Core and 'Crust': Consumer Prices and the Term Structure of Interest Rates *

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Abstract

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Abstract

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1 Introduction

A general view in the empirical macro-finance literature is that financial variables do little to help forecast consumer prices. In particular, most empirical studies find that there is limited or no marginal information content in the nominal interest rate term structure for future inflation (Stock and Watson (2003)). The challenge to reconcile yield curve dynamics with inflation has become even harder during the recent financial crisis due to the wild fluctuations in consumer prices, largely driven by short-lived shocks to food and, especially, energy prices (Figure 1). There is hardly any trace of these fluctuations in the term structure of interest rates. Core price indices, which exclude the volatile food and energy components, have been more stable. Nonetheless, attempts to forecast core inflation using Treasury yields data have also had limited success.

We propose a dynamic term structure model (DTSM) that fits inflation and yields data well, both in and out of sample. We price both the real and nominal Treasury yield curves using no-arbitrage restrictions. In the tradition of the affine DTSM literature (e.g., Duffie and Kan (1996), Piazzesi (2010), Duffie, Pan, and Singleton (2000)), we assume that the real spot rate is a linear combination of latent and observable macroeconomic factors. The macroeconomic factors are the three main determinants of consumer prices growth: core, food, and energy inflation. We model them jointly with the latent factors in a vector autoregression (VAR). Nominal and real bond prices are linked by a price deflator that grows at the total inflation rate, given by the weighted average of the individual core, food, and energy measures.

This framework easily accommodates the properties of the different inflation components. Shocks to core inflation are much more persistent and less volatile compared to shocks to food and, especially, energy inflation (the 'crust' in the total CPI index). The model fits these features by allowing for different degrees of persistence and volatility of the shocks to each of the three inflation measures, and for contemporaneous and lagged dependence among the factors. We recombine the three individual components to obtain dynamics of total inflation that capture fluctuations at different frequencies. We then embed this information in the pricing of the nominal and real yield curves.

When we estimate the model on a panel of nominal Treasury yields and the three inflation measures, we find a considerable improvement in the fit compared to DTSM specifications that rely on a single inflation factor (either total or core). In particular, we see a big improvement in the out-of-sample performance of the model when forecasting inflation. This is most evident in CPI core forecasts, which we find to systematically outperform the forecasts of various univariate time series models, including the ARMA(1,1) benchmark favored by Ang, Bekaert, and Wei (2007) and Stock and Watson (1999). Our model does well on total CPI too, often improving on the ARMA and other benchmarks. Remarkably, it is at par with the Survey of Professional Forecasters (SPF) on total inflation and it outperforms the University of Michigan survey forecasts. Finally, total inflation forecasts from our preferred no-arbitrage DTSM are more precise than forecasts from unconstrained VAR models estimated on interest rate and inflation data, including specifications that use core, food, and energy inflation series.

These results underscore the advantages of modeling the dynamics of the individual inflation components. A DTSM that prices bonds out of a single measure of inflation delivers forecasts for the specific proxy of inflation used for estimation (e.g., total, core, or a principal component of several price series). In contrast, jointly modeling the three inflation factors (core, food, and energy) produces forecasts for total inflation as well as each of its components. Moreover, this approach proves to be more robust to the extreme fluctuations observed in some price indices. In particular, the estimation finds shocks to energy inflation to be short lived and to have limited impact on the yield curve and long-run inflation expectations.

Our inflation forecasts not only reflect information from past price realizations, but also from yield curve dynamics. In fact, we find that the latent factors explain a large fraction of the variation in both nominal yields and core inflation. In particular, we allow the latent factors to shape the conditional mean of core inflation, and model estimation supports such dependence. When we decompose the variance of the forecasting error for core inflation, we find that the latent factors explain more than 60% of it at the five-year horizon. This fraction remains sizeable even at the short one-year horizon (>15%), and it increases even further when we perform an unconditional variance decomposition.

A related analysis shows that the latent factors are the main drivers in bond yields' variation and crowd out inflation variables in explaining the term structure of interest rates. This result is consistent with the model of Joslin, Priebsch, and Singleton (2010), who impose restrictions on the model coefficients such that the loadings of the yields (or their linear combinations) on macroeconomic variables are zero. In contrast, we do not impose such conditions a priori. We estimate an unconstrained model and find factor loadings on the inflation series that are nearly zero. We show that our model replicates the empirical linkage between yields and inflation data extremely well.

The model produces estimates for the real term structure of interest rates. We find a spot real rate pattern that is tightly linked to the history of monetary policy intervention. Longer maturity real yields show a much smoother behavior. At all maturities, real rates exhibit a declining pattern since the 1980s.

While we do not use data on Treasury Inflation Protected Securities (TIPS), we compare our real rates estimates to TIPS yields during the sub-sample for which those data are available. In the early years of TIPS trading, TIPS rates are systematically higher than model-implied real rates, with a spread of approximately 150bps at the ten-year maturity in the first quarter of 1999. The spread progressively shrinks to near zero by 2004. This evidence is consistent with the presence of a liquidity premium in the TIPS market as documented by D'Amico, Kim, and Wei (2010), Fleckenstein, Longstaff, and Lustig (2010), Haubrich, Pennacchi, and Ritchken (2009), and Pflueger and Viceira (2012). More interestingly, the TIPS-real-rate spread widens again during the financial crisis, with a peak immediately after the collapse of Lehman Brothers. This is related to disruptions in the TIPS market, where liquidity dried up in fall 2008 and remained scarce for several months.[1] In contrast, long-term real rates implied by our model remain smooth; only the spot real rate shows a moderate increase in fall 2008 due to heightened short-term deflationary expectations. We obtain these results by estimating our model *solely* on nominal yields and inflation data, without relying on survey- or market-based measures of real rates and expected inflation.

Similar to real rates, the model-implied inflation risk premium is high in the 1980s and declines over time, consistent with Ang, Bekaert, and Wei (2008) but at odds with Haubrich, Pennacchi, and Ritchken (2009). We find a negative inflation risk premium at times since the late 1990s. Most notably, the premium turns negative after 2005, a period during which long-term yields are low in spite of prolonged restrictive monetary policy. Greenspan (2005) refers to this development as a 'conundrum'; our model associates it with a reduction in inflation risk. The inflation risk premium turns negative again during the financial crisis. These results suggest that Treasuries carry significant inflation risk in the 1980s, while they behave like inflation hedges in recent times, providing insurance against recessions in which deflation risk is high. The real rate risk premium shares a pattern similar to that of the inflation risk premium, turning negative at times in the 2000s.[2]

The model provides a natural setting to study the pass-through effect of shocks in energy prices on core inflation and the yield curve. We find that energy shocks have had a limited impact on core inflation through the early 2000s. The effect was stronger in the 1980s and declining ever since. A similar pattern applies to conditional and unconditional correlations in shocks to energy and core inflation, except for a moderate increase in these measures in recent years. Not surprisingly, bond yields are largely unaffected by energy shocks.

Finally, we perform a number of robustness checks and explore some technical issues. First, we perform maximum-likelihood estimation using different methods to extract the latent factors (inverting them from a subset of the yields as in Chen and Scott (1993), or estimating them via the Kalman filter). Second, we explore model estimation on different data sets of yields (CRSP zero-coupon rates with maturity up to five years vs. constant-maturity Treasury yields with maturity up to 20 years) and inflation (CPI vs. PCE data). Third, we perform estimation directly on the yields, or on their principal components (as in, e.g., Adrian and Moench (2010), Hamilton and Wu (2011), and Joslin, Singleton, and Zhu (2011)). Fourth, we explore estimation over different sample periods (a long sample going back to 1962Q1 vs. the post-1984 period).

Related Literature Ang, Bekaert, and Wei (2007, 2008) estimate nominal and real term structures for U.S. Treasury rates with no-arbitrage models that include latent factors and

[1]For instance, a panel of inflation risk professionals convened in New York to discuss developments in the market of inflation-linked products (Risk Magazine 2009). The panel noted that the TIPS market was disrupted to a point that trading took place only 'by appointment'.

[2]This is consistent with the evidence in Campbell, Sunderam, and Viceira (2011), who estimate the covariance between stock and bond returns to be positive in the 1980s and negative in the 2000s, and with Campbell, Shiller, and Viceira (2009), who show that the TIPS beta with stock returns is negative in the downturns of 2001-2003 and 2008-2009.

one inflation factor (measured by either total or core realized inflation). The authors consider specifications with and without regime switches in the inflation dynamics. They find that term structure information does not generally lead to better inflation forecasts and often leads to inferior forecasts compared to those produced by models that use only aggregate activity measures. Their evidence confirms the results in Stock and Watson (1999), and extends them to a wide array of specifications that combine inflation, real activity, and yield dynamics. The relatively poor forecasting performance of term structure models applies to simple regression specifications, iterated long-horizon VAR forecasts, no-arbitrage affine models, and non-linear no-arbitrage models. They conclude that while inflation is very important for explaining the dynamics of the term structure (e.g., Ang, Bekaert, and Wei, 2008), yield curve information is less important for forecasting future inflation. Yet, the yield curve should reflect market participants' expectations of future consumer price dynamics, and our DTSM framework helps us to extract them to produce more accurate inflation forecast.

Several studies incorporate market expectations in fitting real and nominal term structures of interest rates. For instance, Adrian and Wu (2010), Campbell, Sunderam, and Viceira (2011), Christensen, Lopez, and Rudebusch (2010), D'Amico, Kim, and Wei (2010), and Grishchenko and Huang (2010) combine nominal off-the-run yields constructed in Gürkaynak, Sack, and Wright (2007) with TIPS zero-coupon rates from Gürkaynak, Sack, and Wright (2010). Chen, Liu, and Cheng (2010) use raw U.S. TIPS data, while Barr and Campbell (1997) and Hördahl and Tristani (2010) focus on European index-linked bonds. Kim and Wright (2005) and Pennacchi (1991) rely on survey forecasts, while Haubrich, Pennacchi, and Ritchken (2009) introduce inflation swap rates to help identify real rates and expected inflation. In these studies, estimation typically forces the model to match survey- and market-based measures of real rates and expected inflation (TIPS data, survey inflation forecasts, or inflation swaps) up to a measurement error. Hence, model-implied real rates and inflation forecasts inherit the properties of these inputs by construction. In contrast, we propose a model that relies *entirely* on nominal U.S. Treasury and inflation data to jointly estimate real rates, expected inflation for total, core, food, and energy price indices, and the inflation and real rates risk premia. Remarkably, our inflation forecasts are in line with SPF forecasts and outperform the University of Michigan survey; nominal yields forecasts improve upon the SPF. Our estimates for real rates, inflation and real risk premia are also consistent with related market-based measures.

A vast related literature explores the relation between *nominal* interest rates and the macroeconomy. Early works directly relate current bond yields to past yields and macroeconomic variables using a vector auto-regression approach (e.g., Estrella and Mishkin (1997), and Evans and Marshall (1998, 2007)). This literature has successfully established an empirical linkage between shocks to macroeconomic variables and changes in yields. More recently, several studies have explored similar questions using no-arbitrage dynamic term structure models (e.g., Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2006), Diebold, Rudebusch, and Aruoba (2006), Duffee (2006), Hördahl, Tristani, and Vestin (2006), Moench (2008), Diebold, Piazzesi, and Rudebusch (2005), Piazzesi (2005), Rudebusch and Wu (2008)). Other contributions have extended these models to include market expectation in the form of survey forecasts (e.g., Chernov and Mueller (2008), Chun (2010), and Kim and Orphanides (2005)).

Recent work explores the role of no-arbitrage and dynamic restrictions in canonical Gaussian affine term structure models (e.g., Joslin, Singleton, and Zhu (2011), Duffee (2011), and Joslin, Le, Singleton (2011)). These studies question whether no-arbitrage restrictions affect out-of-sample forecasts of yields and macroeconomic factors relative to the forecasts produced by an unconstrained factor model. In our framework, no-arbitrage restrictions allow us to identify market prices of risk (both real and inflation risk premia) and therefore to compute real rates, which are an important part of our analysis. Moreover, our model departs from the canonical Gaussian DTSM class. First, we impose additional restrictions on the physical factor dynamics (Calvet, Fisher, and Wu (2010)) as well as on the interactions between latent and inflation factors. Second, we fix some of the risk premia coefficients at zero. Further, similar to Duffee (2010) we estimate the model under the constraint that conditional maximum Sharpe ratios stay close to their empirical realizations.[3] We confirm that with these restrictions our preferred DTSM outperforms unconstrained VAR models estimated on interest rate and inflation data, including specifications that use core, food, and energy inflation series.

Several scholars study the link between bond risk premia and the macroeconomy (e.g., Cieslak and Povala (2010), Cochrane and Piazzesi (2005), Duffee (2011), Joslin, Priebsch, and Singleton (2010)). This literature focuses on the predictability of bond returns. We concentrate on no-arbitrage models of the nominal and real term structures, and explore their implications for expected inflation and the inflation and real rate risk premia.

The rest of the paper proceeds as follows. Section 2 presents the model. We discuss data and the estimation method in Section 3. The empirical results are in Section 5, while Section 6 concludes the paper.

2 The Model

We assume that K_1 latent factors $L_t = \left[\ell_t^1, ..., \ell_t^{K_1}\right]$ and K_2 inflation factors $\Pi_t = \left[\pi_t^1, ..., \pi_t^{K_2}\right]$ describe the time t state of the economy. Collecting the state variables in a vector $F_t = [L_t, \Pi_t]'$, we define the state dynamics via a Gaussian vector auto-regression (VAR) system with p lags,

$$F_t = \phi_0 + \phi_1 F_{t-1} + ... + \phi_p F_{t-p} + \Sigma u_t \,, \tag{1}$$

[3] Joslin, Singleton, and Zhu (2011) conclude that improvements in the conditional forecasts of the pricing factors in Gaussian dynamic term structure models are due to the combined structure of no-arbitrage and $\mathbb{P}$-distribution restrictions. An example of such auxiliary constraints is the number of risk factors that determine risk premia. Duffee (2011) and Joslin, Le, Singleton (2011) reach similar conclusions. We discuss restrictions on factor dynamics, model Sharpe ratios, and risk premia in more detail in Sections 2.3, 3, and 5.

where ϕ_0 is a $(K_1+K_2)\times 1$ vector of constants and ϕ_i, $i=1,...,p$, are $(K_1+K_2)\times(K_1+K_2)$ matrices with the autoregressive coefficients. The $(K_1+K_2)\times 1$ vector of independent and identically distributed (i.i.d.) shocks Σu_t has Gaussian distribution $N(0,V)$, with $V=\Sigma\Sigma'$.

We stack the contemporaneous unobservable factors, $X_t^u = L_t = \left[\ell_t^1,...,\ell_t^{K_1}\right]$, together with the contemporaneous and lagged observable inflation factors, $X_t^o = [\Pi_t,...,\Pi_{t-(p-1)}]$, in a $K\times 1$ vector $X_t = [X_t^u, X_t^o]'$, where $K = K_1 + K_2 \times p$. With this notation, we introduce the VAR dynamics in first-order compact form,

$$X_t = \Phi_0 + \Phi X_{t-1} + \Omega\varepsilon_t\,, \tag{2}$$

where $\varepsilon_t = [u_t', 0, ..., 0]'$, and the $K\times K$ matrix Ω contains the matrix Σ and blocks of zeros that correspond to the elements of the lagged inflation factors.

2.1 Real Bond Prices

The one-period short real rate, r_t^*, is an affine function of the state vector X_t,

$$r_t^* = \delta_0 + \delta_1' X_t\,. \tag{3}$$

The coefficient δ_1 has dimensions $K\times 1$ and is subject to the identifying restrictions, $\delta_1^{\ell^1},...,\delta_1^{\ell^{K_1}} = 1$ (e.g., Dai and Singleton (2000)). Moreover, we impose the constraint that the short rate depends only on contemporaneous factor values. That is, we fix the elements of the δ_1 coefficient corresponding to lagged inflation variables at zero, $\delta_1 = \left[\left(\delta_1^{\ell^1},...,\delta_1^{\ell^{K_1}}\right), \left(\delta_1^{\pi^1},...,\delta_1^{\pi^{K_2}}\right), 0, ..., 0\right]'$.

We follow Ang, Bekaert, and Wei (2007, 2008) and specify the real pricing kernel m_{t+1}^* as

$$m_{t+1}^* = \exp\left(-r_t^* - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\,, \tag{4}$$

where the market price of risk λ_t is affine in the state vector X_t,

$$\lambda_t = \lambda_0 + \lambda_1 X_t\,, \tag{5}$$

for a $K\times 1$ vector λ_0 and the $K\times K$ matrix λ_1. Combining equations (3)-(4), we obtain

$$m_{t+1}^* = \exp\left[-\frac{1}{2}\lambda_t'\lambda_t - \delta_0 - \delta_1' X_t - \lambda_t'\varepsilon_{t+1}\right]. \tag{6}$$

Given the pricing kernel m_{t+1}^*, the time t price of a real zero-coupon bond with $(n+1)$ periods to maturity is the present expected value of the time $(t+1)$ price of an n-period bond:

$$p_t^{*n+1} = E_t\left[m_{t+1}^* p_{t+1}^{*n}\right]\,. \tag{7}$$

Since the model is affine, equation (7) has solution

$$p_t^{*n} = \exp\left(\bar{A}_n^* + \bar{B}_n^{*\prime} X_t\right)\,, \tag{8}$$

where the coefficients $\bar{A}^*_n$ and $\bar{B}^*_n$ solve the ordinary difference equations (ODEs):

$$\begin{aligned} \bar{A}^*_{n+1} &= -\delta_0 + \bar{A}^*_n + \bar{B}^{*\prime}_n (\Phi_0 - \Omega\lambda_0) + \frac{1}{2}\bar{B}^{*\prime}_n \Omega\Omega' \bar{B}^*_n \\ \bar{B}^{*\prime}_{n+1} &= -\delta_1' + \bar{B}^{*\prime}_n (\Phi - \Omega\lambda_1) \, . \end{aligned} \tag{9}$$

The real short rate equation (3) yields the initial conditions $\bar{A}^*_1 = -\delta_0$ and $\bar{B}^{*\prime}_1 = -\delta_1'$ for the ODEs (9). Thus, the real yield on an n-period zero-coupon bond is

$$y^{*n}_t = -\frac{\log(p^{*n}_t)}{n} = A^*_n + B^{*\prime}_n X_t \, , \tag{10}$$

where $A^*_n = -\frac{\bar{A}^*_n}{n}$ and $B^*_n = -\frac{\bar{B}^*_n}{n}$.

2.2 Nominal Bond Prices

If we define Q_t to be the price deflator, then the time t price of a nominal $(n+1)$-period zero-coupon bond, p^{n+1}_t, is given by

$$p^{n+1}_t = p^{*n+1}_t Q_t = E_t \left[m^*_{t+1} \frac{Q_t}{Q_{t+1}} p^{*n}_{t+1} Q_{t+1} \right] = E_t \left[m_{t+1} p^n_{t+1} \right] \, , \tag{11}$$

where, as in Ang, Bekaert, and Wei (2007, 2008), we have defined the nominal pricing kernel m_{t+1} to be

$$m_{t+1} = m^*_{t+1} \frac{Q_t}{Q_{t+1}} = m^*_{t+1} \exp(-\pi_{t+1}) = \exp\left(-r^*_t - \pi_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} \right) . \tag{12}$$

We assume that the inflation rate $\pi_t \equiv \log(Q_t/Q_{t-1})$ at which investors deflate nominal asset prices is a weighted sum of the inflation factors in Π_t, $\pi_t = \sum_{j=1}^{K_2} \omega^j \pi^j_t$, where $0 \leq \omega^j \leq 1$. The Ang, Bekaert, and Wei (2007, 2008) model without regime switches is a special case of this setting, in which the factor Π_t contains a single measure of inflation (either total or core inflation). We obtain this case by fixing the weight associated to a specific inflation factor at one, and setting all other weights at zero.

Considering the state dynamics in equation (2), we define $\Phi^\pi_0 = \sum_{j=1}^{K_2} \omega^j \Phi^{\pi^j}_0$, where $\Phi^{\pi^j}_0$ is the element of the vector Φ_0 that corresponds to the inflation factor π^j, $j = 1, \ldots, K_2$. Similarly, consider the $1 \times K$ vectors $\Phi^\pi = \sum_{j=1}^{K_2} \omega^j \Phi^{\pi^j}$ and $\Omega^\pi = \sum_{j=1}^{K_2} \omega^j \Omega^{\pi^j}$, where Φ^{π^j} and Ω^{π^j} are the rows of the Φ and Ω matrices that correspond to the inflation factor π^j. Then, Appendix A shows that nominal bond prices are an affine function of the state vector X:

$$p^n_t = \exp\left(\bar{A}_n + \bar{B}_n' X_t \right) \, , \tag{13}$$

where the coefficients $\bar{A}_n$ and $\bar{B}_n'$ solve the ODEs:

$$\begin{aligned} \bar{A}_{n+1} &= -\delta_0 + \bar{A}_n + \bar{B}_n' (\Phi_0 - \Omega\lambda_0) - \Phi^\pi_0 + \frac{1}{2}\bar{B}_n'\Omega\Omega'\bar{B}_n + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi\lambda_0 - \bar{B}_n'\Omega\Omega^{\pi\prime} \\ \bar{B}_{n+1}' &= -\delta_1' - \Phi^\pi + \bar{B}_n' (\Phi - \Omega\lambda_1) + \Omega^\pi\lambda_1 \, , \end{aligned} \tag{14}$$

with initial conditions $\bar{A}_1 = -\delta_0 - \Phi_0^\pi + \Omega^\pi\lambda_0 + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime}$ and $\bar{B}_1' = -\delta_1' - \Phi^\pi + \Omega^\pi\lambda_1$. Thus, the yield on a nominal n-period zero-coupon bond is affine in the state vector,

$$y_t^n = -\frac{\log\left(p_t^n\right)}{n} = A_n + B_n' X_t \,, \tag{15}$$

where $A_n = -\frac{\bar{A}_n}{n}$ and $B_n = -\frac{\bar{B}_n}{n}$.

2.3 Model Restrictions

We explore models with different sets of latent and inflation factors. In this section we focus on the most general factor dynamics in equation (2). We will then restrict some of the element in the Φ and Ω matrices based on the specification tests discussed in Section 5.

2.3.1 Restrictions on Latent Factors Dynamics

$K_1 = 2$ Latent Factors The first specification assumes the existence of two latent factors ℓ_t^1 and ℓ_t^2 with mean equal to zero. The two factors follow a joint AR(1) process where the submatrix of ϕ_1 that pertains to the latent factors, ϕ_1^l, can be written as:

$$\phi_1^l = \begin{pmatrix} \phi_1^{\ell^1,\ell^1} & 0 \\ \phi_1^{\ell^1,\ell^2} & \phi_1^{\ell^2,\ell^2} \end{pmatrix}, \tag{16}$$

and where lagged inflation has no direct effect on the latent factors. Similarly, we assume that shocks to ℓ_t^1 and ℓ_t^2 are each orthogonal to the other random disturbances that perturb the states X_t. Taken together, these two assumptions allow for the identification of the unobservable variables, ℓ_t^1 and ℓ_t^2.

$K_1 \geq 3$ Latent Factors When raising the number of latent factors to $K_1 \geq 3$, we adopt a 'recursive' structure for $\ell_t^1, ..., \ell_t^{K_1}$, as in Calvet, Fisher, and Wu (2010). This specification assumes the presence of K_1 correlated latent factors, with the k^{th} latent factor mean-reverting to the lagged realization of the $(k-1)^{th}$ factor. We retain the orthogonality condition for the shocks $u_t^{\ell^k}$, as in the $K_1 = 2$ case. Taken together, these restrictions yield the following dynamics for the latent variable ℓ^k:

$$\ell_t^k = \left(1 - \phi_1^{\ell^k,\ell^k}\right)\ell_{t-1}^{k-1} + \phi_1^{\ell^k,\ell^k}\,\ell_{t-1}^k + \sigma_{\ell^k} u_t^{\ell^k} \,. \tag{17}$$

Moreover, as in Calvet, Fisher, and Wu (2010), we impose a non-linear decay structure on the auto-regressive coefficients, $\phi_1^{\ell^k,\ell^k} = \exp\{-\beta_k\}$, $\beta_k = \beta_1\, b^{k-1}$, with $\beta_1 > 0$, $b > 1$ and $k = 1, \ldots, K_1$. This parsimonious representation naturally ranks the latent factors in order of persistence and therefore avoids issues related to possible factors rotations (e.g., Collin-Dufresne, Goldstein, and Jones (2008), Dai and Singleton (2000), Hamilton and Wu (2010), Joslin, Priebsch, and Singleton (2010)).[4]

[4] While there are common elements with Calvet, Fisher, and Wu (2010) term structure model, there are also significant differences. First, our vector of state variables includes inflation series in addition to latent

2.3.2 Restrictions on Inflation Factors Dynamics

$K_2 = 1$ Inflation Factor In our first configuration for the inflation dynamics, we model the evolution of a single inflation factor, either core or total inflation. That is, $K_2 = 1$ with either $\Pi_t = \pi_t^{tot}$ or $\Pi_t = \pi_t^c$. We assume that lagged latent factors, ℓ_{t-1}^1,..., $\ell_{t-1}^{K_1}$, have a direct impact on inflation and that inflation follows an AR(4) process, so that the expected value of π_t conditional on information at time $t-1$ is:

$$E_{t-1}[\pi_t] = \phi_0^{\pi} + \sum_{k=1}^{K_1} \phi_1^{\pi,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi,\pi} \pi_{t-i}\,. \tag{18}$$

The shock u_t^{π} to the inflation process is orthogonal to the latent factors shocks $u_t^{\ell^k}$, $k = 1, \ldots, K_1$.

$K_2 = 2$ Inflation Factors In this second specification, the vector of inflation factors contains both total and core inflation, $\Pi_t = [\pi_t^{tot}, \pi_t^c]$, and market participants deflate nominal asset prices in equation (12) at the total inflation rate, $\pi_t = \pi_t^{tot}$. That is, π_t is the weighted sum of π_t^{tot} and π_t^c with weights $\omega^{tot} = 1$ and $\omega^c = 0$.

We assume that the conditional mean of core inflation π_t^c follows an AR(1) process and is driven by a combination of the latent factor $\ell_t^1, ..., \ell_t^{K_1}$. Similarly, total inflation, π_t^{tot}, mean-reverts to core inflation, π_t^c, and a linear combination of the same latent factors. In particular, we model the conditional means of core and total inflation as:

$$\begin{aligned}
E_{t-1}\left[\pi_t^{tot}\right] &= \phi_0^{\pi^{tot}} + \sum_{k=1}^{K_1} \phi_1^{\pi^{tot},\ell^k} \ell_{t-1}^k + \left(1 - \phi_1^{\pi^{tot},\pi^{tot}}\right) \pi_{t-1}^c + \phi_1^{\pi^{tot},\pi^{tot}} \pi_{t-1}^{tot} \\
E_{t-1}\left[\pi_t^c\right] &= \phi_0^{\pi^c} + \sum_{k=1}^{K_1} \phi_1^{\pi^c,\ell^k} \ell_{t-1}^k + \phi_1^{\pi^c,\pi^c} \pi_{t-1}^c\,.
\end{aligned} \tag{19}$$

We also consider a special case of this model with two additional restrictions. First, we set $\phi_1^{\pi^c,\ell^1} = \left(1 - \phi_1^{\pi^c,\pi^c}\right)$ and, second, we assume that the AR(1) coefficients of core and total inflation follow a non-linear decay structure. In particular, we set $\phi_1^{\pi^c,\pi^c} = \exp\{-\beta_{core}\}$, where $\beta_{core} = \beta_1 b^{\pi}$ and $b^{\pi} > 1$. In turn, for total inflation we have $\phi_1^{\pi^{tot},\pi^{tot}} = \exp\{-\beta_{tot}\}$, where $\beta_{tot} = \beta_{core} b^{\pi} = \beta_1 (b^{\pi})^2$. This specification resembles the recursive structure adopted for the latent factors in the $K_1 \geq 3$ case, with the additional restriction that the first latent factor determines the central tendency of core inflation. In turn, total inflation reverts back to the more persistent core-inflation series. With these restrictions, fitting the conditional mean of core and total inflation requires the estimation of a single new coefficient, b^{π}, as β_1 is the same coefficient that determines the speed of mean reversion of the first latent factor ℓ^1, $\phi_1^{\ell^1,\ell^1} = \exp\{-\beta_1\}$ in equation (17).

factors. Second, we price both the nominal and real term structures. Third, we allow the real spot rate to depend on all latent factors as well as the inflation variables. This is in contrast to their assumption that the nominal spot rate equals the least persistent latent factor.

The variance matrix V allows for non-zero cross-correlations among shocks that hit the two inflation processes. Moreover, we allow shocks to the inflation variables to correlate with latent factors shocks.

$K_2 = 3$ Inflation Factors In the third model specification, the vector of inflation factors contains core, food, and energy inflation, $\Pi_t = [\pi_t^c, \pi_t^f, \pi_t^e]$. Market participants deflate nominal asset prices in equation (12) at the total inflation rate, computed as the weighted sum of the three inflation series. That is, $\pi_t = \pi_t^{tot} = \omega^c \pi_t^c + \omega^f \pi_t^f + \omega^e \pi_t^e$, where ω^c, ω^f, and ω^e represent the relative importance of core, food, and energy prices in the total price index. Similarly, the terms Φ_0^π, Φ^π, and Ω^π in the ODEs (14) become

$$\begin{aligned}
\Phi_0^\pi &= \omega^c \Phi_0^{\pi^c} + \omega^f \Phi_0^{\pi^f} + \omega^e \Phi_0^{\pi^e} \\
\Phi^\pi &= \omega^c \Phi^{\pi^c} + \omega^f \Phi^{\pi^f} + \omega^e \Phi^{\pi^e} \\
\Omega^\pi &= \omega^c \Omega^{\pi^c} + \omega^f \Omega^{\pi^f} + \omega^e \Omega^{\pi^e}.
\end{aligned} \tag{20}$$

We assume that the conditional means of the three inflation factors can be expressed as:

$$\begin{aligned}
E_{t-1}\left[\pi_t^c\right] &= \phi_0^{\pi^c} + \sum_{k=1}^{K_1} \phi_1^{\pi^c,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^c,\pi^c} \pi_{t-i}^c + \sum_{i=1}^{4} \phi_i^{\pi^c,\pi^e} \pi_{t-i}^e \\
E_{t-1}\left[\pi_t^f\right] &= \phi_0^{\pi^f} + \sum_{k=1}^{K_1} \phi_1^{\pi^f,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^f,\pi^f} \pi_{t-i}^f + \sum_{i=1}^{4} \phi_i^{\pi^f,\pi^e} \pi_{t-i}^e \\
E_{t-1}\left[\pi_t^e\right] &= \phi_0^{\pi^e} + \sum_{k=1}^{K_1} \phi_1^{\pi^e,\ell^k} \ell_{t-1}^k + \sum_{i=1}^{4} \phi_i^{\pi^e,\pi^e} \pi_{t-i}^e\,,
\end{aligned} \tag{21}$$

where the three inflation series follow a VAR(4) process. Similar to the univariate case, lagged realization of the latent factors, ℓ_{t-1}^k also enter in the conditional mean for the inflation factors. We allow core and food inflation to respond to lagged realizations of energy inflation, π_{t-i}^e, $i = 1, \ldots, 4$.

The covariances between shocks to the three inflation series, $\left(\sigma_{\pi^c,\pi^f}, \sigma_{\pi^c,\pi^e}, \sigma_{\pi^e,\pi^f}\right)$, in the matrix V are non-zero. Moreover, we allow shocks to the inflation variables to correlate with latent factors shocks.

2.4 Benchmark Models

In the empirical part of the paper we explore the in- and out-of-sample performance of our term structure models. Since we focus on their ability to forecast inflation, it is useful to establish a comparison with the forecasts produced by other models that fall outside of the affine term structure class. The literature has proposed a wide array of models (e.g., Stock and Watson (1999, 2003, and 2007)). Of these, the ARMA(1,1) and random walk models have proven particularly resilient in predicting consumer price dynamics over different sample periods. Thus, we consider both of these univariate models for comparison with our term structure specifications.

The ARMA(1,1) model for an inflation series π^i is

$$\pi^i_t = \mu + \rho\, \pi^i_{t-1} + \varepsilon_t + \theta\, \varepsilon_{t-1}\,. \tag{22}$$

In addition to fitting model (22) to each inflation series separately (total, core, food, and energy), we also construct forecasts for total inflation as a weighted sum of the ARMA(1,1) forecasts of each component, $E_t[\pi^{tot}_{t+n,n}] = \omega^c E_t[\pi^c_{t+n,n}] + \omega^f E_t[\pi^f_{t+n,n}] + \omega^e E_t[\pi^e_{t+n,n}]$, where $\pi^i_{t+n,n}$ denotes inflation realized from t to $t+n$. We term such forecast ARMA$_W$. As in Atkeson and Ohanian (2001), the random walk (RW) forecast for an inflation series at any future horizon is the average of the realizations during the past four quarters.

Ang, Bekeart, and Wei (2007) argue that inflation surveys outperform other popular forecasting methods (see also consistent evidence in Faust and Wright (2009)). Surveys are conducted for a limited number of price series. Whenever available, we include them as additional benchmarks, as described in Section 3 below.

Recent work explores the role of no-arbitrage and dynamic restrictions in canonical Gaussian affine term structure models (e.g., Joslin, Singleton, and Zhu (2011), Duffee (2011), and Joslin, Le, Singleton (2011)). These studies question whether no-arbitrage restrictions affect out-of-sample forecasts of yields and macroeconomic factors relative to the forecasts produced by an unconstrained factor model. Therefore, as a final benchmark we also consider an unconstrained VAR estimated on interest rates, core, food, and energy inflation data. To obtain $E_t[\pi^{tot}_{t+n,n}]$ forecasts, we weigh the forecasts for the individual inflation components, as in the ARMA$_W$ case.

3 Data and Estimation

We jointly use U.S. Treasury yield and inflation data for model estimation. We consider two sample periods, both ending in December 2009. The first starts in January 1985. The second is a longer period that includes the Fed's monetary experiment of the early 1980s. It begins in January 1962, since this is the first date from which all data series described below become available.

1. We consider two data sets of U.S. Treasury yields.

   (a) The first data set comprises quarterly observations on zero-coupon yields with maturities of 1, 4, 12, and 20 quarters. The bond yields (4, 12, and 20 quarters maturities) are from the Fama Center for Research in Security Prices (CRSP) zero coupon files, while the 1-quarter rate is from the Fama CRSP Treasury Bill files. All bond yields are continuously compounded. This data set is very popular in the empirical term structure literature. However, it does not contain longer-maturity yields that could contain useful information about investors' inflation expectations.

(b) The second data set extends the maturity of available yields up to 30 years; it consists of daily constant-maturity par yields computed by the U.S. Treasury and distributed by the Board of Governors in the H.15 data release. Prior to analysis, we interpolate the par yields into zero-coupon yields using a smoothed spline interpolation, as described in Section A.1 of the Online Appendix.[5] On each day, we construct the term structure of zero-coupon rates from all available yield maturities. However, for model estimation we only use yields with maturities of 1, 3, 5, 10, and 20 years. We then aggregate the daily series at the quarterly frequency. The 1-quarter par yield in the H.15 release becomes available from September 1, 1981. Thus, to allow estimation over a long sample period, we combine the interpolated zero-coupon yield series with maturities from 1 to 20 years with the 1-quarter rate from the Fama CRSP Treasury Bill files. When estimating the model with data post 1984, we confirm that using our interpolation of the 1-quarter zero-coupon rate from the H.15 constant-maturity par yields gives similar results.

2. We focus on two widely used measures of inflation:

   (a) First, we collect monthly data on four Consumer Price Indices (CPI) constructed by the Department of Labor, Bureau of Labor Statistics (BLS): (1) the total CPI for all Urban Consumers (all items CPI-U); (2) the core CPI (all items less food and energy); (3) the food CPI; and (4) the energy CPI. For all series, 1984 is the base year.

   (b) Second, we separately repeat the analysis with Personal Consumption Expenditure (PCE) data released by the Bureau of Economic Analysis (BEA). Similar to the CPI series, we consider total, core, food, and energy PCE indices. The base year is 2005.

All price series are seasonally adjusted. We compute quarterly price indices by averaging over the monthly observations. Growth rates are quarter over quarter logarithmic differences in the index levels. Appendix B explains how we construct a measure of the weights ω^c, ω^f, and ω^e associated to the core, food, and energy components.

Table 1 contains CPI and PCE summary statistics for the long (Panel A) and short sample periods (Panel B). The CPI- and PCE-weighted series are the total inflation series computed from their core, food, and energy components using the relative importance weights. Summary statistics for CPI- and PCE-weighted are nearly identical to those computed for the total CPI and PCE inflation series released by the BLS and the BEA. Moreover, we find that the correlation between CPI and CPI-weighted total inflation series is 99.73% in the post 1984 sample period, while it is 99.58% in the long sample period. For PCE data, the correlation is higher than 99.9% in both sample periods. This evidence suggests that our

[5] We confirm that our estimation results are unchanged when we compute zero-coupon rates using a linear term structure interpolation (similar to the unsmoothed Fama-Bliss method).

measure of total inflation constructed as a weighted average of the various components is a close proxy to the actual inflation series computed from the total CPI index.

Table 1 also shows the difference in persistence across inflation series. For both sample periods, the first-order auto-correlation for CPI-core inflation exceeds 0.82; higher-order auto-correlations remain high. The CPI-food series is much less persistent, with a first-order auto-correlation of 0.46 and 0.62 in the two sample periods, and declining at longer lags. In contrast, the shocks to CPI-energy series are short lived, with a first-order auto-correlation of 0.19. Shocks die away quickly, resulting in second- and third-order correlations that are close to zero or even negative. Consequently, total CPI inflation is less persistent than core inflation, especially since 1985 when shocks to both food and energy prices have become less persistent (Stock and Watson (2007)). This is also evident from Figure 1, which plots the four inflation series over the full sample period. PCE inflation shares similar properties with the CPI series.

For both CPI and PCE series, the core inflation component has a predominant weight. The average relative importance of CPI core is 0.73 in the long sample period, compared to 0.77 since 1984. The average weights for the PCE-core series are slightly higher and remain stable across sample periods (0.82 and 0.86, respectively). The food CPI component has average weights of 0.15 and 0.19 in the two samples, while the average energy weight is 0.08. In the PCE series, the relative importance of food and energy is slightly lower. The weights show limited time variation, with a standard deviation that is very small and nearly zero after 1984. Related, the auto-correlation for these series is high at all lags. In our analysis, we consider three alternative approaches in using the weights series to compute a proxy for total inflation. First, we fix the weights at their average value over the sample period. Second, we fix them at the value observed at the end of the sample. Third, we allow the weights to vary over time.[6] Since there is little time variation in the weights, the three approaches yield similar results. In what follows, we report findings based on the third approach, which allows us to use the most current information at the time we price the bonds in the sample.

As discussed in Section 2.4, we also collect data on survey forecasts of inflation and nominal U.S. Treasury yields that we use to assess the performance of our models. The survey data are:

1. First, the Michigan survey forecasts based on the Survey of Consumers conducted by the University of Michigan's Survey Research Center. The Center randomly contacts approximately 500 households monthly and asks them about expected changes in key macroeconomic indicators in the next twelve months. We use the median inflation forecast, which is available since January 1978.

2. Second, the survey of professional forecasters (SPF), currently conducted by the Federal

[6]The bond pricing formula derived in Section 2.2 still holds when weights are time varying, under the assumption that over the life span of the bond the weights remain equal to the value observed at the time we compute their prices. This is a reasonable approximation since there is little time variation in the weights series.

Reserve Bank of Philadelphia. The SPF is a quarterly survey of 9-40 professional forecasters that collects forecasts for the current and the next four quarters. We use median forecasts for total CPI; the three-month Treasury bill rate; and the 10-year Treasury bond rate. These series are available since the third quarter of 1981 (CPI inflation and three-month rate) and the first quarter of 1992 (the 10-year rate). We do not use CPI core, PCE total and core forecasts since they become available only recently, in the first quarter of 2007.

We estimate the benchmark ARMA models by maximum likelihood and the VAR by ordinary least squares (OLS). As for the term structure models, we consider two alternative estimation methods:

1. The transition density for the state vector X in equation (2) is multivariate Gaussian. Thus, maximum likelihood estimation is feasible given a measure of the latent factors in X. We assume that a subset of the bonds in the sample is priced without error and solve for the latent states.

2. We apply the Kalman filter to the state-space representation of the model and estimate its coefficients via maximum likelihood. The observable variables are nominal bond yields with different maturities n, y_t^n, and inflation factors Π_t. We assume that the inflation factors are measured without error, while the nominal yields are observed with measurement error. We consider two approaches. First, we assume that all yields are measured with i.i.d. Gaussian errors with mean zero and constant standard deviation. Second, we include the first four principal components extracted from the panel of yields in the measurement equation, rather than the yields themselves (e.g., Adrian and Moench (2010), Hamilton and Wu (2011), Joslin, Singleton, and Zhu (2011)). We assume i.i.d. zero-mean Gaussian measurement errors. The errors variance matrix is diagonal with $\sigma_i^2 = \overline{\sigma}^2 \neq \sigma_4^2$ for $i = 1, 2$, and 3. The state dynamics (2) for the vector X complete the state-space system.

The first estimation method is widely used in the empirical term structure literature (e.g., Chen and Scott (1993) and many others since them). Thus, we employ it for model estimation to facilitate a comparison of our results with previous studies. However, this approach requires arbitrary assumptions concerning what bonds are priced without error. Moreover, it becomes difficult to implement for models that have a high number of latent factors, possibly higher than the number of bonds in the sample. Thus, we turn to the Kalman filter method, which avoids these problems.

Duffee (2010) argues that conditional maximum Sharpe ratios implied by fully flexible four-factor and five-factor Gaussian term structure models are astronomically high. To solve this problem, he estimates the model coefficients with the constraint that the sample mean of the filtered conditional maximum Sharpe ratios does not exceed an upper bound. Similar to Duffee, during estimation we penalize the likelihood function when model parameters produce conditional maximum Sharpe ratios that deviate from empirical realizations. The penalty takes the form of a gamma density for the model-implied conditional maximum Sharpe ratio, computed as a function of the model coefficients. We fix the mean of the gamma distribution at 0.25, a value that Duffee finds to be consistent with the Sharpe ratios of U.S. Treasury returns, and its standard deviation at 0.025. In the model estimation, we maximize the sum of the logarithmic likelihood function and its penalty.[7]

4 Model Specifications and Fit

Section 2.3 lays out the various model configurations. Each specification is characterized by the number of latent factors, K_1, and inflation factors, K_2. Moreover, there are possible restrictions on the VAR coefficients, ϕ_i, $i = 1, \ldots, p$, and Σ. Specifically, for each variable we need to determine the dependence on its lagged realizations (i.e., the order of the auto-regressive component, p) as well as on lagged realizations of the other state variables. Of particular interest is the dependence of core inflation on lagged realizations of energy inflation. Similarly, allowing for non-zero off-diagonal elements in Σ captures possible contemporaneous correlations in the disturbances to the observable variables. Another important ingredient is the specification of the risk premia coefficients. This is not only critical to obtain a good fit for the yields' term structure, but it can also be helpful to improve the out-of-sample performance of the model (e.g., Joslin, Singleton, and Zhu (2011)).

There are several issues concerning model estimation. We need to make assumptions about the properties of measurement errors. Moreover, we can conduct estimation on a panel of yields' with different subsets of maturities, as discussed in Section 3. Alternatively, we can fit the first few principal components of the yields' series (as in, e.g., Adrian and Moench (2010), Hamilton and Wu (2011), and Joslin, Singleton, and Zhu (2011)) in an attempt to decrease the incidence of short-lived fluctuations in the data, possibly due to noise. Much of the macro-finance term-structure literature focuses on estimation over a long sample period, often starting as early as in the 1950s. A drawback of this approach is that such period includes different regimes of monetary policy (e.g., the monetary experiment of the early 1980s).[8] For this reason, we also consider estimation over shorter samples beginning in 1985. We explore these issues in more detail below.

The $\mathbf{DTSM}_{2,1}$ Model We start with a model that has two latent factors, $K_1 = 2$, and one inflation factor, $K_2 = 1$, which we label $\text{DTSM}_{2,1}$. This specification has been previously studied in the literature. For instance, Ang, Bekaert, and Wei (2008) explore it as a special case of their model with regime shifts with the restriction that all risk premia coefficients in

[7]This is similar to an approach commonly used in the empirical macro literature for the estimation of state space models via Bayesian methods, e.g., An and Schorfheide (2007).

[8]Consistent with this interpretation, Ang, Bekaert, and Wei (2008) find evidence of regime shifts in inflation and latent factors over the 1952-2004 period. Nonetheless, accounting for such regimes produces only moderate improvements to the out-of-sample model performance; see, e.g., Ang, Bekaert, and Wei (2007).

equation (5) are zero, except for $\lambda_{0,2}$ and $\lambda_{1,1}$, which they estimate as free parameters.[9] Also, they fix the off-diagonal element in the auto-regressive matrix given in equation (16) at zero, $\phi_1^{\ell^1,\ell^2} = 0$. We adopt the same approach. Moreover, in this and all other models discussed below, we fix ϕ_0^π at a value such that the unconditional mean of the inflation process matches the sample mean of the realized inflation series. We check in unreported results that treating ϕ_0^π as a free parameter produces a similar fit.

We estimate this model on quarterly CRSP zero-coupon yields with 1, 4, 12, and 20 quarters maturity. As is customary in the literature, we assume that the shortest- and longest-maturity (1- and 20-quarter) yields are measured without error and use them to obtain a proxy for the two latent factors. We then proceed to estimate the model by maximum likelihood using total and core inflation as alternative measures of the single inflation factor.

The DTSM$_{2,3}$ Model To explore the role of the various inflation components on the yield curve, we think of total inflation as the weighted average of the core, food, and energy series. We introduce these three inflation variables, $K_2 = 3$, in a model that has two latent factors, $K_1 = 2$, with dynamics identical to those of the DTSM$_{2,1}$ specification. We label this case DTSM$_{2,3}$.

To facilitate comparison with the results of previous studies, we first focus on estimation via maximum likelihood on the same sample of CRSP zero-coupon yields we used in the DTSM$_{2,1}$ case. Next, we extend the panel of yields to include zero-coupon yields with maturities up to ten years (the second yield data set in Section 3). We explore two estimation approaches. First, as in the DTSM$_{2,1}$ case we back out the two latent factors from the shortest- and longest-maturity yields, which we assume to be measured without error. Second, we complement this approach with Kalman filter estimation of the latent states. In this case we fit the model to the series of individual yields, or their first four principal components, up to a measurement error, as discussed in Section 3.

As in the DTSM$_{2,1}$ case, we treat $\lambda_{0,2}$ and $\lambda_{1,1}$ as the only free parameters in equation (5). We explore different configurations for the auto-regressive coefficients ϕ_i, $i = 1, \ldots, p$. To asses the fit of different flavors of the model, we use the Bayesian information criterion, which is less likely to prefer over-parameterized models compared to other criteria, e.g., Akaike and Hannan-Quinn.

Through this exercise, we restrict the core, food, and energy series to follow AR(3), AR(1), and AR(1) processes, respectively. We also find dependence of core inflation on lagged realizations of the two latent factors. This suggests that the latent factors play a dual role in the model. On the one hand, they explain the yield term structure. On the other hand, they have a significant influence on the conditional mean of core inflation and price dynamics.

Moreover, using the same approach we explore the dependence of core and food on energy inflation. We do not find dependence of current food inflation on lagged energy realizations,

[9]See, in particular, Table A.1 of their September 2003 working paper version. When estimating the model on an identical sample period, we found the same results.

i.e., $\phi_i^{\pi^f,\pi^e} = 0$, $i = 1, \ldots, 4$, in equation (21). In contrast, the link between energy and core changes across sample periods. Realizations of energy inflation with one quarterly lag have an impact on current core inflation. The effect is positive and significant in sample periods that start in the early 1960s and end on or after 1985. However, the magnitude of the coefficient declines steadily as the end date of the sample increases. This result indicates the presence of limited pass-through of energy shocks on core inflation that has gradually declined since the 1980s. Realizations of energy inflation with lags higher than one quarter do not have a significant impact on core inflation, i.e., $\phi_i^{\pi^c,\pi^e} = 0$, $i = 2, \ldots, 4$.[10] These results extend previous findings in the literature. For instance, using Phillips curve models Hooker (2002) argues that oil shocks contributed substantially to core inflation until 1981, but since that time pass-through has been largely absent. More recently, Stock and Watson (2010) and Clark and Terry (2010) reach similar conclusions. Our results are consistent with these studies and extend their analysis to a no-arbitrage term structure model. We return to these issues in Section 5.5 below.

The DTSM$_{K_1,3}$ Model When extending the model to include a higher number of latent factors, we focus on the specification of Calvet, Fisher, and Wu (2010) for the latent factors. We combine this structure with core, food, and energy inflation dynamics, a specification that we label DTSM$_{K_1,3}$. As K_1 increases, it becomes impractical to extract a proxy for the factors from a panel of bonds with limited number of maturities. Thus, in what follows we concentrate on maximum likelihood estimation of the model via the Kalman filter. Moreover, we consider panels of yields with maturities up to 20 years.

We first explore the $K_1 = 3$ case with the same restrictions on the risk premia coefficients as in the DTSM$_{2,1}$ and DTSM$_{2,3}$ cases, i.e., $\lambda_{0,2} \neq 0$ and $\lambda_{1,1} \neq 0$. Information criteria suggest that the presence of an additional latent factor gives the model more flexibility necessary to fit the data. Intuitively, in the DTSM$_{3,3}$ case the three latent factors span a wider array of frequencies in inflation and yields than the two latent variables in DTSM$_{2,3}$. With this configuration, a combination of the first lag in core inflation along with lagged realizations of the first and third latent factors captures the range of frequencies in core inflation well. The first factor becomes highly persistent, delivering a distinct tent shape to the conditional mean of the core process. The second and third ones accommodate shorter-lived fluctuations in prices and interest rates. In this setting, we also find that food and energy inflation follow AR(1) processes, as in the DTSM$_{2,3}$ case.

Next, we explore specifications that allow for a richer interaction between latent and inflation factors along with alternative risk premia configurations. Cochrane and Piazzesi (2008) argue that market prices of risk are earned only in compensation for exposure to shocks in the 'level' factor. In particular, they find them to depend on only one variable, the Cochrane and Piazzesi (2005) tent-shaped return forecasting factor. While we do not have their return forecasting variable in the model, we explore risk premia configurations

[10] We also consider models in which current core inflation depends on the average of the past four quarterly energy realizations. Our specification tests reject this restriction.

in which we only price shocks to the first, most persistent latent factor, and use model specification tests to determine which state variables drive time variation in risk premia. Moreover, we allow for contemporaneous correlations between shocks to core and the latent factors. Specification tests favor a model in which core correlates with the first latent factor but not with the others. We label this model $\text{DTSM}^*_{3,3}$.

The $\text{DTSM}_{K_1,2}$ Model Our last model specification contains two macro factors that are measures of total and core inflation. In this setting, we find that core provides a good measure of central tendency for total inflation, i.e., specification tests favor the restrictions $\phi_1^{\pi^{tot},\ell^k} = 0$, $k = 1, \dots, K_1$, in equation (19). In turn, we cannot reject the restriction that core mean-reverts to the first latent factor ℓ^1, similar to the higher-order latent factors ℓ^k, $k = 2, \dots, K_1$. We model this dependence with the cascade structure described in Section 2.3. Moreover, we allow for contemporaneous correlations between shocks to inflation and the latent factors. Finally, specification tests support a dependence of the first latent factor on lagged realizations of total inflation.

This cascade structure captures different frequencies in core and total inflation fluctuations, similar to the $\text{DTSM}_{K_1,3}$ case. However, here we explicitly introduce total inflation in the state dynamics instead of obtaining it from its three individual components (core, food, and energy) as in the $\text{DTSM}_{K_1,3}$ specification. That is, we model the crust as short-lived fluctuations of total inflation around core. Below we explore whether this parsimonious representation fares well out of sample when $K_1 = 3$. In doing so, we consider the same two risk premia configurations used in $\text{DTSM}_{3,3}$ and $\text{DTSM}^*_{3,3}$.

5 Empirical Results

Here we report the main empirical findings. First, we use the events of the recent financial crisis to contrast the implications of our preferred $\text{DTSM}_{3,3}$ to those of other models. Second, we discuss the out-of-sample performance of the models in forecasting inflation. Third, we examine the implications of our preferred model for the term structure of real rates and model risk premia. Fourth, we explore the determinants of interest rates and inflation in our preferred DTSM. Fifth, we assess the pass-through effect of energy shocks on core inflation. Sixth, we briefly discuss nominal yields forecasts.

5.1 Dynamic Term Structure Models and the U.S. Financial Crisis

The U.S. financial crisis took a dramatic turn in fall 2008 after the bankruptcy of Lehman Brothers. The total CPI index decreased by 1% in October and 1.7% in November 2008. Energy prices were the main determinant of this decline, with the CPI energy index falling by 8.6% and 17% in the months of October and November. This extreme drop continued the downward pattern in energy prices observed since the previous summer, resulting in a 32.4% total fall from their July 2008 peak. In contrast, core CPI prices declined 0.1% in October, and remained flat in November. These fluctuations in consumer prices produce the extreme drop in total inflation that we report in Table 2 for the fourth quarter of 2008, expressed in percent per annum. These extreme events provide a useful framework to develop intuition for the working of different model specifications.

We fit several flavors of our term structure models using data through the end of 2008 and forecast inflation for year 2009.[11] First, we focus on the $\text{DTSM}_{2,1}$, which we estimate on CRSP zero-coupon rates and either total or core CPI as a measure of the inflation factor. This model forecasts total and core CPI inflation to be -5.67% and -0.25% in 2009, respectively (Table 2). Both values are far from the subsequent realizations observed in 2009 (1.45% and 1.72%, respectively). That is, when estimated with total CPI inflation, the model extrapolates the -9.64% inflation rate realized in the fourth quarter of 2008, and predicts strong deflation in 2009. Fitting the model with the less volatile core inflation series produces a much less dramatic deflation scenario.

This analysis underscores several advantages of modeling the dynamics of the individual inflation components. With the $\text{DTSM}_{2,1}$ we are forced to choose whether bonds are priced out of total or core inflation. Either choice produces forecasts for one series but not for the other. In contrast, jointly modeling three inflation factors, CPI core, food and energy, yields forecasts for total inflation as well as each of its components. Moreover, this approach proves to be more robust to the extreme energy price fluctuations observed during this period. For instance, the $\text{DTSM}_{2,3}$ produces much higher total CPI forecasts for 2009, 0.14% compared to -5.67% for the $\text{DTSM}_{2,1}$ when estimated on the same panel of CRSP yields. This is because the model finds shocks to energy inflation to be short lived. It expects energy prices to decline moderately in 2009, with only a limited pass-though effect on total CPI inflation.

Finally, we estimate the $\text{DTSM}_{3,3}$ on a sample of zero-coupon rates interpolated from CMT par yields with maturity up to 10 years. The sample period starts in the first quarter of 1985, i.e., it omits the monetary experiment of the early 1980s. In this case, the model downplays the effect of energy shocks even more when forecasting total and core CPI inflation. The 2009 forecasts are 1.32% and 1.08%, respectively. These forecasts are very close to the 2009 observations: The last column of Table 2 shows realized total and core CPI rates of 1.45% and 1.72%. Energy prices show a moderate rebound (a 0.25% projected increase in 2009), while food prices are expected to grow at 3.25%. Both series are much less persistent and more volatile than core CPI. This is consistent with a higher forecast error, as seen from the last column of Table 2.

The bottom row of Table 2 shows model-implied estimates of the five-year real rate as of the end of the sample period, and contrasts them to two popular market-based estimates of real rates, (1) the average five-year zero-coupon rate on TIPS over the fourth quarter of 2008;[12] and (2) the difference between the five-year zero-coupon nominal yield in the fourth

[11] Table 2 reports results computed with the most recent CPI data releases, which includes small revisions since fall 2008. Model estimation with real time data as of the beginning of 2009 has given similar results.

[12] The data are from the Federal Reserve Board; their staff compute daily TIPS zero-coupon rates using the approach of Gürkaynak, Sack, and Wright (2010).

quarter of 2008 and trailing 2008 inflation.

Real rates estimates from the $\mathrm{DTSM}_{2,1}$ are as high as 2.94%, in line with the stark deflation outlook predicted by this model. The 2.73% TIPS rate is also consistent with high deflation risk. However, many market participants noticed that the TIPS market was greatly disrupted by the poor liquidity conditions prevalent in financial markets in fall 2008 and deemed TIPS rates to be an unreliable measure of inflationary expectations.[13] Thus, the second market-based real rate estimate of 0.56% in the last column of Table 2 is arguably a more accurate forecast than the TIPS yield. This value is very close to the real rates estimated by our $\mathrm{DTSM}_{2,3}$ and $\mathrm{DTSM}_{3,3}$ with separate core, food, and energy inflation factors.

5.2 Inflation Forecasts

The specification tests discussed in Section 4 indicate that the $\mathrm{DTSM}_{K_1,3}$, with $K_1 \geq 3$, does a good job at jointly fitting Treasury yields and inflation data. Moreover, the stylized evidence in Section 5.1 suggests that the same model might fare well at forecasting inflation. Hence, here we explore the out-of-sample performance of the model in more detail. Along the way, we compare its behavior to that of other DTSM flavors, including some specifications previously studied in the literature. As additional benchmarks, we include inflation forecasts from other univariate time series models as well as survey forecasts.

The sample period beginning in 1985Q1 is a natural starting point for this analysis. It avoids the monetary experiment of the early 1980s and is therefore less likely to include different regimes in inflation and interest rates dynamics. Thus, we repeatedly estimate the $\mathrm{DTSM}_{K_1,3}$ using quarterly yields data over the period beginning in January 1985 and ending on date t, where t ranges from December 1998 through December 2008. For each set of coefficients estimated with data up to and including quarter t, we forecast core, food, and energy inflation at quarter $t+j$, $j = 1, \ldots, 4$. As in Ang, Bekaert, and Wei (2007), for each series i we sum the four quarterly forecasts to estimate inflation realization over the next year, $\pi^i_{t+4,4} = \pi^i_{t+1} + \pi^i_{t+2} + \pi^i_{t+3} + \pi^i_{t+4}$, where $\pi^i_{t+j} = \log(Q_{t+j}/Q_{t+j-1})$. Moreover, we use the weights ω^c, ω^f, and ω^e to compute a forecast for the realization of total inflation during the next year, $E_t[\pi^{tot}_{t+4,4}] = \omega^c_t E_t[\pi^c_{t+4,4}] + \omega^f_t E_t[\pi^f_{t+4,4}] + \omega^e_t E_t[\pi^e_{t+4,4}]$. We assess the forecast error against realized inflation based on the root mean squared error criterion (RMSE),

$$\mathrm{RMSE} = \sqrt{E[(E_t(\pi^i_{t+4,4}) - \pi^i_{t+4,4})^2]} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(E_t(\pi^i_{t+4,4}) - \pi^i_{t+4,4})^2}\,, \tag{23}$$

[13]For instance, on November 9, 2009 Paul Krugman writes in his New York Times blog, The Conscience of a Liberal: "The yield on TIPS shot up after Lehman fell; ordinary bond yields plunged over the same period. Was this a collapse in expected inflation? Not really, or at any rate not mostly: TIPS are less liquid than regular 10-year bonds, so in the rush for liquidity they became very underpriced for a while. Correspondingly, as markets calmed down there was a fall in TIPS yields and a rise in ordinary bond yields; this probably didn't have much to do with changing inflation expectations."

for each inflation series i. In particular, for total inflation we compare the forecast $E_t[\pi^{tot}_{t+4,4}]$ with the actual realization, $\pi^{tot}_{t+4,4}$, not with the weighted proxy $\omega^c_t \pi^c_{t+4,4} + \omega^f_t \pi^f_{t+4,4} + \omega^e_t \pi^e_{t+4,4}$. Table 3 reports RMSEs for the DTSM$_{K_1,3}$ cases with $K_1 = 3$ and 4, estimated on CMT yields with maturity up to 10 years or their first four principal components. Panel A focuses on CPI inflation measures, while Panel B shows results for PCE data. In all cases, maximum likelihood estimation relies on the Kalman filter. For comparison, the table also includes the RMSE corresponding to the ARMA(1,1), the weighted ARMA$_W$, and the good old random walk, RW. Moreover, we report RMSEs for SPF and University of Michigan Survey forecasts.[14]

We first look at the RMSEs for the DTSM$_{3,3}$ estimated on the yields principal components. The results are particularly favorable for both total and core CPI inflation. On these two series, the 1.36% and 0.4% RMSEs (expressed in percent per annum) are systematically lower than the RMSE produced by each of the univariate models. For total inflation, the DTSM$_{3,3}$ produces an 11% improvement over the RMSE for the ARMA$_W$ model; the DTSM$_{3,3}$ performs even better when compared to the other univariate specifications. Similarly, there is a 22% improvement in the RMSE for core inflation compared to both the ARMA and RW cases. The model improves the RMSE for food inflation by at least 10%, while it is at par with the ARMA for energy.

Not surprisingly, professional forecasters do quite well at forecasting inflation (e.g., Ang, Bekaert, and Wei (2007), Faust and Wright (2009)): The SPF does better than each of the univariate models with a 1.38% RMSE for total inflation. Remarkably, the DTSM$_{3,3}$ is at par with these results.

To assess whether the difference in RMSEs is statistically significant we choose the ARMA(1,1) as a benchmark (Stock and Watson (1999) and Ang, Bekaert, and Wei (2007)). We test for equal forecast accuracy using the approach of West (1996), which accounts for parameters estimation error.[15] We compute p-values under the null that the RMSE for the ARMA model equals the DTSM$_{3,3}$ RMSE. The alternative hypothesis is that the RMSE for the ARMA model exceeds the DTSM$_{3,3}$ RMSE. The test rejects the null and favors the DTSM$_{3,3}$ for core inflation. For total inflation the test cannot statistically distinguish the DTSM$_{3,3}$ from the ARMA model. This is not entirely surprising due to the higher volatility of total vs. core inflation.

Table 3, Panel A, further shows that estimation of the DTSM$_{3,3}$ on yields gives results nearly identical to the estimation on the yields' first four principal components. Moreover, increasing the number of latent factors to 4 does not improve the forecasting ability of the

[14]Thomas (1999), Mehra (2002), and Souleles (2004) document systematic biases in survey forecasts. Along similar lines, Ang, Bekaert, and Wei (2007) show that, while there are some significant biases in inflation survey forecasts, these biases must be small, relative to the total amount of forecast error in predicting inflation. In fact, they find that raw survey forecasts outperform bias-corrected forecasts. Given their findings, we report results based on raw survey forecasts.

[15]In all tests for equal forecast accuracy, we compare non-nested models. Thus, West (1996) asymptotic results hold. Note in particular that the four-quarter RW that we use here (e.g., Atkeson and Ohanian (2001)) is not nested in the ARMA(1,1).

model. The table also reports RMSEs for the $\mathrm{DTSM}^*_{3,3}$, in which level is the only priced risk factor. While there is some deterioration in the model forecasts, the results are qualitatively similar to the $\mathrm{DTSM}_{3,3}$ case.

The $\mathrm{DTSM}_{3,2}$ does slightly worse, but is largely in line with the $\mathrm{DTSM}_{3,3}$ on core: the RMSEs are 0.44% and 0.40%, respectively. This still represents a considerable improvement over the ARMA benchmark. In contrast, the $\mathrm{DTSM}_{3,2}$ performance deteriorates considerably on total inflation (1.59% vs. 1.36%) and is at par with the ARMA (1.61%). These results confirm that separating the frequencies in total inflation helps extract predictive content for inflation from yields data. However, in the $\mathrm{DTSM}_{3,2}$ case the improvement is limited to core forecasts. This is not surprising as the latent factors affect core dynamics in a way similar to the $\mathrm{DTSM}_{3,3}$ case. In contrast, the $\mathrm{DTSM}_{3,3}$ does a better job at modeling the crust by treating food and energy as distinct processes. Similar conclusions apply to the $\mathrm{DTSM}^*_{3,2}$, in which investors demand compensation only for bearing risk associated to shocks to the first latent factor.

The results for PCE inflation series are largely consistent with the evidence on CPI inflation. Namely, for total PCE inflation all flavors of the $\mathrm{DTSM}_{3,3}$ produce a 13.4% decrease in the RMSE compared to the ARMA case. There is an 11% improvement in the food inflation RMSE, while the RMSEs for core and energy are roughly at par with the ARMA RMSEs. The $\mathrm{DTSM}_{3,2}$ does slightly worse than $\mathrm{DTSM}_{3,3}$ on both core and total inflation; yet, it outperforms the ARMA on total.

Now we turn to the out-of-sample performance of the $\mathrm{DTSM}_{K_1,3}$ estimated over a longer sample period that starts in 1962 (the earliest date for which we have CMT yields). Panel A of Table 4 reports RMSEs for the 1985Q4-2008Q4 forecasting window. The $\mathrm{DTSM}_{3,3}$ estimated on the PCs of CMT yields with maturities up to 10 years gives a 1.36% RMSE for total CPI inflation. The 17% improvement over the ARMA benchmark is statistically significant according to the West (1996) test p-value. The RMSE for core inflation is 0.67%, which is lower than the 0.71% RMSE for the ARMA model but higher than the RMSE for the RW forecasts. The model does well at forecasting energy inflation, while the ARMA outperforms it in forecasting food inflation.

Next, we compare these results to the $\mathrm{DTSM}_{2,1}$ case. When estimated on total inflation and CRSP yields data starting from 1962Q1, the $\mathrm{DTSM}_{2,1}$ produces a 1.69% RMSE, which exceeds the 1.63% RMSE for the ARMA model. This confirms the results of Ang, Bekaert, and Wei (2007) and extends them to a sample that includes more recent data. The $\mathrm{DTSM}_{3,3}$ improves on these forecasts considerably. It produces a 1.36% RMSE for total inflation, which is 20% lower than in the $\mathrm{DTSM}_{2,1}$ case. We record a similar improvement on core inflation: The 0.67% RMSE for the $\mathrm{DTSM}_{3,3}$ is 20% lower than the 0.84% value registered in the $\mathrm{DTSM}_{2,1}$ case.

There might be a concern that these results are explained by the choice of the long sample period, which spans different monetary policy regimes and includes shifts in inflation dynamics (e.g., Ang, Bekaert, and Wei (2008)). However, Ang, Bekaert, and Wei (2007) show that accounting for these features does not improve the out-of-sample performance of the $\text{DTSM}_{2,1}$ significantly.

This evidence suggests that jointly modeling the three inflation components greatly improves the forecasting performance of the model. A reader might also wonder the extent to which our results can be explained, at least in part, by the choice of the yields data set; the autocorrelation structure in the inflation series; the estimation method; the inclusion of a higher number of latent factor; and the out-of-sample window over which we assess the performance of the forecasts.

To address these issues, we first estimate the $\text{DTSM}_{2,3}$ on CRSP data by maximum likelihood with the Chen-Scott method. When using an AR(1) process on all inflation series the model produces a 1.51% RMSE for total inflation, a 11% improvement over the $\text{DTSM}_{2,1}$ case. Allowing for an AR(3) process for core inflation (an extension favored by our specification tests) lowers the $\text{DTSM}_{2,3}$ RMSE for total inflation to 1.40%, only slightly higher than the $\text{DTSM}_{3,3}$ RMSE. On core, the $\text{DTSM}_{2,3}$ produces 0.99% and 0.84% RMSEs in the AR(3-1-1) and AR(1-1-1) cases, respectively. The latter is at par with the $\text{DTSM}_{2,1}$ results.

Second, we fit the $\text{DTSM}_{2,3}$ AR(3-1-1) model via maximum likelihood and the Kalman filter. The best results are for the case in which the observables are the first four principal components of the CRSP yields. In that case, the RMSEs for total and core are 1.39% and 0.79%. Either value is lower than the $\text{DTSM}_{2,1}$ RMSEs.

Third, we further increase the number of latent factors. To improve identification, in this exercise we include the 20-year yield. The best results are for $K_1 = 4$, with RMSEs that are similar to those for the $\text{DTSM}_{3,3}$ estimated with yields up to 10-year maturity. We do not find additional improvement when raising K_1 to 5.

Fourth, Table 4, Panel B, confirms that, when estimating the model with the long sample period from 1962Q1, the results for the 1998Q4-2008Q4 forecasting window are similar to those for the 1985Q4-2008Q4 window. A comparison with Table 3, Panel A, illustrates that estimation over the shorter sample period that starts in 1985Q1 leads to smaller RMSEs for the same 1998Q4-2008Q4 forecasting window.

To further examine the sensitivity to the forecasting period, we compute RMSEs over a grid of out-of-sample windows with start date ranging from 1996Q4 to 1999Q4 and end date from 2002Q4 to 2008Q4. The discussion so far indicates that the $\text{DTSM}_{3,3}$ estimated on the principal components of yields with maturities up to 10 years, sampled from 1985Q1, produces the best inflation forecasts. Thus, we focus on this model during this exercise. For each window in the grid, we compute RMSEs for core and total inflation associated to the $\text{DTSM}_{3,3}$ and ARMA models. Figure 2 plots their percentage ratio, $100 \times (\text{RMSE DTSM}_{3,3}/\text{RMSE ARMA} - 1)$. That is, negative numbers in the plot signal that the DTSM outperforms the ARMA. The improvement is most visible for the core inflation forecasts in the top panel. In this case, the DTSM does better than the ARMA nearly 98% of the times. The reduction in RMSEs is sizeable except for out-of-sample windows that have an early start date, possibly due to the limited length of the estimation period. For total inflation the evidence is more mixed, with the DTSM outperforming the

ARMA about half of the times.

Figure 3 illustrates the improvement compared to a $DTSM_{2,1}$ estimated on univariate CPI inflation (either core or total). The level of the RMSE ratios is much higher than those reported in Figure 2, with the ARMA significantly outperforming the $DTSM_{2,1}$ in nearly all cases. This clearly shows that our core and 'crust' framework for modeling inflation shocks produces an improvement in forecasting performance over the $DTSM_{2,1}$ case that is robust to the choice of the out-of-sample window.

Finally, Figure 4 compares percentage RMSE ratios for the $DTSM_{3,3}$ (left panels) and the unconstrained VAR (right panels) relative to the ARMA benchmark. We estimate the VAR on the first four principal components of CMT yields with maturities up to 10 years, along with core, food, and energy inflation data. In the top panels, we recursively estimate the model with data starting from 1985Q1; in the bottom panels we consider longer samples that start in 1962Q1. In all panels, RMSEs are for total CPI inflation. The color coding and the grid of out-of-sample windows are as in Figures 2-3. Overall, we find that $DTSM_{3,3}$ does well relative to the unconstrained VAR, especially when estimation relies on the long sample period.

5.3 The Determinants of Interest Rates and Inflation

In this section, we examine the determinants of interest rates and inflation. We first decompose the variance of the forecasting errors into components associated with shocks to the latent factors and inflation. We then study the contemporaneous linkage between yields and inflation.

5.3.1 Variance Decomposition

We now investigate what proportion of the variance of the yields and inflation forecasts is explained by shocks to the latent factors versus the inflation factors. Table 5 shows variance decompositions for CPI inflation and yields with one-quarter, five- and ten-year maturity, computed as in Hamilton (1994, p. 323-324).[16] For illustration, we consider the same models discussed in Section 5.1 and estimated over a sample period ending in 2009Q4.

Panel A shows results for CPI inflation. At the short one-year horizon, innovations in CPI inflation explain most of the variation in CPI forecast errors. As the forecasting horizon increases, innovations to the latent factors become prevalent in driving the variance of the errors. In particular, the latent factors explain more than 50% of the unconditional variation of inflation. This proportion is higher for CPI core than for total CPI. Moreover, the three latent factors in the $DTSM_{3,3}$ explain close to 80% of the unconditional variance of CPI core inflation, with the first, most persistent, factor accounting for nearly 70%.

[16]Shocks to the inflation factors are correlated. Thus, as customary we rely on a Cholesky factorization of the covariance matrix when computing the forecasting errors at different horizons. We order the state variables with the latent factors first, then the core, food, and energy inflation factors. Any other ordering of the state variables gives us virtually identical results.

Panels B-D report the variance decomposition for yields with maturity of one quarter, five and ten years. In all cases, the latent factors account for the majority of the variation in yields' dynamics. For short horizon forecasts, in the preferred $\mathrm{DTSM}_{3,3}$ much of the variation is explained by innovations to the less persistent higher-order factors. As the forecasting horizon increases, the first latent factor takes over, especially in long-maturity yields for which the first latent factor explains up to 94% of the unconditional variation.

Taken together, the results in Table 5 suggest that inflation and interest rates share a common structure of latent factors. This is particularly evident in the $\mathrm{DTSM}_{3,3}$, in which inflation shocks play a minimal role in explaining yields dynamics.

5.3.2 Unspanned Inflation Risk

Next, we examine the immediate response of the nominal yield curve to a shock in the state vector. Figure 5 displays the B_n coefficients in equation (15) for $\mathrm{DTSM}_{3,3}$, annualized and rescaled to correspond to one standard deviation movement in the factors. The loadings $B_n^{\ell^k}$, $k = 1, \ldots, 3$, on the latent factors far exceed B_n^{core}, B_n^{food}, and B_n^{energy} in magnitude, which confirms that shocks to the latent factors are the main driving force in yield changes. Consistent with much of the empirical DTSM literature, ℓ^1 plays the role of a 'level' factor that has an even impact on the yield curve (the $B_n^{\ell^1}$ coefficient is fairly flat across yields' maturities). In contrast, shocks to ℓ^2 affect the two-year yield the most and have a lower impact on the short and long end of the term structure, while shocks to ℓ^3 mostly affect short term rates. These features are specific to the cascade structure (17), in which factors are ranked based on their frequencies and response functions to latent factors' shocks are hump shaped, except for the highest frequency latent factor (Calvet, Fisher, and Wu (2010)).

In contrast, the loadings on the core, food, and energy variables are close to zero across bond maturities. This suggests that, in the model, the contemporaneous relation between innovations in yields and inflation is tenuous, consistent with the variance decomposition evidence discussed in Section 5.3.1.

To clarify these results, we examine the empirical relation between interest rates and inflation and compare the evidence to the predictions of our model. For each yield maturity n, Table 6 shows results for the OLS regressions:

$$\text{Levels}: \quad y_t^n = \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n \tag{24}$$

$$\text{Changes}: \quad \Delta y_t^n = \beta^c \Delta\pi_t^c + \beta^f \Delta\pi_t^f + \beta^e \Delta\pi_t^e + \varepsilon_t^n \,. \tag{25}$$

We first estimate the regressions for each yield series y_t^n with maturity n equal to one quarter, one, three, five, and ten years against core, food, and energy inflation sampled quarterly from 1985Q1 to 2009Q4. For each regression, we report coefficient estimates and Newey-West heteroskedasticity and autocorrelation robust standard errors (in brackets). Next, we simulate 10,000 samples of quarterly yields with the same maturities as well as core, food, and energy inflation series from the preferred $\mathrm{DTSM}_{3,3}$ using the scheme described in Section A.2 of the Online Appendix. We estimate the same regressions on each simulated sample and report mean, 5th, 50th, and 95th percentiles of the estimated coefficients. The results in the left-hand side of the table are for regressions in levels (the table omits the estimate of the intercept α); those in the right-hand side are for regressions in changes.

Table 6 shows that the $\text{DTSM}_{3,3}$ predicts a link between inflation and yields that closely matches the features of the data:

1. When estimating regressions of the levels of yields on inflation (equation (24)), we find the coefficient β^c to be significant, while β^f and β^e are not statistically different from zero. The point estimates of these coefficients are close to the mean of the coefficients computed in simulated data and always fall within the simulated 90% confidence intervals.

2. When estimating regressions of the changes in yields on changes in inflation (equation (25)), all coefficients β^c, β^f, β^e are insignificant, with point estimates that are close to zero in magnitude. Model simulations lead to identical conclusions.

3. In the data, the adjusted R^2 coefficients for regressions in levels always exceed 50%, and are nearly 70% for long-maturity yields. Interpreting these results requires caution, as the regressions include persistent variables; autocorrelation in the residuals could produce spuriously high R^2 coefficients (Granger and Newbold (1974)). Indeed, regressions in changes have adjusted R^2 coefficients that are nearly zero. Model simulations reproduce this evidence closely: 90% confidence intervals include the R^2 estimated in the data. Figure 6 provides a visual illustration of the R^2 distributions.

To accommodate these features, Joslin, Priebsch, and Singleton (2011) propose a DTSM with unspanned macroeconomic risk. They impose restrictions on the model coefficients such that the loadings of the yields (or their linear combinations) on macroeconomic variables are zero. In contrast, we do not impose such conditions a priori. We estimate an unconstrained model and find factor loadings on the inflation series that are nearly zero. The evidence we provide above shows that our model replicates the empirical linkage between yields and inflation data extremely well.[17]

5.4 Real Rates and Risk Premia

Here we explore the time-series and term-structure properties of real rates computed using our model and compare them to those of TIPS rates. We then illustrate the patterns in model-implied inflation and real rates risk premia.

5.4.1 The Time-Series of Real Rates: Model vs. TIPS

Figure 7 shows the one-quarter (spot) real rate estimated with the $\text{DTSM}^*_{3,3}$ (in which level is the only priced risk factor), while Figure 8 depicts five- and ten-year real rates. Although we

[17] The notion of unspanned macroeconomic risk is also supported by the evidence in Duffee (2011), who detects the presence of a 'hidden' factor that is not explained by the term structure of yields and correlates weakly with macroeconomic variables.

do not include TIPS in the data set used for estimation, Figure 8 also provides a comparison between model-implied real yields and matching-maturity TIPS rates during the sub-sample for which TIPS data are available.[18]

The pattern in the spot real rate is quite intuitive and tightly linked to monetary policy intervention. Just like the Federal funds rate, the spot real yield increases during periods of expansion and declines during recessions. In particular, there is a pronounced rise since the mid-2000s followed by a decline during the most recent crisis.

The long real yields in Figure 8 show a smoother pattern with a common downward trend. DTSM yields are considerably lower than TIPS rates during the early part of the TIPS sample period. In 1999Q1, the spread is approximately 150 bps at the ten-year maturity. This is consistent with a high liquidity premium embedded in TIPS when their trading began in the late 1990s. As TIPS liquidity conditions improve, the spread narrows and is near zero around 2004. These results are in line with the findings of D'Amico, Kim, and Wei (2010) for the 1999-2007 period and point to a liquidity premium that is lower than the one estimated by Pflueger and Viceira (2012). The bottom panel in Figure 8 shows that the five- and ten-year liquidity premia share similar patterns. However, the five-year premium exceeds the ten-year one on average. The two diverge even more during the last recession, indicating that liquidity disruptions had different impacts on the two segments of the TIPS market. Taken together, these results suggest that expected inflation measures backed out from nominal and TIPS yields (break-even inflation rates) can be biased. Moreover, the liquidity differential at five- and ten-year maturities can also bias forward measures of expected inflation (e.g., the so-called five-year five-year forward break-even rate).

TIPS rates increase starkly in 2008Q4. As mentioned previously, market participants attribute this pattern to dislocation in TIPS markets following Lehman Brothers' bankruptcy, rather than to an extreme increase in deflationary expectations with a prolonged impact that extends to the five- and ten-year horizons. The model agrees with this interpretation and, as discussed in Section 5.1, it downplays the impact of the big negative shock in energy prices observed in 2008Q4 on expected inflation. Thus, long-maturity real DTSM yields in Figure 8 continue their decline through the recession. In contrast, the spot real rate in Figure 7 has a moderate uptick in 2008Q4, consistent with a short-run expected decline in consumer prices associated to a negative energy shock.

5.4.2 The Term Structure of Real Rates

Figure 9 shows the nominal and real term structures computed using a $\text{DTSM}^*_{3,3}$. The nominal term structure is upward sloping, consistent with well-known empirical evidence. Of more interest, we find the real term structure to be upward sloping as well. This is consistent with empirical evidence from U.S. TIPS data[19] and the theoretical implications

[18]The TIPS rates are zero-coupon yields interpolated by the staff at the Federal Reserve Board, using the approach of Gürkaynak, Sack, and Wright (2010).

[19]Data on nominal and inflation-indexed U.K. government bonds, however, tell a different story. For instance, Pflueger and Viceira (2011; Table 2, Panel B) find downward sloping term structures for *both*

of several asset pricing models. For instance, Campbell and Cochrane (1995) and Wachter (2006) find an upward sloping real term structure in an exchange-economy in which the representative agent displays habit persistence. The long-run risk model of Bansal and Yaron (2004) predicts a downward sloping real term structure; however, Yang (2011) finds the opposite when durable consumption contains a persistent predictable component, while nondurables and services follow a random walk.

5.4.3 Model Risk Premia

Figure 10, top panel, displays the five- and ten-year inflation risk premium computed as

$$\mathrm{IRP}_t^n \equiv y_t^n - E_t[\pi_{t+n,n}^{tot}] - y_t^{*n}, \tag{26}$$

where $E_t[\pi_{t+n,n}^{tot}] = \sum_{i=1}^{K_2} \omega_t^i E_t[\pi_{t+n,n}^i] = \sum_{i=1}^{K_2} \sum_{j=1}^{n} \omega_t^i E_t[\pi_{t+j}^i]$ is the time-t expectation of total inflation over the next n periods, $n = 20$ and 40 quarters (5 and 10 years). As in prior studies (e.g., Ang, Bekaert, and Wei (2008) and Buraschi and Jiltsov (2005)), the IRP is positive on average and has a downward pattern since the mid-1980s.[20] This is consistent with the Federal Reserve's effort to control inflation and its success in shaping market's expectations on consumer price dynamics. In recent years, at times the risk premium turns negative. A notable example is the period of prolonged monetary tightening following 2004. In spite of an increase in nominal and real spot rates (Figure 7), nominal long-maturity yields remained low during that period, a development that Greenspan (2005) famously described as a 'conundrum.' The model fits the shift in the slope of the yield curve and associates it with a reduction in long term inflation risk premia. This mechanism is at play again during the recent financial crisis, when the inflation risk premium has often been negative. These findings support the view that the risk profile of Treasuries has changed over time. In the early 1980s, long-maturity bonds carry a high risk premium, possibly associated to uncertainty about future inflation. More recently, there are times when Treasuries act as hedges, providing safe-haven protection against recessions in which deflationary risk is perceived to be high.

The bottom panel of Figure 10 depicts the five- and ten-year real rate risk premium computed as

$$\mathrm{RRRP}_t^n \equiv y_t^{*n} - y_t^{*n,LEH}, \tag{27}$$

where $y_t^{*n,LEH}$ is the n-quarter real yield under the local expectation hypothesis (e.g., Piazzesi (2010), Section 3.3). The plots show that the decline in risk is not limited to the inflation component. In fact, the real rate risk premium also turns negative at times during the 2000s.

The risk premia measures in Figure 10 are obtained from a DTSM model estimated on nominal yields and inflation data alone. Yet, the pattern in our estimates is consistent with

nominal and real yields over the 1985/4-2009/12 sample period.

[20]This is at odds with the evidence in Haubrich, Pennacchi, and Ritchken (2009), who find small fluctuations in the ten-year inflation risk premium around a constant positive level, and a negative two-year inflation risk premium throughout their sample period. Our results also differ from Adrian and Wu (2010), who report a positive inflation risk premium that peaks in fall 2008.

the evidence in studies that focus on the comovements between Treasuries and stock market returns. For instance, Campbell, Sunderam, and Viceira (2011) find the covariance between nominal U.S. Treasury bond returns and stock returns to be unusually high in the early 1980s and negative in the 2000s, and Campbell, Shiller, and Viceira (2009) show the TIPS beta with stock returns to be negative in the downturns of 2001-3 and 2008-9.

5.5 Energy Pass Through

To expand on the discussion in Section 4, Figure 11 shows various measures of correlation between energy and core inflation. We estimate the $\text{DTSM}_{3,3}$ over samples with start date of 1962Q1 and end dates ranging from 1985Q1 to 2009Q4. For each sample period, in the bottom left panel we report the estimate for the $\phi_1^{\pi^c,\pi^e}$ coefficient that links lagged realization of energy inflation to core inflation, along with 90% confidence bands. In sample periods ending in the 1980s through the early 2000s, the coefficient is positive and significant. However, it is small in size and declining as the end date of the sample period increases. For a 100bps increase in lagged energy inflation, there is at most a 3-4bps pass through to core inflation.

The right panel complements these results by showing unconditional and conditional correlations between energy and core inflation. Both measures are positive. The unconditional correlation estimate shares a downward trend with the $\phi_1^{\pi^c,\pi^e}$ coefficient. However, it remains positive over the entire period and shows an uptick when the model is estimated including the most recent data. Such increase is driven by a higher estimate for the conditional correlation. In recent times, shocks to energy inflation show a larger direct impact on core dynamics.

The top two panels of Figure 11 depict the same correlation measures estimated using data samples starting in 1985Q1 and with end dates ranging from 1995Q1 to 2009Q4. The shorter sample size results in a less precise estimate of the $\phi_1^{\pi^c,\pi^e}$. Nonetheless, we observe a similar decrease in the energy pass through on core over time. Moreover, extreme energy shocks in the 2000s weigh more heavily in the estimates for the conditional correlations between core and energy inflation, resulting in a larger uptick in the unconditional correlations at the end of the sample.

These results extend the analysis of, e.g., Clark and Terry (2010), Hooker (2002), and Stock and Watson (2010) to a DTSM setting. One can interpret the decline in the energy pass through as a results of multiple factors. For instance, Stock and Watson (2010) point out that energy is a smaller share of expenditures than it was during the oil price shocks of the 70s, labor union membership has declined sharply over the past forty years, and there has been a shift from production of goods to production of services.

5.6 Nominal Yields Forecasts

Our preferred $\text{DTSM}_{3,3}$ does well at forecasting nominal Treasury yields when compared to other DTSM specifications, time-series models such as the ARMA and the random walk, as well as SPF forecasts. To save on space, we provide detailed results in Section A.4 of the Online Appendix. It is plausible that extending our framework to include other factors (e.g., a measure of real activity or the Cochrane and Piazzesi (2005, 2008) tent-shaped linear combination of forward rates) would further improve the nominal yields' forecasts (e.g., Ang and Piazzesi (2003), Joslin, Priebsch, and Singleton (2010)). Since this is not the focus of our analysis, we point the reader to those studies for more details.

6 Conclusions

Much of the empirical macro-finance literature finds that financial variables contain little predictive content for consumer price inflation. Nonetheless, this conclusion goes in the face of the intuition that the yield curve reflects market participants' expectations of future price dynamics. This leaves us with the challenge to improve conventional models and estimation methods to jointly fit term structure and inflation data. Our DTSM makes a step in this direction.

A key feature of the model is to separately specify the dynamics of the three main components of total inflation: core, food, and energy. These dynamics combine together to produce a measure of total expected inflation that investors use to price Treasury bonds. Thus, the model captures the different degree of persistence and volatility in shocks to the three inflation components. In particular, it downplays the role of short-lived fluctuations in energy prices in determining expectations of future inflation.

When we estimate the model on a panel of nominal Treasury yields and the three inflation measures, we find a considerable improvement in the fit compared to DTSM specifications that rely on a single inflation factor (either total or core). The model does especially well at forecasting CPI core inflation and it often outperforms an ARMA(1,1) model on total inflation.

Energy shocks have a limited pass-through on inflation forecasts and interest rates. In contrast, a common structure of latent factors explains most of the variance of the forecasting error for core inflation, as well as for bond yields. Taken together, all this evidence suggests that our framework helps us to extract predictive content from the yield curve to forecast future inflation.

While we do not use market-based expectations of inflation and real rates during estimation, the predictions of our model are consistent with such measures. Our inflation forecasts are in line with the Survey of Professional Forecasters and outperform the University of Michigan inflation survey. Moreover, we estimate real rates that agree with TIPS data when accounting for the liquidity premium in TIPS markets. Finally, inflation and real rates risk premia show a pattern consistent with the evidence on time-varying covariances between stock returns and nominal/real Treasury bond returns by Campbell, Sunderam, and Viceira (2011) and Campbell, Shiller, and Viceira (2009).

A Nominal Bond Prices

The price of a one-period nominal zero-coupon bond is:

$$\begin{aligned} p_t^1 &= E_t\left[m_{t+1}\right] = E_t\left[\exp\left(-r_t^* - \pi_{t+1} - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right] \\ &= E_t\left[\exp\left(-\delta_0 - \delta_1' X_t - \sum_{j=1}^{K_2}\omega^j \pi_{t+1}^j - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1}\right)\right] \\ &= \exp\left(-\delta_0 - \delta_1' X_t - \Phi_0^\pi - \Phi^\pi X_t - \frac{1}{2}\lambda_t'\lambda_t\right) E_t\left[\exp\left(-(\lambda_t' + \Omega^\pi)\varepsilon_{t+1}\right)\right]. \end{aligned} \tag{28}$$

Since $\varepsilon_{t+1} \sim N(0, I)$, then $E_t[\exp\left(-(\lambda_t' + \Omega^\pi)\varepsilon_{t+1}\right)] = \exp\left(\frac{1}{2}(\lambda_t' + \Omega^\pi)(\lambda_t' + \Omega^\pi)'\right)$. Substituting in equation (28) and rearranging terms we obtain

$$p_t^1 = \exp\left(-\delta_0 - \delta_1' X_t - \Phi_0^\pi - \Phi^\pi X_t + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi(\lambda_0 + \lambda_1 X_t)\right) = \exp\left(\bar{A}_1 + \bar{B}_1' X_t\right), \tag{29}$$

where $\bar{A}_1 = -\delta_0 - \Phi_0^\pi + \Omega^\pi\lambda_0 + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime}$ and $\bar{B}_1 = -\delta_1' - \Phi^\pi + \Omega^\pi\lambda_1$.

Assume now that equation (13) prices a nominal n-period zero-coupon bond. Then, the same formula prices an $(n+1)$-period bond. To verify this claim, combine equations (11)-(13):

$$\begin{aligned} p_t^{n+1} &= E_t\left[\exp\left(-r_t^* - \sum_{j=1}^{K_2}\omega^j\pi_{t+1}^j - \frac{1}{2}\lambda_t'\lambda_t - \lambda_t'\varepsilon_{t+1} + \bar{A}_n + \bar{B}_n' X_{t+1}\right)\right] \\ &= \exp\left(-\delta_0 - \Phi_0^\pi + \bar{B}_n'\Phi_0 - \frac{1}{2}\lambda_t'\lambda_t + \bar{A}_n + (-\delta_1' - \Phi^\pi + \bar{B}_n'\Phi)\, X_t\right) \\ &\quad \times E_t\left[\exp\left((-\lambda_t' - \Omega^\pi + \bar{B}_n'\Omega)\,\varepsilon_{t+1}\right)\right] \\ &= \exp\Bigg(-\delta_0 + \bar{A}_n + \bar{B}_n'\left(\Phi_0 - \Omega\lambda_0\right) - \Phi_0^\pi + \frac{1}{2}\bar{B}_n'\Omega\Omega'\bar{B}_n + \frac{1}{2}\Omega^\pi\Omega^{\pi\prime} + \Omega^\pi\lambda_0 - \bar{B}_n'\Omega\Omega^{\pi\prime} \\ &\quad + (-\delta_1' - \Phi^\pi + \bar{B}_n'\left(\Phi - \Omega\lambda_1\right) + \Omega^\pi\lambda_1)\, X_t\Bigg). \end{aligned} \tag{30}$$

We collect terms linear in X_t and independent of X_t to obtain the ODEs (14).

B Core, food, and energy weights

Market participants deflate nominal asset prices in equation (12) at the total inflation rate, π_t. In the model that has three inflation factors, we compute π_t as the weighted sum of the core, food, and energy inflation series. That is, $\pi_t = \pi_t^{tot} = \omega^c\pi_t^c + \omega^f\pi_t^f + \omega^e\pi_t^e$, where the weights ω^c, ω^f, and ω^e represent the relative importance of core, food, and energy prices in the total price index. This appendix describes how we construct such weights.

B.1 Consumer price index weights

For the CPI weights we use the relative importance of core, food, and energy in the CPI reported by the Bureau of Labor Statistics (BLS). The relative importance of a component is the percentage share of the expenditure on that component relative to the expenditure on all items within an area. The BLS conducts a Consumer Expenditure Survey to determine how these shares change over time to reflect fluctuations in the consumption patterns of the population. Each year since 1987, the BLS releases the December value of these series based on the core, food, and energy consumption baskets for that year. Monthly fluctuations in prices result in changes in the relative importance shares for these baskets compared to the values reported the previous December. To account for this pattern, we update the value of the December shares to obtain monthly series that reflect the changes in the cost to purchase the same food, core, and energy baskets. The BLS Internet site at http://www.bls.gov/cpi/cpi_riar.htm explains in details how to do that. The BLS does not make relative importance shares broadly available for years prior to 1987. We thank the BLS for sharing such data with us.

B.2 Personal consumption expenditures weights

Similar to the CPI weights, the PCE weights are the shares of the expenditures on the core, food, and energy baskets relative to total personal consumption expenditures. To compute these shares, we use data from the national income and product account (NIPA) Table 2.3.5U, Personal Consumption Expenditures by Major Type of Product and by Major Function. The variables are (1) Personal consumption expenditures; (2) Personal consumption expenditures excluding food and energy; (3) Food and beverages purchased for off-premises consumption; and (4) Energy goods and services.

Figures and Tables

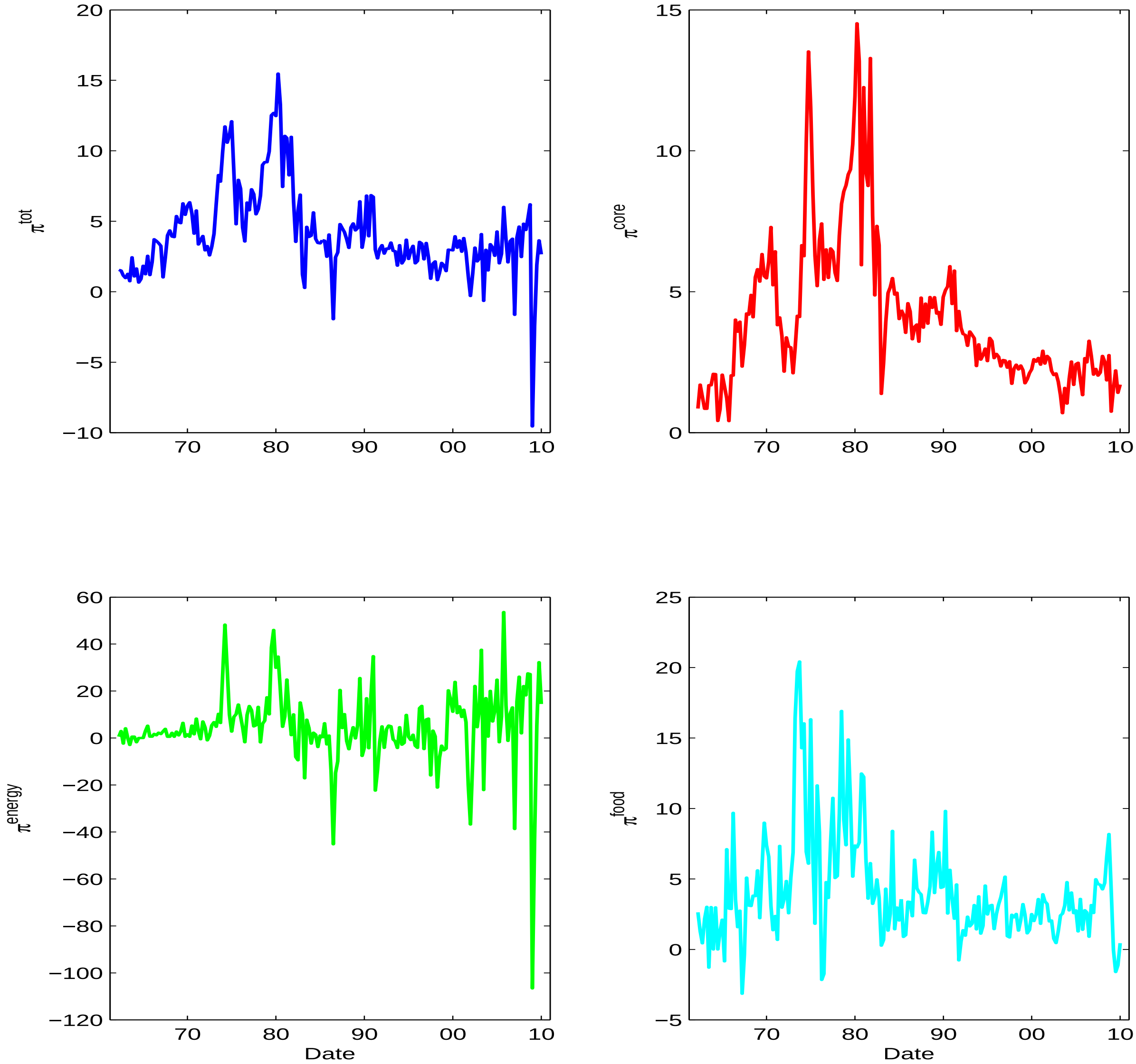

Figure 1: **CPI Inflation Series.** The plots depict total, core, food, and energy quarterly CPI inflation series. The sample period is 1962Q1-2009Q4.

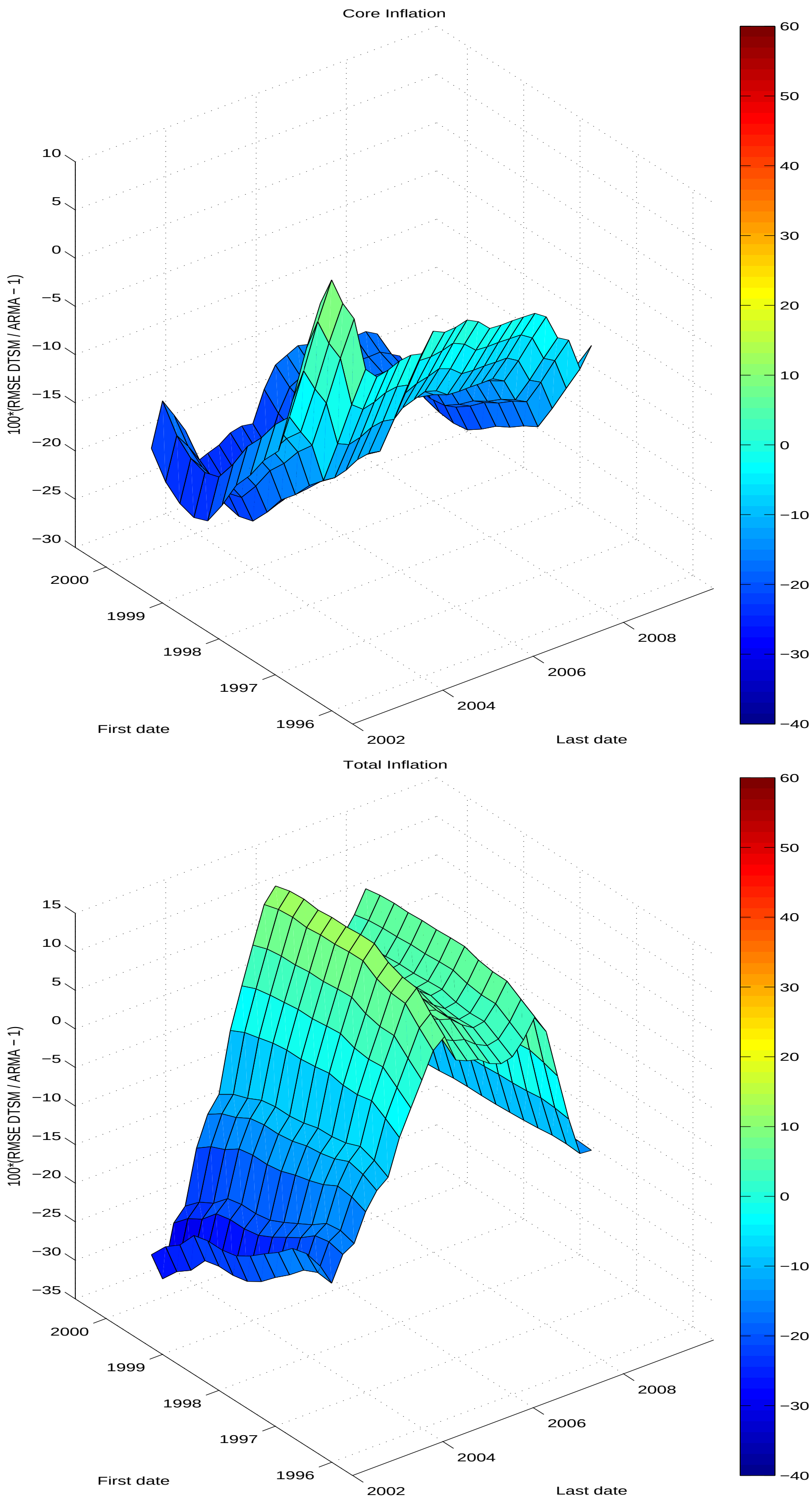

Figure 2: **RMSE Percentage Ratios: DTSM$_{3,3}$.** For the DTSM$_{3,3}$ and ARMA models, we compute RMSEs over a grid of out-of-sample windows with start date ranging from 1996Q4 to 1999Q4 and end date from 2002Q4 to 2008Q4. The figure displays their percentage ratio, $100 \times (\text{RMSE DTSM}_{3,3}/\text{RMSE ARMA} - 1)$. Negative numbers in the plot signal that the DTSM outperforms the ARMA.

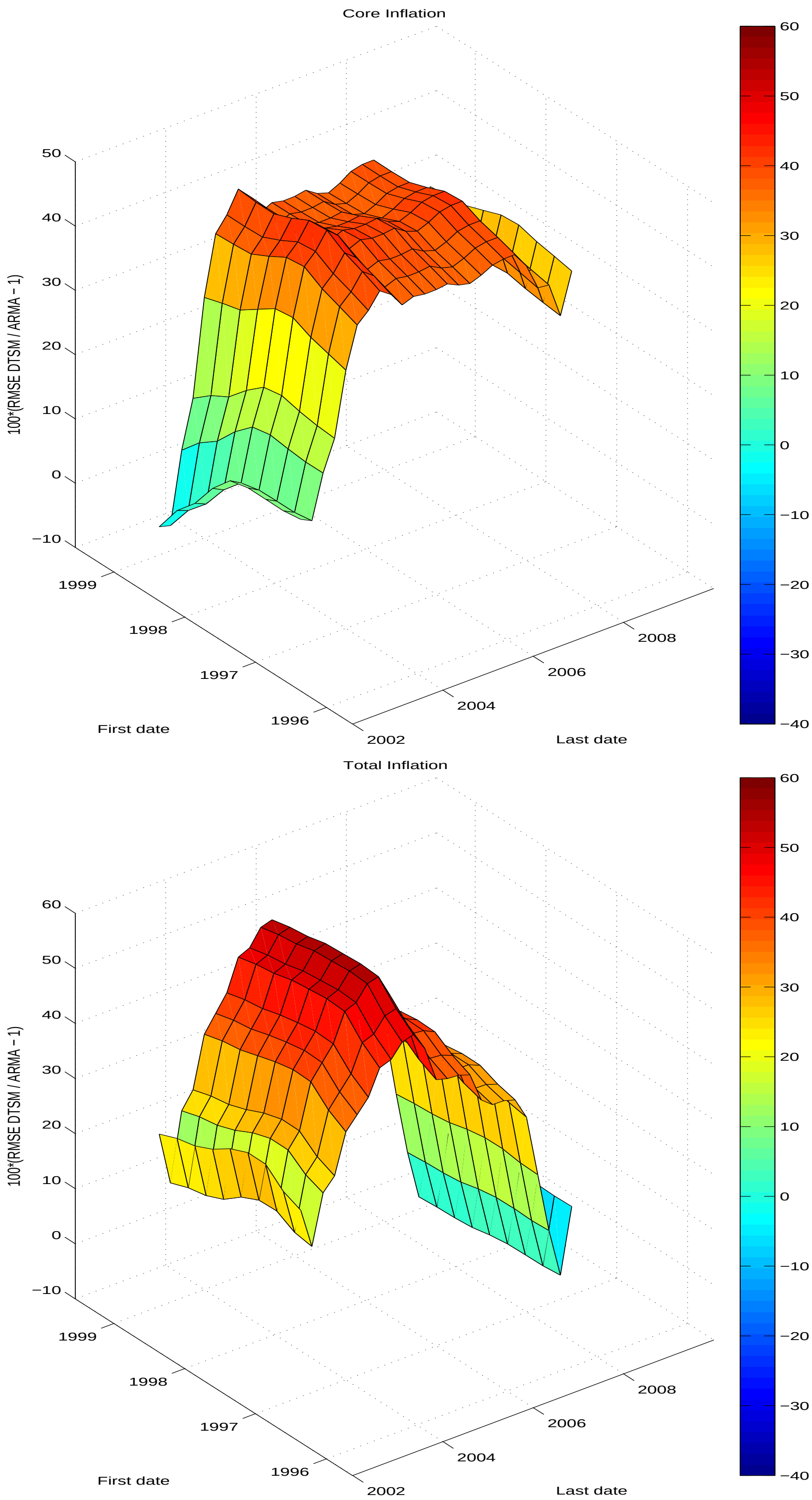

Figure 3: **RMSE Percentage Ratios: DTSM$_{2,1}$.** For the DTSM$_{2,1}$ and ARMA models, we compute RMSEs over a grid of out-of-sample windows with start date ranging from 1996Q4 to 1999Q4 and end date from 2002Q4 to 2008Q4. The figure displays their percentage ratio, $100 \times$ (RMSE DTSM$_{3,3}$/RMSE ARMA $- 1$). Negative numbers in the plot signal that the DTSM outperforms the ARMA.

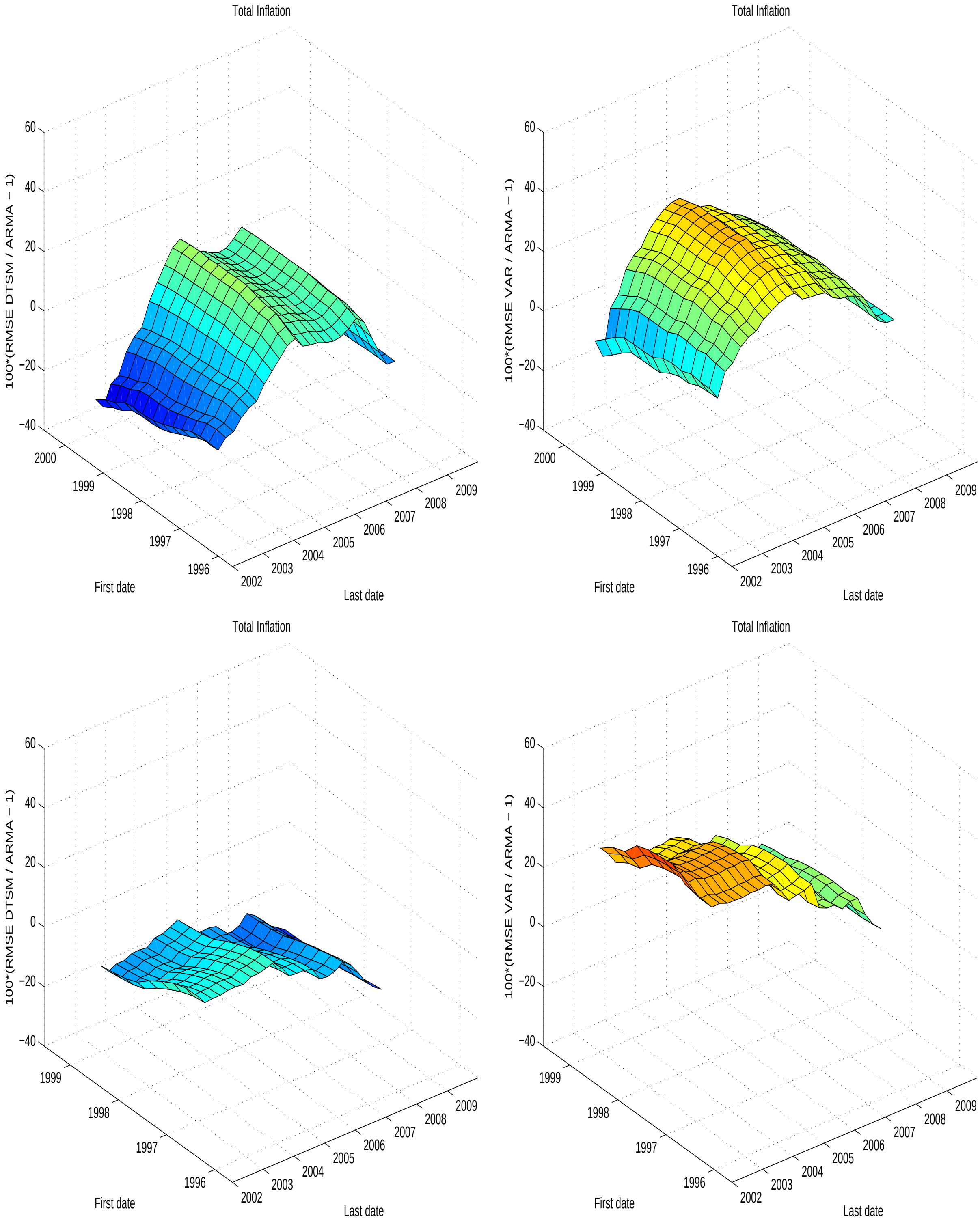

Figure 4: **RMSE Percentage Ratios: $\mathbf{DTSM_{3,3}}$ vs. VAR.** We compute RMSEs for the $DTSM_{3,3}$, the unconstrained VAR, and the ARMA(1,1) total inflation forecasts over a grid of out-of-sample windows with start date ranging from 1996Q4 to 1999Q4 and end date from 2002Q4 to 2008Q4. The figure displays the percentage ratios $100 \times$ (RMSE $DTSM_{3,3}$ or VAR/RMSE ARMA $- 1$). In the left panels, the percentage ratio has the RMSE for $DTSM_{3,3}$ in the numerator; in the right panels, it has the RMSE for the VAR. Negative numbers in the plot signal that the $DTSM_{3,3}$ / VAR models outperforms the ARMA. In the top panels, the sample period is 1985Q1-2009Q4; in the bottom panels, it is 1962Q1-2009Q4

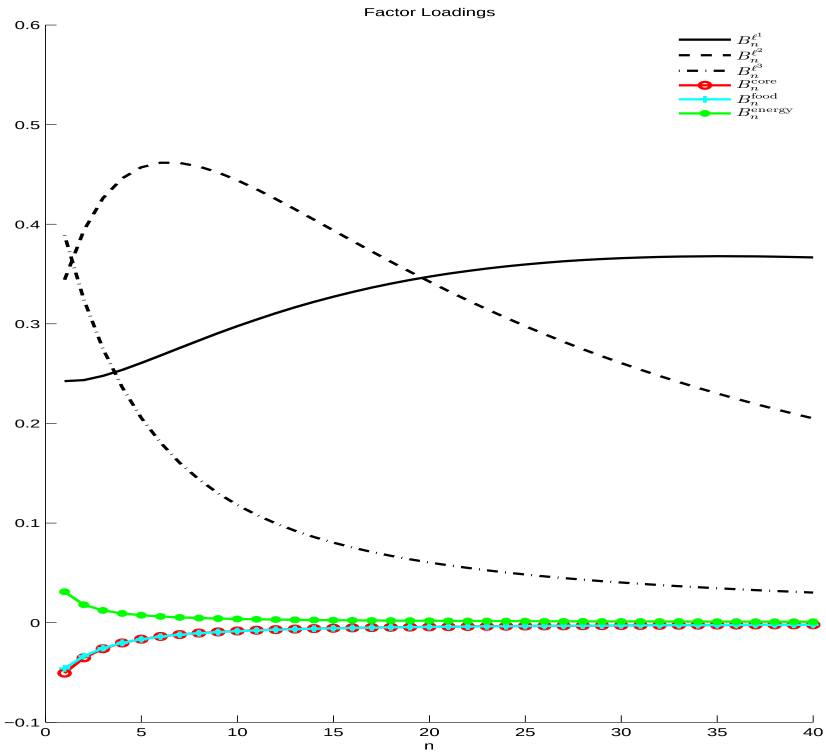

Figure 5: **Factor Loadings.** The plot depicts the factor loading for nominal yields on the latent factors ($B_n^{\ell k}, k = 1, \ldots, K_1$) and inflation factors (B_n^{core}, B_n^{food}, and B_n^{energy}). We scale the factor loadings to correspond to one standard deviation movement in the factors and we annualize them by multiplying by 400. The sample period is 1985Q4-2009Q4.

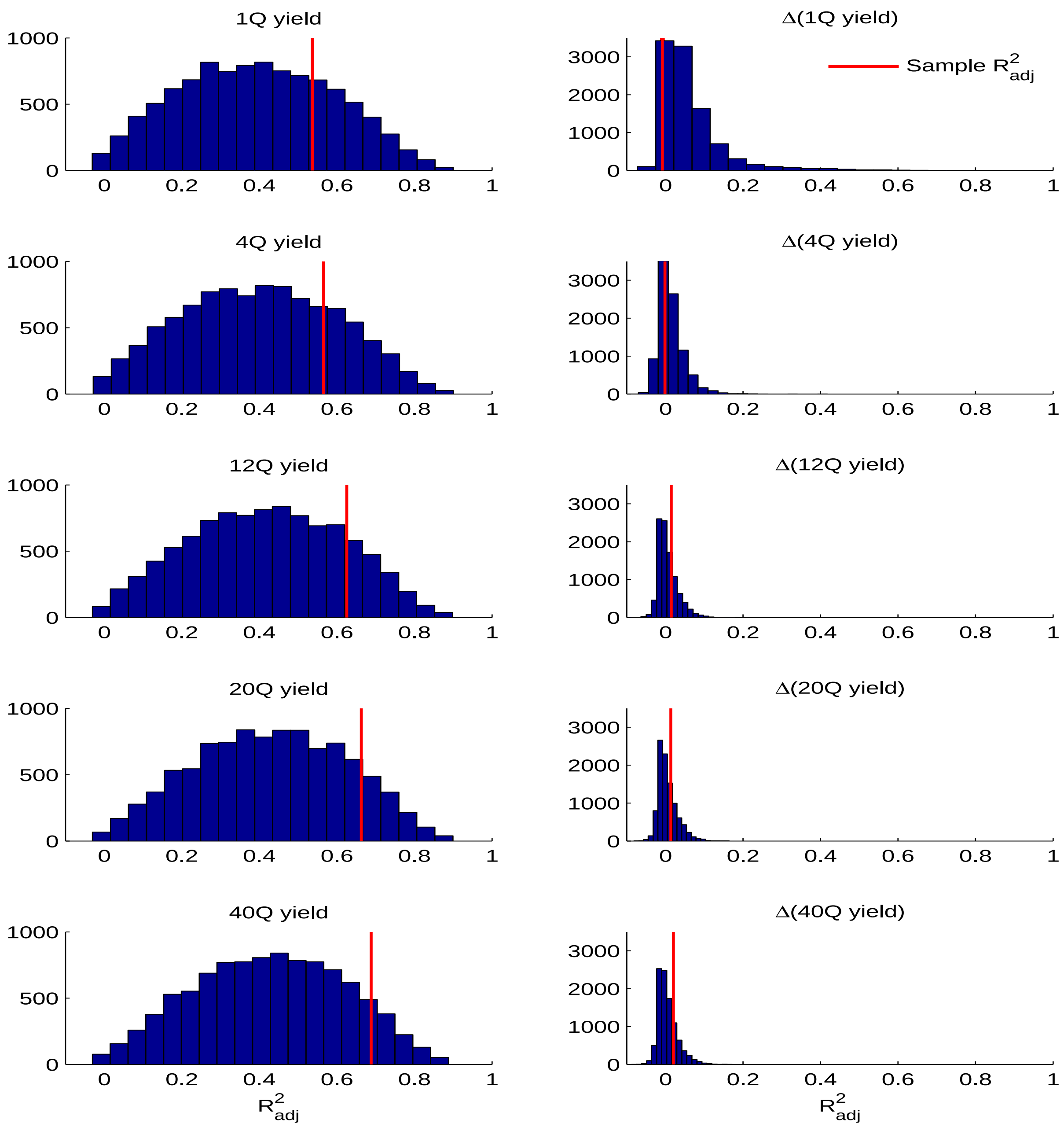

Figure 6: **Linear Spanning Regressions.** For each yield maturity n, the figure shows the adjusted R^2 coefficients for the OLS regressions:

$$\text{Left panels}: \quad y_t^n = \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n$$

$$\text{Right panels}: \quad \Delta y_t^n = \beta^c \Delta\pi_t^c + \beta^f \Delta\pi_t^f + \beta^e \Delta\pi_t^e + \varepsilon_t^n\ .$$

The vertical red line marks the adjusted R^2 coefficient for the regressions of quarterly yields against core, food, and energy inflation data from 1985Q1 to 2009Q4. The histograms show the distribution of the adjusted R^2 coefficients for regressions estimated on 10,000 samples of quarterly yields and inflation series simulated from the preferred $DTSM_{3,3}$ using the scheme described in Section A.2 of the Online Appendix. The results in the left panels are for regressions in levels; those in the right panels are for regressions in changes.

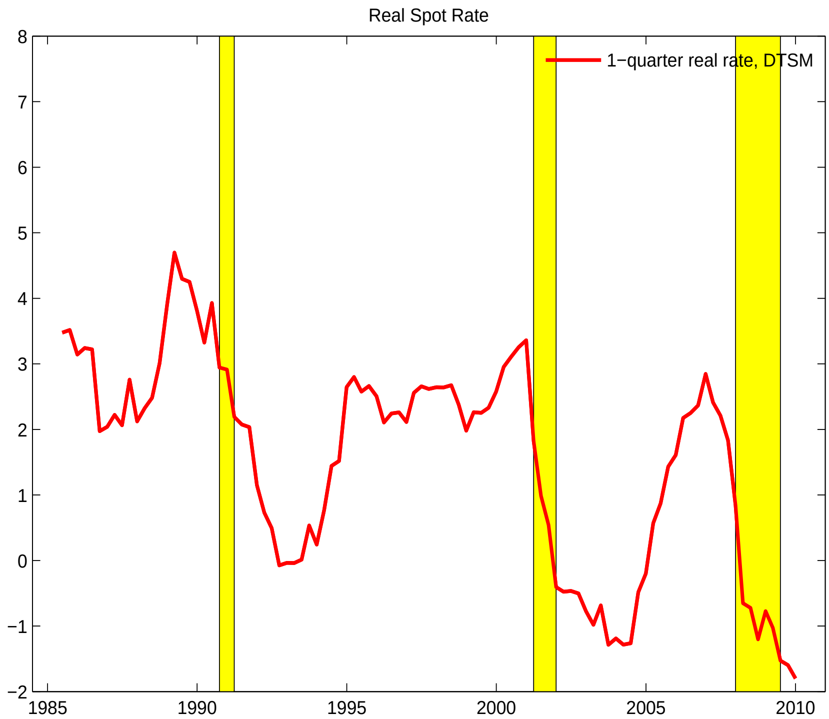

Figure 7: **Real Spot Rate.** The plot shows the model-implied real one-quarter rate computed using a $DTSM^*_{3,3}$ in which level is the only priced risk factor. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. The sample period is 1985Q1-2009Q4. The shading corresponds to NBER recessions.

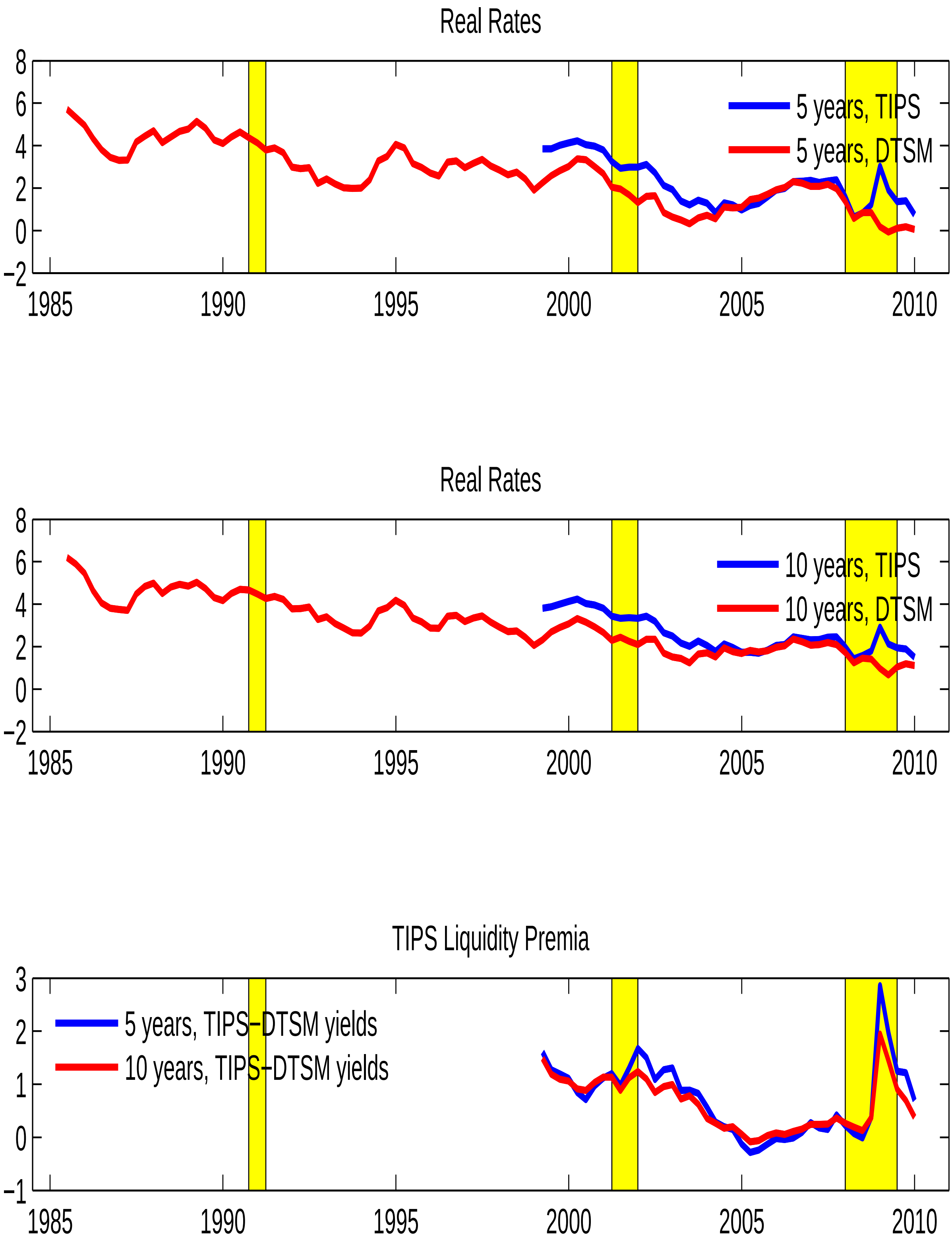

Figure 8: **Real Rates: TIPS vs. DTSM.** The plots contrast model-implied and TIPS real rates with five and ten years maturity. We compute real rates using a DTSM$^*_{3,3}$ in which level is the only priced risk factor. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. The sample period is 1985Q1-2009Q4. The shading corresponds to NBER recessions.

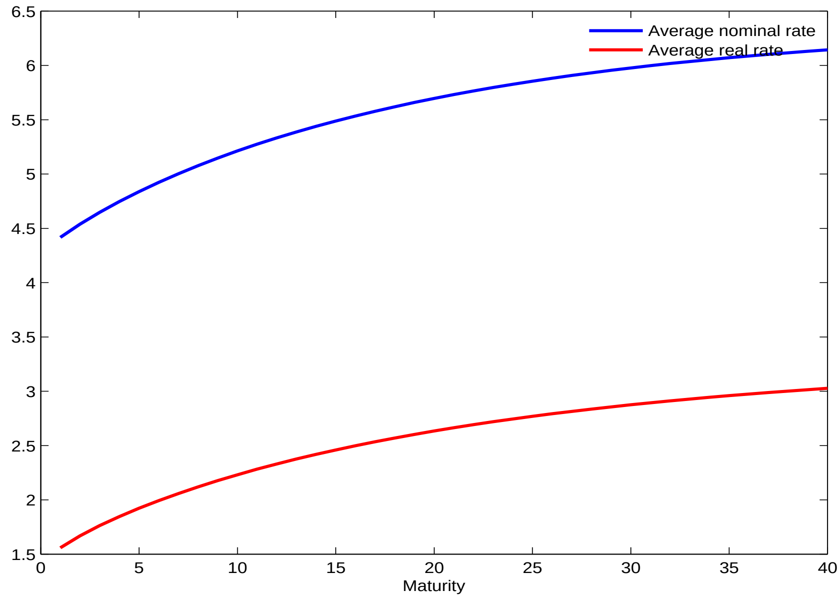

Figure 9: **Nominal and Real Term Structures.** The plots shows the nominal and real term structures computed using a $\text{DTSM}^*_{3,3}$ in which level is the only priced risk factor. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. The sample period is 1985Q1-2009Q4.

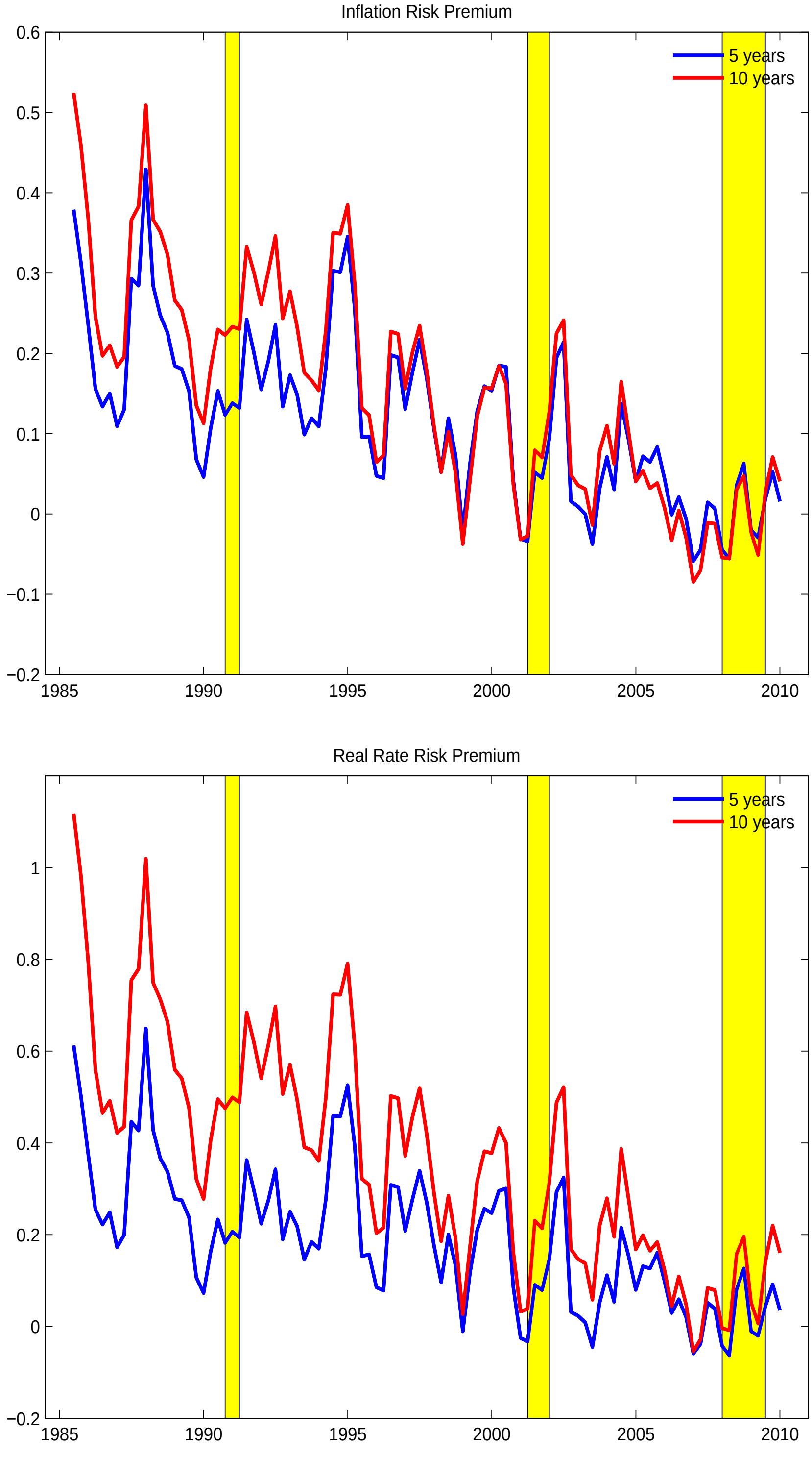

Figure 10: **Inflation and Real Rate Risk Premium.** The plots depict the five- and ten-year inflation and real rate risk premia implied by a $DTSM^{*}_{3,3}$ in which level is the only priced risk factor. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. The sample period is 1985Q1-2009Q4. The shading corresponds to NBER recessions.

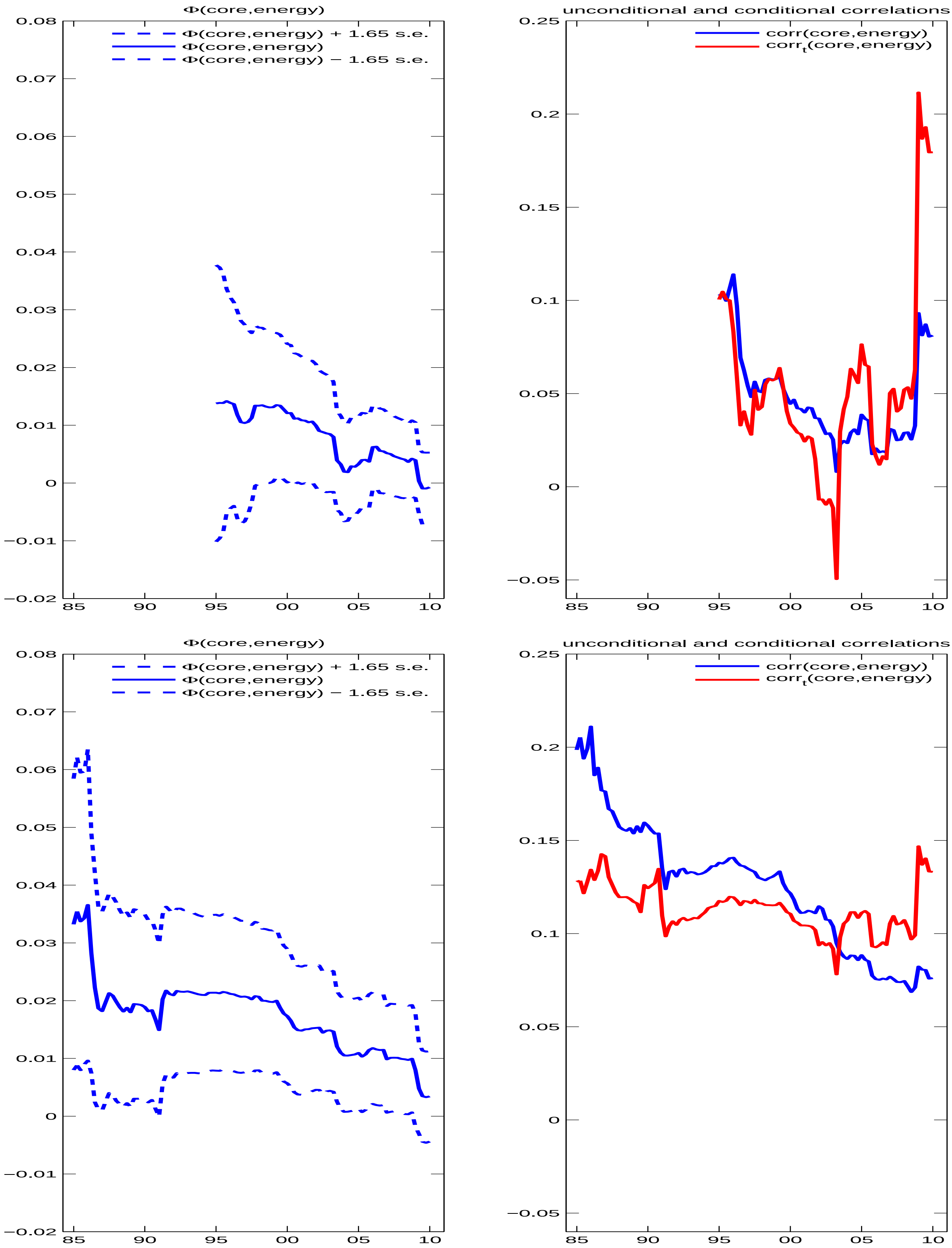

Figure 11: **Energy Pass Through.** The plots depict measures of correlation between energy and core inflation for a $\mathrm{DTSM}_{3,3}$. We estimate the model on the first four principal components of CMT yields with maturities up to ten years and CPI inflation data. In the top panels, the sample period starts in 1985Q1 and the end date ranges from 1995Q1 to 2009Q4. In the bottom panels, the sample period starts in 1962Q1 and the end date ranges from 1985Q1 to 2009Q4. The left panels show the estimate for the $\phi_1^{\pi^c,\pi^e}$ coefficient and its 90% confidence bands. The right panels depict conditional and unconditional correlations between core and energy inflation.

Table 1: **Summary Data Statistics.** The table reports summary statistics for CPI and PCE inflation series on core, food, energy and total consumer price indices; as well as CPI and PCE measures of relative importance weights for the core, food, and energy price indices. CPI- and PCE-weighted are the total inflation series computed from their core, food, and energy components using the relative importance weights. Panel A focuses on the 1962Q1-2009Q4 sample period; Panel B uses data from 1985Q1 to 2009Q4.

	Central moments				Autocorrelations		
	Mean	Std. Dev.	Skewness	Kurtosis	AC(1)	AC(2)	AC(3)
Panel A: Long Sample Period: 1962Q1-2009Q4							
CPI	4.12	3.11	0.65	6.01	0.75	0.64	0.66
CPI-weighted	4.15	3.08	0.82	5.50	0.76	0.66	0.67
CPI-core	4.08	2.67	1.59	5.79	0.86	0.80	0.77
CPI-food	4.11	3.81	1.76	7.10	0.63	0.46	0.49
CPI-energy	4.63	15.99	-1.69	15.63	0.29	-0.01	0.05
Weight-core	0.74	0.04	-0.26	1.73	0.98	0.95	0.92
Weight-food	0.19	0.04	0.69	2.30	0.98	0.95	0.92
Weight-energy	0.08	0.02	1.07	3.04	0.98	0.94	0.92
PCE	3.68	2.57	0.87	4.82	0.83	0.75	0.73
PCE-weighted	3.69	2.55	0.95	4.67	0.84	0.76	0.74
PCE-core	3.61	2.18	1.07	3.71	0.92	0.88	0.84
PCE-food	3.61	4.02	1.48	5.94	0.55	0.40	0.45
PCE-energy	4.68	16.51	-1.72	16.49	0.29	-0.02	0.03
Weight-core	0.82	0.04	0.02	1.36	0.99	0.98	0.98
Weight-food	0.12	0.03	0.18	1.56	0.99	0.98	0.98
Weight-energy	0.06	0.01	0.40	2.62	0.99	0.97	0.95
Panel B: Short sample period: 1985Q1-2009Q4							
CPI	2.89	2.04	-2.43	15.96	0.26	0.03	0.10
CPI-weighted	2.93	1.94	-1.92	12.63	0.27	0.03	0.12
CPI-core	2.91	1.09	0.57	2.89	0.82	0.83	0.78
CPI-food	2.95	1.90	0.74	4.64	0.46	0.34	0.23
CPI-energy	2.89	19.55	-1.88	12.11	0.19	-0.13	-0.06
Weight-core	0.77	0.02	-2.33	8.35	0.91	0.79	0.68
Weight-food	0.15	0.01	0.91	4.02	0.95	0.89	0.83
Weight-energy	0.08	0.01	1.35	4.14	0.90	0.76	0.68
PCE	2.53	1.55	-1.61	10.86	0.41	0.17	0.20
PCE-weighted	2.55	1.51	-1.31	9.07	0.42	0.18	0.22
PCE-core	2.53	1.05	0.64	2.61	0.80	0.74	0.72
PCE-food	2.46	2.22	0.49	3.92	0.39	0.29	0.10
PCE-energy	3.08	20.30	-1.89	12.45	0.19	-0.12	-0.06
Weight-core	0.86	0.02	-0.76	2.73	0.96	0.92	0.88
Weight-food	0.09	0.01	0.51	1.84	0.98	0.96	0.95
Weight-energy	0.05	0.01	0.58	3.54	0.90	0.79	0.70

Table 2: **Expected inflation and real rates.** The top panel reports 2009 CPI forecasts computed using data through 2008Q4. In the last two columns are the annualized 2008Q4 and the 2009 CPI growth rates. The bottom panel reports the five-year real yield computed with data through 2008Q4. In the last two columns are the 2008Q4 five-year TIPS rate and the difference between the five-year nominal yield and the 2008 CPI growth rate. Sections 4 and 5.1 explain the different model specifications.

	2009 CPI forecasts computed as of 2008Q4				Realized	
	$DTSM_{2,1}$		$DTSM_{2,3}$	$DTSM_{3,3}$	2008Q4*	2009
	total	core				
CPI	-5.67		0.14	1.32	-9.64	1.45
CPI-core		-0.25	-0.58	1.08	0.61	1.72
CPI-food			4.20	3.25	4.14	-0.59
CPI-energy			-0.28	0.25	-106.52	2.38
	5-year real yield computed as of 2008Q4				TIPS	5Y nom. yield minus 2008 infl.
	$DTSM_{2,1}$		$DTSM_{2,3}$	$DTSM_{3,3}$		
	total	core				
5Y real yield	2.94	1.33	0.42	0.34	2.73	0.56

* The 2008Q4 rates are annualized

Table 3: **Forecasts of Annual Inflation Series: Post-1984 Estimation**. We use data from 1985Q1 for model estimation and forecast annual inflation out of sample over 1998Q4 to 2008Q4. Section 4 explains the different model specifications. For each model, the table shows the p-value for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

Panel A: Estimation on CPI data

	CPI		CPI-core		CPI-food		CPI-energy	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models								
ARMA	1.61		0.51		1.54		14.25	
ARMA_W	1.52	0.31						
RW	1.92	0.98	0.51	0.71	1.82	0.94	19.69	0.98
Survey forecasts								
U. of M.	1.72	0.75						
SPF	1.38	0.16						
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.36	0.17	0.40	0.00	1.38	0.19	14.25	0.52
$\text{DTSM}_{4,3}$ AR(1-1-1)	1.39	0.17	0.43	0.02	1.38	0.18	14.23	0.42
$\text{DTSM}^*_{3,3}$ AR(1-1-1)	1.37	0.17	0.43	0.06	1.38	0.19	14.26	0.53
$\text{DTSM}_{3,2}$	1.59	0.48	0.44	0.07				
$\text{DTSM}^*_{3,2}$	1.63	0.53	0.49	0.43				
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.37	0.19	0.40	0.00	1.37	0.19	14.25	0.49

Table 3, continued

Panel B: Estimation on PCE data

	PCE		PCE-core		PCE-food		PCE-energy	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models								
ARMA	1.27		0.43		1.91		15.07	
ARMA_W	1.12	0.14						
RW	1.42	0.97	0.41	0.24	2.27	0.90	20.67	0.98
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.10	0.19	0.47	0.82	1.70	0.18	15.10	0.59
$\text{DTSM}_{4,3}$ AR(1-1-1)	1.10	0.15	0.42	0.35	1.70	0.18	15.07	0.50
$\text{DTSM}^*_{3,3}$ AR(1-1-1)	1.09	0.15	0.49	0.91	1.70	0.18	15.11	0.59
$\text{DTSM}_{3,2}$	1.21	0.39	0.49	0.82				
$\text{DTSM}^*_{3,2}$	1.22	0.41	0.52	0.86				
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.11	0.18	0.50	0.94	1.69	0.18	15.09	0.57

Table 4: **Forecasts of Annual Inflation Series**. We estimate each model using yields data starting from 1962Q1 and forecast annual inflation over the 1985Q4-2008Q4 and 1998Q4-2008Q4 out-of-sample periods. Section 4 explains the different model specifications. For each model, the table shows the p-value for a test of equal forecast accuracy (West (1996)) computed under the null that the RMSE for that model equals the RMSE for the ARMA(1,1), when the alternative is that the RMSE for the ARMA(1,1) exceeds the RMSE for that model.

Panel A: 1985Q4 to 2008Q4 out-of-sample period

	CPI		CPI-core		CPI-food		CPI-energy	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models								
ARMA	1.63		0.71		1.59		11.29	
ARMA_W	1.40	0.05						
RW	1.52	0.08	0.52	0.00	1.64	0.68	14.98	1.00
Survey forecasts								
U. of M.	1.36	0.01						
SPF	1.11	0.03						
DTSMs, CRSP yields, Chen-Scott estimation								
$\text{DTSM}_{2,1}$ tot	1.69	0.65						
$\text{DTSM}_{2,1}$ core			0.84	0.94				
$\text{DTSM}_{2,3}$ AR(1-1-1)	1.51	0.32	0.99	1.00	1.78	0.99	11.25	0.47
$\text{DTSM}_{2,3}$ AR(3-1-1)	1.40	0.17	0.84	0.94	1.78	0.99	10.91	0.13
DTSMs, CRSP yields, Kalman Filter estimation on yields and their PCs								
$\text{DTSM}_{2,3}$ AR(3-1-1) yields	1.53	0.25	1.00	1.00	1.77	0.98	10.84	0.06
$\text{DTSM}_{2,3}$ AR(3-1-1) PCs	1.39	0.09	0.79	0.87	1.76	0.99	10.91	0.10
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.36	0.09	0.67	0.16	1.73	0.98	10.79	0.07
DTSMs, CMT yields $\leq$ 20Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.44	0.13	0.91	0.98	1.75	0.99	10.84	0.07
$\text{DTSM}_{4,3}$ AR(1-1-1)	1.35	0.06	0.73	0.62	1.77	0.99	11.00	0.18
$\text{DTSM}_{5,3}$ AR(1-1-1)	1.48	0.23	0.99	1.00	1.78	0.99	11.03	0.21

Table 4, continued

Panel B: 1998Q4 to 2008Q4 out-of-sample period

	CPI		CPI-core		CPI-food		CPI-energy	
	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.	RMSE	p-val.
Univariate models								
ARMA	2.05		0.69		1.70		14.54	
ARMA_W	1.64	0.03						
RW	1.92	0.13	0.51	0.01	1.82	0.79	19.69	1.00
Survey forecasts								
U. of M.	1.72	0.00						
SPF	1.38	0.07						
DTSMs, CRSP yields, Chen-Scott estimation								
$\text{DTSM}_{2,1}$ tot	2.15	0.65						
$\text{DTSM}_{2,1}$ core			0.90	0.90				
$\text{DTSM}_{2,3}$ AR(1-1-1)	1.57	0.14	1.02	0.98	1.77	0.72	13.81	0.10
$\text{DTSM}_{2,3}$ AR(3-1-1)	1.53	0.10	0.87	0.90	1.78	0.72	13.86	0.10
DTSMs, CRSP yields, Kalman Filter estimation on yields and their PCs								
$\text{DTSM}_{2,3}$ AR(3-1-1) yields	1.75	0.11	1.15	1.00	1.77	0.71	13.93	0.10
$\text{DTSM}_{2,3}$ AR(3-1-1) PCs	1.64	0.07	0.92	1.00	1.76	0.71	13.92	0.10
DTSMs, CMT yields $\leq$ 10Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.53	0.06	0.67	0.37	1.74	0.66	13.86	0.11
DTSMs, CMT yields $\leq$ 20Y, Kalman Filter estimation on the yields' PCs								
$\text{DTSM}_{3,3}$ AR(1-1-1)	1.70	0.10	1.08	1.00	1.76	0.70	13.88	0.10
$\text{DTSM}_{4,3}$ AR(1-1-1)	1.60	0.07	0.86	0.90	1.77	0.72	13.95	0.12
$\text{DTSM}_{5,3}$ AR(1-1-1)	1.62	0.10	0.98	0.95	1.78	0.73	13.96	0.12

Table 5: **Variance Decomposition**. We decompose the proportion of inflation and term structure movements attributable to core, food, energy inflation observable shocks, and other latent variables. Sections 4 and 5.1 explain the different model specifications. In Panel A, we decompose the variance of core inflation, except for the first $DTSM_{2,1}$ case, for which the decomposition applies to total inflation. Panels B-D show a similar decomposition for the variance of the one-quarter, five- and ten-year yield.

	$DTSM_{2,1}$ total			$DTSM_{2,1}$ core			$DTSM_{2,3}$			$DTSM_{3,3}$		
Horizon:	4	20	∞	4	20	∞	4	20	∞	4	20	∞
Panel A: CPI												
ℓ_1	2.74	21.94	35.63	2.91	31.13	49.73	4.06	35.55	53.79	11.14	47.02	67.70
ℓ_2	13.93	26.63	21.99	12.76	24.55	18.01	11.75	20.09	14.48	1.00	11.18	7.30
ℓ_3										5.38	3.30	1.98
CPI	83.33	51.43	42.38									
CPI-core				84.33	44.31	32.26	83.52	43.94	31.43	82.43	38.47	23.01
CPI-food							0.01	0.00	0.00	0.00	0.00	0.00
CPI-energy							0.66	0.41	0.29	0.06	0.03	0.02
Panel B: 1-quarter yield												
ℓ_1	47.09	72.19	81.34	38.65	67.76	77.91	38.20	67.78	78.01	18.95	43.39	67.60
ℓ_2	51.73	27.30	18.32	54.11	28.61	19.60	54.37	28.46	19.42	60.79	51.08	29.36
ℓ_3										19.79	5.41	2.97
CPI	1.18	0.51	0.34									
CPI-core				7.24	3.63	2.49	6.94	3.56	2.43	0.24	0.06	0.03
CPI-food							0.40	0.15	0.10	0.16	0.04	0.02
CPI-energy							0.09	0.05	0.04	0.07	0.02	0.01

Table 5, continued

	$DTSM_{2,1}$ total			$DTSM_{2,1}$ core			$DTSM_{2,3}$			$DTSM_{3,3}$		
Horizon:	4	20	∞	4	20	∞	4	20	∞	4	20	∞
Panel C: 5-year yield												
ℓ_1	90.15	96.30	97.74	87.56	95.20	96.93	86.87	94.82	96.65	54.54	75.12	86.22
ℓ_2	9.78	3.67	2.24	11.19	4.33	2.77	11.69	4.63	2.99	44.75	24.68	13.68
ℓ_3										0.70	0.19	0.10
CPI	0.07	0.02	0.01									
CPI-core				1.26	0.47	0.30	1.41	0.54	0.35	0.00	0.00	0.00
CPI-food							0.01	0.00	0.00	0.00	0.00	0.00
CPI-energy							0.02	0.01	0.01	0.00	0.00	0.00
Panel D: 10-year yield												
ℓ_1										78.44	88.77	93.71
ℓ_2										21.32	11.16	6.26
ℓ_3										0.24	0.06	0.03
CPI												
CPI-core										0.00	0.00	0.00
CPI-food										0.00	0.00	0.00
CPI-energy										0.00	0.00	0.00

Table 6: **Linear Spanning Regressions.** For each yield maturity n, the table shows results for the OLS regressions:

$$\begin{aligned} \text{Levels}: \quad y_t^n &= \alpha + \beta^c \pi_t^c + \beta^f \pi_t^f + \beta^e \pi_t^e + \varepsilon_t^n \\ \text{Changes}: \quad \Delta y_t^n &= \beta^c \Delta\pi_t^c + \beta^f \Delta\pi_t^f + \beta^e \Delta\pi_t^e + \varepsilon_t^n \,. \end{aligned}$$

We first estimate the regressions for each yield series y_t^n with maturity n equal to one quarter, one, three, five, and ten years against core, food, and energy inflation sampled quarterly from 1985Q1 to 2009Q4. For each regression, we report coefficient estimates and Newey-West heteroskedasticity and autocorrelation robust standard errors (in brackets). Next, we simulate 10,000 samples of quarterly yields with the same maturities as well as core, food, and energy inflation series from the preferred DTSM$_{3,3}$ using the scheme described in Section A.2 of the Online Appendix. We estimate the same regressions on each simulated sample and report mean, 5th, 50th, and 95th percentiles of the estimated coefficients. The results in the left-hand side of the table are for regressions in levels (the table omits the estimate of the intercept α); those in the right-hand side are for regressions in changes.

	Data		Model				Δ Data		Δ Model			
			Mean	5%	50%	95%			Mean	5%	50%	95%
Panel A: One-quarter yield												
β^c	1.41	(0.23)	1.37	0.48	1.36	2.25	0.05	(0.05)	-0.10	-0.26	-0.10	0.06
β^f	0.13	(0.12)	-0.06	-0.26	-0.06	0.14	0.00	(0.02)	-0.02	-0.08	-0.02	0.04
β^e	0.00	(0.01)	-0.01	-0.02	-0.01	0.01	0.00	(0.00)	0.00	-0.01	-0.00	0.01
R^2_{adj}	53.66%		39.30%	7.72%	39.02%	71.85%	-0.81%		6.17%	-1.50%	3.92%	21.58%
Panel B: One-year yield												
β^c	1.46	(0.23)	1.34	0.49	1.34	2.18	0.12	(0.05)	-0.04	-0.19	-0.04	0.11
β^f	0.12	(0.11)	-0.05	-0.24	-0.05	0.14	0.00	(0.02)	-0.01	-0.06	-0.01	0.04
β^e	-0.00	(0.01)	-0.01	-0.02	-0.01	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.01
R^2_{adj}	56.56%		40.01%	7.78%	39.90%	72.45%	-0.16%		1.20%	-2.22%	0.47%	7.18%

Table 6, continued

	Data		Model				Δ Data		Δ Model			
			Mean	5%	50%	95%			Mean	5%	50%	95%
Panel C: Three-year yield												
β^c	1.55	(0.22)	1.24	0.48	1.24	1.98	0.14	(0.06)	-0.01	-0.15	-0.02	0.13
β^f	0.03	(0.10)	-0.04	-0.21	-0.04	0.13	-0.00	(0.02)	-0.00	-0.05	-0.00	0.05
β^e	-0.00	(0.01)	-0.01	-0.02	-0.01	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.00
R^2_{adj}	62.56%		41.68%	9.41%	41.71%	73.44%	1.49%		0.59%	-2.32%	0.00%	5.62%
Panel D: Five-year yield												
β^c	1.52	(0.20)	1.16	0.46	1.16	1.84	0.13	(0.06)	-0.01	-0.14	-0.01	0.12
β^f	-0.02	(0.09)	-0.04	-0.19	-0.04	0.12	-0.00	(0.02)	-0.00	-0.05	-0.00	0.04
β^e	-0.00	(0.00)	-0.01	-0.02	-0.01	0.01	0.00	(0.00)	0.00	-0.00	0.00	0.00
R^2_{adj}	66.28%		42.80%	10.60%	42.96%	73.97%	1.40%		0.55%	-2.35%	-0.04%	5.45%
Panel E: Ten-year yield												
β^c	1.45	(0.19)	0.99	0.39	0.99	1.59	0.13	(0.06)	-0.01	-0.12	-0.01	0.11
β^f	-0.05	(0.08)	-0.03	-0.16	-0.03	0.10	0.00	(0.02)	-0.00	-0.04	-0.00	0.04
β^e	-0.00	(0.00)	-0.00	-0.02	-0.00	0.00	0.00	(0.00)	-0.00	-0.00	-0.00	0.00
R^2_{adj}	68.86%		42.60%	10.73%	42.85%	73.48%	2.04%		0.56%	-2.40%	-0.01%	5.69%

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