since (B2) is satisfied by the function $\phi_2(\cdot)$, we know that[35]

$$\begin{aligned} & u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2) \\ & \geq u_1(\hat{a}_1, \hat{a}_2) + u_2(\hat{a}_1, \hat{a}_2) - \int_{\bar{a}_1}^{\hat{a}_1} \frac{\partial u_2(s, \phi_2(s))}{\partial a_1} ds \\ & \geq u_1(\hat{a}_1, \hat{a}_2) + u_2(\hat{a}_1, \hat{a}_2) - \int_{\bar{a}_1}^{\hat{a}_1} \frac{\partial u_2(s, \hat{a}_2)}{\partial a_1} ds \\ & = u_1(\hat{a}_1, \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2) \\ & \geq u_1(a_1, \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2) \\ & = u_1(a_1, \hat{a}_2) + [u_2(a_1, \hat{a}_2) - u_2(a_1, \hat{a}_2)] + u_2(\bar{a}_1, \hat{a}_2) \\ & = u_1(a_1, \hat{a}_2) + u_2(a_1, \hat{a}_2) - \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \hat{a}_2)}{\partial a_1} ds \\ & = u_1(a_1, \tilde{\phi}_2(a_1)) + u_2(a_1, \tilde{\phi}_2(a_1)) - \int_{\bar{a}_1}^{a_1} \frac{\partial u_2(s, \tilde{\phi}_2(a_1))}{\partial a_1} ds. \end{aligned}$$

Hence, in their attempt to implement any $(\bar{a}_1, \bar{a}_2) \in B_1$ the players can restrict attention to such "two-point" contracts. Note, however, that for such a contract the right-hand side of (B2) equals $u_1(a_1, \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2)$ and is therefore maximized over all $a_1 > \bar{a}_1$ at $a_1 = b_1(\hat{a}_2)$, at which point it takes the value $u_1(b_1(\hat{a}_2), \hat{a}_2) + u_1(\bar{a}_1, \hat{a}_2)$. Hence, the existence of $\hat{a}_2 \leq \bar{a}_2$ such that (1) holds is a necessary and sufficient condition for $(\bar{a}_1, \bar{a}_2)$ to be implementable. In fact, it is easy to see that (1) can hold only if $\hat{a}_2 < \bar{a}_2$.

We next use (1) to argue that the players can, without loss, restrict attention in their search for a contract that dominates the best complete contract to (incomplete) contracts that implement an action pair in the set A° (out of all the action pairs in set B_1). First, we show that one can rule out all contracts supporting action pairs $(\bar{a}_1, \bar{a}_2) \in B_1$ for which $(\bar{a}_1, a_2^\circ(\bar{a}_1)) \notin B_1$. In such cases, $\bar{a}_2 < b_1^{-1}(\bar{a}_1) \leq a_2^\circ(\bar{a}_1)$; the concavity

[34] The existence of $\hat{a}_2$ follows from three facts: $\phi_2(\cdot)$ is nonincreasing, $b_1(\cdot)$ is continuous, and A_i is bounded for $i = 1, 2$.

[35] The second weak inequality follows because $\phi_2(a_1) > \hat{a}_2$ for all $a_1 \in [\bar{a}_1, \hat{a}_1]$ and $(\partial^2 u_2(\cdot)/\partial a_1 \partial a_2) < 0$. The third weak inequality follows because $\hat{a}_1 = b_1(\hat{a}_2)$.

of aggregate utility therefore implies that the outcome $(\bar{a}_1, b_1^{-1}(\bar{a}_1))$ is superior to $(\bar{a}_1, \bar{a}_2)$. But $(\bar{a}_1, b_1^{-1}(\bar{a}_1))$ is always supportable through a complete contract.

Second, we show that one can rule out all contracts supporting pairs $(\bar{a}_1, \bar{a}_2) \in B_1$ for which $(\bar{a}_1, a_2^\circ(\bar{a}_1)) \in B_1$, but $\bar{a}_2 \neq a_2^\circ(\bar{a}_1)$. Assume that such a pair is implementable. Then there exists an $\hat{a}_2 < \bar{a}_2$ such that $(\bar{a}_1, \bar{a}_2)$ and $\hat{a}_2$ satisfy (1). Note that $\hat{a}_2 < a_2^\circ(\bar{a}_1)$: this is immediate if $\bar{a}_2 < a_2^\circ(\bar{a}_1)$; for $\bar{a}_2 \geq a_2^\circ(\bar{a}_1)$, $\hat{a}_2 \in [a_2^\circ(\bar{a}_1), \bar{a}_2)$ would imply

$$u_1(\bar{a}_1, \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2)$$
$$> u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)$$
$$\geq u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2),$$

which is a contradiction. Note also that

$$u_1(\bar{a}_1, a_2^\circ(\bar{a}_1)) + u_2(\bar{a}_1, a_2^\circ(\bar{a}_1))$$
$$> u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2),$$

from which it follows that $(\bar{a}_1, a_2^\circ(\bar{a}_1)) \in B_1$ and $\hat{a}_2 < a_2^\circ(\bar{a}_1)$ satisfy (1). The outcome $(\bar{a}_1, a_2^\circ(\bar{a}_1))$ is therefore implementable and superior to $(\bar{a}_1, \bar{a}_2)$.

These first two steps allow us to restrict attention to contracts supporting action pairs $(\bar{a}_1, \bar{a}_2) \in B_1$ with $\bar{a}_2 = a_2^\circ(\bar{a}_1)$. We now show that we can rule out all such points except those that lie in A°. Assume that $(\bar{a}_1, \bar{a}_2) \in B_1$ is implementable, that $\bar{a}_2 = a_2^\circ(\bar{a}_1)$, and that $a_1 < a_1^*$. Then there exists an $\hat{a}_2 < \bar{a}_2$ such that $(\bar{a}_1, \bar{a}_2)$ and $\hat{a}_2$ satisfy (1). Note that $\hat{a}_2 < a_2^*$: if not, then one obtains the following contradiction:

$$u_1((a_2^\circ)^{-1}(\hat{a}_2), \hat{a}_2) + u_2((a_2^\circ)^{-1}(\hat{a}_2), \hat{a}_2)$$
$$> u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)$$
$$\geq u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2)$$
$$\geq u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2((a_2^\circ)^{-1}(\hat{a}_2), \hat{a}_2)$$

[where the first inequality can be shown to follow from concavity of aggregate payoffs, and where the final inequality follows from $(a_2^\circ)^{-1}(\hat{a}_2) > (a_2^\circ)^{-1}(\bar{a}_2) = \bar{a}_1$]. But then, since

$$u_1(a_1^*, a_2^*) + u_2(a_1^*, a_2^*)$$
$$> u_1(\bar{a}_1, \bar{a}_2) + u_2(\bar{a}_1, \bar{a}_2)$$
$$\geq u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(\bar{a}_1, \hat{a}_2)$$
$$> u_1(b_1(\hat{a}_2), \hat{a}_2) + u_2(a_1^*, \hat{a}_2)$$

(where the last inequality follows from $a_1^* > \bar{a}_1$), we know that (a_1^*, a_2^*) and $\hat{a}_2 < a_2^*$ satisfy (1). Thus, (a_1^*, a_2^*) is implementable and is superior to $(\bar{a}_1, \bar{a}_2)$.

PROOF OF PROPOSITION 7:

Here we show that whenever there is a unique interior Nash equilibrium in the unrestricted game, (a_1^N, a_2^N), we can create multiple Pareto-unranked equilibria in the stage 2 game through an appropriately designed partially incomplete contract.

To begin, for $\varepsilon > 0$ define $y_i(\varepsilon)$ for $i = 1, 2$ such that

$$u_i(y_i(\varepsilon), x_j^N + \varepsilon) = u_i(x_i^N + \varepsilon, x_j^N + \varepsilon).$$

Note, first, that, by the assumption that (x_1^N, x_2^N) is interior, for small enough $\varepsilon > 0$ both $x_1^N + \varepsilon$ and $x_2^N + \varepsilon$ are feasible and such a $y_i(\varepsilon)$ exists. In addition, note that $x_i^N + \varepsilon > b_i(x_j^N + \varepsilon)$ for $i = 1, 2$ and $j \neq i$ by the strategic substitutes condition and so, by strict concavity of $u_i(\cdot)$ in a_i we have $y_i(\varepsilon) < b_i(x_j^N + \varepsilon)$ for $i = 1, 2$ and $j \neq i$.

Next, note that

$$u_i(y_i(\varepsilon), y_j(\varepsilon)) < u_i(x_i^N + \varepsilon, y_j(\varepsilon))$$

for $i = 1, 2$ and $j \neq i$. Now, there exists a $\delta \in (0, x_i^N - y_i(\varepsilon))$ such that

$$u_i(y_i(\varepsilon) + \delta, x_j^N + \varepsilon)$$
$$> u_i(x_i^N + \varepsilon, x_j^N + \varepsilon),$$

and

$$u_i(y_i(\varepsilon) + \delta, y_j(\varepsilon) + \delta)$$
$$< u_i(x_i^N + \varepsilon, y_j(\varepsilon) + \delta),$$

where the first inequality follows from the fact that $y_i(\varepsilon) < b_i(x_j^N + \varepsilon) < x_i^N$, from the feasibility of all points in the set $[y_i(\varepsilon), x_i^N]$, and from the concavity of player i's objective function in a_i, and where the second inequality follows from the continuity of $u_i(\cdot)$.

Now suppose the players sign a contract that gives each player i a choice in the stage 2 game between the two actions $y_i(\varepsilon) + \delta$ and $x_i + \varepsilon$. It is immediate that $(y_1(\varepsilon), x_2^N + \varepsilon)$ and $(x_1^N + \varepsilon, y_2(\varepsilon))$ are the pure strategy equilibria of the restricted game. To see that they are Pareto unranked note that for $i = 1, 2$ and $j \neq i$ we have $u_i(y_i(\varepsilon) + \delta, x_j^N + \varepsilon) < u_i(y_i(\varepsilon) + \delta, y_j(\varepsilon) + \delta) < u_i(x_i^N + \varepsilon, y_j(\varepsilon) + \delta)$.

APPENDIX C

Imagine that two entrepreneurs have formed a partnership to start a new business, and that this requires an expenditure of both effort and money (in a sense, we modify Example 2 so that both parties invest both effort and money). Effort is observable but not contractible, while money is fully contractible. After an initial start-up phase (stage 1) involving effort, the parties anticipate the need to make a substantial monetary investment (stage 2). We will be relatively vague about the details of stage 1, and focus attention primarily on the investment problem.

In stage 2, both parties simultaneously supply money to the enterprise. We will use p_i to denote party i's payment. So that structural intertemporal linkages are absent, we assume that the returns to the stage 2 investment are independent of effort levels in stage 1. Specifically, the net payoff received by player i in stage 2 is given by

$$u_i(p_1, p_2) = I(p_1 + p_2 > k)\cdot v + f(\max\{p_1 + p_2 - k, 0\}) - p_i.$$

In this expression $I(\cdot)$ denotes an indicator function (equal to unity when the conditional is satisfied, and zero otherwise). Thus, the investment generates no return unless it achieves some minimum scale, k. At k, the gross value of the investment is v for each party. Investments beyond k add value, as indicated by $f(\cdot)$. We assume that $2v > k > v$ (so that no party would be willing to undertake the minimum investment by itself, even though it is in their joint interests), $\frac{1}{2} < f'(0) < 1$ (so that no party would be willing to undertake further investments unilaterally, even though it is in their interests to do so jointly), $f(0) = 0$, and $f''(\cdot) < 0$.

The efficient aggregate level of investment, $P^* > k$, is defined by the equation $2f'(P^* - k) = 1$. A complete contract would specify a total investment of P^* and associated payments in advance of stage 1. Since this would provide the parties with no opportunity to impose punishments contingent on stage 1 performance, they may prefer to retain discretion with respect to stage 2 funding. We assume, for simplicity, that in renegotiating an explicit contract the parties split evenly any gain in surplus resulting from renegotiation.

Consider an agreement that leaves investment choices completely unspecified. Then there are many continuation equilibria for stage 2. Obviously, $(0, 0)$ (no financing) is a zero-payoff equilibrium. There is also a continuum of equilibria of the form $(p, k - p)$ where $p \in [k - v, v]$; these equilibria yield payoffs of $(v - p, v - k + p)$. While all of these equilibria are inefficient, only the first is Pareto dominated within the set of equilibria; the rest are Pareto unranked. This reflects the fact that the players' choices are strategic complements at the threshold where minimum financing is achieved, but strategic substitutes beyond this point. Renegotiation-proofness rules out the use of the $(0, 0)$ continuation equilibrium as a punishment, just as it rules out all punishments where strategic complementarities exist globally (Proposition 6). One can nevertheless construct contingent rewards and punishments with the remaining equilibria (as in Proposition 7).

In particular, imagine that players impose no contractual restrictions on second-stage choices, and follow an equilibrium where they each play $a^* = (k/2, k/2)$ on the equilibrium path, while using $a^1 = (v, k - v)$ to punish undesired first-stage actions by player 1, and $a^2 = (k - v, v)$ to punish undesired first-stage actions by player 2. Intuitively, this corresponds to an implicit agreement wherein the parties always finance the project at the level k, splitting the cost equally unless one has failed to contribute adequate effort in stage 1 (in which case the shirker must contribute a larger share of the monetary resources). Absent renegotiation of the explicit contract, resulting continuation payoffs would be $\pi^* = (v - k/2, v - k/2)$, $\pi^1 \equiv (0, 2v - k)$, and $\pi^2 \equiv (2v - k, 0)$, respectively. Thus, if renegotiation of the initial (explicit) contract were impossible, each party would face a potential penalty of $v - k/2$ for bad behavior in stage 1.

Of course, under our assumptions, the parties will renegotiate the explicit contract to achieve efficient outcomes and so will agree to a total contribution of P^*. Since the gain in total surplus resulting from renegotiation is the same regardless of the continuation equilibrium anticipated by the players, and since this surplus is split evenly in all cases, the post-renegotiation penalty for bad behavior in stage 1 is still $v - k/2$. Through an incomplete agreement of this type, the parties may therefore improve upon the outcome achievable with a complete contract.

The agreement described above may provide rather modest punishments. Indeed, as k approaches $2v$, the magnitude of the penalty falls to zero. This is because, for such cases, there is relatively little variation in the range of payoffs within the set of Pareto-unranked equilibria. However, it may be possible to expand this variation, thereby increasing the scope for punishment, through the use of *partially* incomplete contracts.

In the context of this example, we will consider the set of partially incomplete contracts that impose a minimum contribution level, l. That is, prior to stage 1, the parties sign a contract wherein they each agree to contribute at least l in the second stage. For $l < k/2$, there is a continuum of equilibria of the form $(p, k - p)$ for $p \in [\max\{l, k - v - l\}, \min\{k - l, v + l\}]$.[36] For the moment, we will assume that l is small, so that $\max\{l, k - v - l\} = k - v - l$, and $\min\{k - l, v + l\} = v + l$. Note that for this range of l, the effect of having $l > 0$ is to raise the maximal amount that a player is willing to invest in an equilibrium by l (to $v + l$) and to lower the minimal amount that a player must invest in an equilibrium by the same amount. By the same logic as above, the parties will renegotiate after stage 1 so that the total stage 2 contribution is P^*, and the post-renegotiation payoffs will be such that use of this partially incomplete contract increases the potential punishment for each party by l, to $v - k/2 + l$.

For small l an increase in l expands the set of Pareto-unranked equilibria, and thereby raises the potential punishment. However, these effects reverse once $\max\{l, k - v - l\} = l$ and $\min\{k - l, v + l\} = k - l$. Thus, the parties can maximize potential punishments (among the class of contracts involving a lower bound on investments) by choosing l to satisfy $l = k - v - l$, or $l = (k - v)/2$. The magnitude of the associated punishment is given by $v - k/2 - l = (k - v)/2$. Notice that this punishment is positive even when $k = 2v$. Thus, the use of the optimal partially incomplete contract significantly expands the scope for meaningful punishment of stage 1 deviations.

[36] There may also be an equilibrium of the form (l, l), but this is Pareto dominated within the class of equilibria for pertinent values of l.

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