MECHANISM DESIGN WITH LIMITED COMMUNICATION: Implications for Decentralization[1]

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Abstract

We develop a theory of mechanism design in a principal-multiagent setting with private information, where communication involves costly delay. The need to make production decisions within a time deadline prevents agents from communicating their entire private information to the principal, rendering revelation mechanisms infeasible. Feasible communication protocols allow only finite number of possible messages sent in a finite number of stages. An extension of the 'Revenue Equivalence Theorem' is obtained, and used to show that an optimal production allocation can be computed by maximizing virtual profits of the Principal subject to communication constraints alone. In this setting decentralization of production decisions is (strictly) valuable, while decentralizing authority to contract is not.

KEYWORDS: communication, mechanism design, decentralization, incentives, principal-agent, organizations

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1 Introduction

This paper develops a theory of mechanism design with limitations on the capacity of agents and Principal to communicate, which prevent implementation of 'complete' contracts or revelation mechanisms. In such mechanisms agents simultaneously send a report of their entire private information to the Principal, instead of communicating directly with one another. Moreover, they are not delegated authority over (verifiable) production decisions: they await instructions from the Principal on what to do (see, e.g., Myerson (1982)). In contrast 'real' organizations involve substantial decentralization of decision-making and direct, interactive communication among agents. The aim of this paper is to explore circumstances where this can be explained by limits on communication abilities on otherwise strategic and rational organization members.

Limitations on communication can be justified by the fact that reading and writing messages are time-consuming tasks. If delays in decision-making are costly, organizations will be forced to restrict communication systems to those that do not allow 'full' revelation of private information held by agents. Even after the exchange of information, agents will still know more about their respective environments than the Principal or any central coordination device. It may then make sense to delegate some decisions to agents. Such ideas can be traced back to Hayek (1945) as well as the 'message space' literature (Hurwicz (1960, 1972), Mount and Reiter (1974), Segal (2006)).

If all agents in the organization share the same goals as the Principal, as assumed in the theory of teams developed by Marschak and Radner (1972), such communicational limitations typically justify decentralized decision-making, as they permit production decisions to be based on better information. In the presence of incentive considerations where agents pursue self-interested goals, this is no longer clear: decentralization may encourage agents to pursue their own agendas at the same time that it brings to bear better information on decisions. There is a need to trade off the potential 'control loss' or 'abuse of power' against the 'flexibility' advantages of delegation.

This paper explores this trade-off in the context of a standard two agent production model with independent private costs. Each agent produces a one-dimensional real-valued input at a real-valued unit cost which it is privately informed about. The inputs of the two agents have to be coordinated to jointly produce a benefit for the Principal. Agents are provided transfers by the Principal to provide them with suitable incentives to report their costs and produce inputs. We pose the problem of design of a mechanism by the Principal to maximize her expected net benefit, subject to communicational, incentive and participation constraints. In particular, agents will have at most a finite number of communicational strategies available, which will be insufficient to communicate their (real-valued) private information.

The representation of communicational constraints poses a significant challenge: how should communicational complexity be measured, and how can this be incorporated in mechanism design theory? How do we trade-off benefits of greater communication with attendant incentive problems?

The classical message-space literature (Hurwicz (1960, 1972), Mount and Reiter (1974)) posed the question of minimal communicational complexity (measured by the dimensionality of message space sizes in iterative communication protocols) required to implement given resource allocations. They abstracted from incentive considerations. More recent literature has examined the effect of using different measures of communicational complexity (e.g., which incorporate both message space size and number of rounds of communication), and how the minimal communicational complexity required to implement given allocations increase in the presence of incentive problems (Reichelstein and Reiter (1988), Segal (2006), Fadel and Segal (2006), van Zandt (2006)).

An alternative approach is to fix a limited communication protocol and ask what is the constrained optimal allocation mechanism. The Marschak-Radner theory of teams can be viewed as belonging to this approach, in the absence of incentive considerations. Some authors have attempted to extend this to contexts with incentive problems (Green and Laffont (1986, 1987), Melumad, Mookherjee and Reichelstein (MMR) (1992, 1997), Laffont and Martimort (1998)).[3] This latter literature, however, typically imposes severe restrictions on communication protocols. For instance, MMR assume only one round of communication, and message sets of given (finite) size. They also restrict attention to comparison across two specific mechanisms: one where contracting, communication and production decisions are all centralized, with another where they are all decentralized.

This paper extends the MMR approach to accommodate a wider range of both communication protocols and mechanisms. We allow an arbitrary (but finite) number of communication rounds and message space sizes at each round, and arbitrary communication network structures (i.e., who communicates with whom). We do not restrict the class of mechanisms either. In particular, 'mixed' mechanisms are allowed, where different components of the mechanism: contracting, communication and production decisions can be independently centralized or decentralized.

We show that despite the lack of much structure imposed on the communication protocols, some general lessons for mechanism design emerge:

(i) Decentralized contracting cannot dominate centralized contracting, if the communicational requirements of the two contracting modes are constrained to be similar. This result implies that some previous papers (e.g., MMR (1992)) arguing for the

[3] Deneckere and Severinov (2003) develop a theory of mechanism design where truthful messages can be sent costlessly, while non-truthful messages may entail some cost.

superiority of decentralized contracting were based on a formulation that implicitly allowed greater communication to the decentralized mode.

(ii) The mechanism design problem can be simplified in a way analogous to methods used for auction design or related problems with quasilinear utilities in the absence of communicational constraints, which rely on a version of the 'Revenue Equivalence' theorem in auction theory. Under the standard assumption of monotone hazard rates for the cost distributions, the problem can be expressed in terms of selecting a production allocation rule to maximize 'virtual' expected profit of the Principal, subject to communicational feasibility restrictions alone. This implies a convenient separation of the incentive and communicational aspects of the design: the former are entirely incorporated in the objective function (in the form of incentive rents of the agents which cause actual costs to be inflated to virtual costs). Communicational algorithms to maximize virtual profit can be devised without worrying about incentive compatibility of the chosen communication strategies.

(iii) Decentralizing production decisions to agents is superior to any system where they are set production targets by others. This is a consequence of the characterization of optimal production allocations. More generally, greater 'flexibility' of decision-making and information flows across agents is desirable. Communication protocols should be designed to maximize the flow of information across agents; this will also typically necessitate direct, iterative communication between agents.

An implication of this theory for organization design is the importance of "mixed" mechanisms in the presence of communication constraints. In particular, the results indicate the value of mechanisms that combine centralized contracting with decentralized decision-making and communication networks. Such mechanisms have been emphasized as distinctive features of 'Japanese' firms (see, e.g., Aoki (1990)).

Our analysis is based on a number of assumptions which are worth mentioning at this stage. We abstract from computational complexity considerations, studied by Radner (1992, 1993), Mount and Reiter (1995) van Zandt (1996,1999, 2003a,b), and van Zandt and Radner (2001). In particular, agents are assumed to have unlimited computational ability; the only restriction is on time they require to read and write messages. It is possible that decentralized contracting may prove superior to centralized contracting if computational abilities of the Principal are limited.

In addition, we abstract from problems of collusion among agents: there are no unobserved side contracts or communication among them. The Principal is assumed to be able to commit to any chosen mechanism. Finally, communication does not entail any noise or errors. The consequences of relaxing these assumptions need to be investigated in future research.

The paper is organized as follows. Section 2 introduces the model, describing both centralized and decentralized contracting. Section 3 shows that decentralized contracting cannot achieve superior outcomes than centralized contracting. Section 4 provides a characterization of optimal allocations attainable via centralized contracts. Section 5 shows that an optimal mechanism must decentralize production decisions, and Section 6 discusses implications for design of communication networks. Finally, Section 7 concludes.

2 Model

There is a Principal (P) and two agents 1 and 2. Agent $i = 1, 2$ produces a one-dimensional nonnegative real valued input q_i at cost $\theta_i q_i$, where θ_i is a real-valued parameter distributed over an interval $\Theta_i \equiv [\underline{\theta}_i, \bar{\theta}_i]$ according to a positive, continuously differentiable density function f_i and associated c.d.f. F_i. The distribution satisfies the standard monotone hazard condition that $\frac{F_i(\theta_i)}{f_i(\theta_i)}$ is nondecreasing, implying that the 'virtual cost' $v_i(\theta_i) \equiv \theta_i + \frac{F_i(\theta_i)}{f_i(\theta_i)}$ is strictly increasing. θ_1 and θ_2 are independently distributed, and these distributions F_1, F_2 are common knowledge among the three players.

The inputs of the two agents combine to produce a gross return $V(q_1, q_2)$ for P, whose net payoff is $V - t_1 - t_2$, where t_i denotes a transfer from P to i. The payoff of i is $t_i - \theta_i q_i$. All are risk-neutral and have autarkic payoffs of 0. We shall assume that V is twice continuously differentiable, strictly concave and satisfies Inada conditions. So the inputs of both agents are essential in production. We shall also assume that the production function is non-separable: $V_{12} \neq 0$ for all q_1, q_2. This creates a need to coordinate production assignments between the agents.

2.1 Centralized Contracting

2.1.1 Timing

In centralized contracting, P selects a *communication protocol* (explained further below) at $t = -1$, and offers contracts to both agents. There is enough time between $t = -1$ and $t = 0$ for agents to read the offered contracts. We abstract from the time needed to read or negotiate contracts at the *ex ante* stage and focus entirely on communication delays arising from the *interim* state onwards. P commits to the communication protocol and the contracts. The agents do not commit to their participation decisions until after they observe their private information.

At $t = 0$, each agent i privately observes the realization of θ_i, and independently decides whether to participate or opt out of the mechanism. If either agent opts out the game ends; otherwise they enter the planning or communication phase which lasts until $t = T$. We treat the deadline T as given; it can be chosen subsequently by the organization designer to trade off the cost of delayed production with the benefit of added communication. Our results do not depend on the specific deadline chosen.

At $t = T$, agent i selects production level q_i. This does not necessarily mean that i 'decides' q_i in any meaningful sense. As discussed further below, someone else may set a target for q_i — this could be a message sent to i by the target-setter during the communication phase — and the incentive scheme for i may effectively force i to meet this target. Finally payments are made according to the contracts signed.

The timeline for centralized contracting is depicted below.

Timeline: Centralized Contract

$t = -1$ P chooses $p \in \mathcal{P}$ and offers $t_i(q_i, q_j, h_i)$ to A_i

$t = 0$ θ_i is realized, and A_i decides whether to participate or not.

$t = 1$

$t = 2$ Communication phase: For each t, agents chooses $m_{ijt} \in \mathcal{M}_{ijt}(h_{it})$

$t = T$ Agent i selects q_i.

Payments are made according to the contracts.

2.1.2 Communication Protocol

A communication protocol is a dynamic process of exchange of messages among agents and the principal. We shall subsequently refer to the two agents $1, 2$ and the principal P as members of the communication network. The protocol is represented by a set of dates $t = 0, 1, \ldots, T$ at which messages are exchanged, with the time interval between successive dates taken up by reading (of messages recently received) and writing (of new messages to be sent at the next date). The protocol specifies a message space $\mathcal{M}_{ijt}(h_{i,t-1})$ for messages that can be sent by each member i to every other member j on the network at any date t, following a history of messages $h_{i,t-1}$ exchanged by i until $t-1$.

Message histories are generated recursively as follows. For member i with history $h_{i,t-1}$ until date $t-1$, h_{it} is generated by adding the messages sent and received by i to and from other members at t. We assume absence of communication noise or errors, so messages sent are received without fail or distortion.

The specification of protocols does not require all agents to simultaneously communicate with one another. Alternating senders and receivers at different dates can be accommodated by assigning null message spaces to receivers. It can incorporate different network structures: if i is not allowed to communicate with j at t, then $\mathcal{M}_{ijt}(h_{i,t-1})$ is empty.

A *communication plan* $c_i(h_{it})$ for i specifies a message selected from $\mathcal{M}_{ijt}(h_{it})$ to be sent to j at t, following history h_{it}. Throughout, we restrict attention to pure strategies.

The message space of each agent is finite at every date, as reading and writing messages are time-consuming tasks. Hence the set of possible communication plans $\mathcal{C}_i$ for every member i is finite. The 'technology' of reading and writing messages is incorporated in the specification of feasible communication plans relative to any given deadline T.

Formally, a feasible communication protocol p is represented by a finite set of communication plans $\mathcal{C} \equiv (\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_P)$, and a history $h_{it}(c)$ of messages exchanged by i until any date t as a function of a tuple of communication plans $c \equiv (c_1, c_2, c_P) \in \mathcal{C}$.

Let $\mathcal{P}$ denote the set of feasible protocols. Also we shall use h_i to denote h_{iT}, the message history of i at the deadline T. And H_{it} will denote $\{\tilde{h}_{it} | \exists c \in \mathcal{C} : \tilde{h}_{it} = h_{it}(c)\}$, the set of all date t histories that could be generated by the protocol.

We shall assume that the message histories h_i for each member, and the input produced by each agent are verifiable by the Principal. Such verification of messages and productions may be time-consuming, but are not subject to any time deadlines. The Principal can audit realized outputs and message histories after production has occurred. Hence transfers can be conditioned on outputs and message histories.

It is also assumed that reading and writing messages do not generate any private costs for members. Hence there are no moral hazard problems associated with communication *per se*, nor is there the possibility of unobserved communication among agents.

Communication plans chosen by each agent $i = 1, 2$ will be a function of their type θ_i. We shall refer to this as i's *communication strategy* $c_i(\theta_i) \in \mathcal{C}_i$.

2.1.3 Contracts and Production Decisions

A centralized contract for i is represented by a transfer rule $t_i(q_i, q_j, h_i)$. There will be no benefit that can be derived by conditioning them on message histories of other agents (i.e., h_j); besides the latter may raise 'privacy' concerns.

At the deadline T each agent i will select a production level q_i. This can depend on their type θ_i and the message history h_i. The restrictions on communication force production decisions to depend on 'coarse' messages about the state of the other agent: the production of agent i must depend on θ_j only through h_i. Specifically, agent i's production strategy is a function $\hat{q}_i(\theta_i, h_i(c_i(\theta_i), c_j(\theta_j), c_P))$. They can be fine-tuned to information about i's own cost θ_i, which constitutes the potential 'flexibility' advantage of decentralizing production decisions.

Formally, we shall say that the *production decision q_i is centralized* if it is measurable with respect to h_P, and *partially decentralized* if it is measurable with respect to $h_j, j \neq i$ but not with respect to h_P. In these cases, production decisions concerning q_i can be thought of as being made by P or j respectively.

In contrast, the production decision q_i is said to be *completely decentralized* if it is not the case that q_i is measurable with respect to h_P or $h_j, j \neq i$. The production decision of i cannot then be predicted by P or j at $t = T$ based on the messages they have sent or read. In other words it is not possible that agent i is assigned a production target by someone else, combined with an incentive scheme that forces i to abide by the target. Instead, i will make the production decision personally, a choice that will be influenced, though not completely determined, by the messages sent by others that enter as arguments of the incentive scheme.

2.2 Decentralized Contracting

2.2.1 Timing

In decentralized contracting, P contracts with the manager (agent i), who subsequently contracts with the employee (agent j). The communication network is also hierarchical: the employee communicates with the manager, and the manager with the Principal. Contracts are offered at $t = -1$: P offers a contract for i and selects an 'upper-level' communication protocol between herself and the manager. The manager then offers a subcontract to j, and selects the 'lower level' communication protocol.[4]

[4]The two networks are linked by the participation of the manager: messages sent by the manager on one network may be based on messages received in previous stages on the other network. For instance,

The rest is as under centralized contracting. At $t = 0$ agents i, j observe the realization of θ_i, θ_j respectively, and each independently decides whether or not to opt out. If neither opts out, they enter the communication phase from $t = 0$ until $t = T$. At T agents i, j decide q_i, q_j respectively, and payments are thereafter made as stipulated in the contracts. The timeline for decentralized contracting is depicted below.

Timeline: Decentralized Contract

$t = -1$ — P chooses $p_1 \in \mathcal{P}^1_D$ and offers $t_i(q_i, q_j, h_{Pi})$ to A_i

A_i chooses $p_2 \in \mathcal{P}^2(p_1)$ and offers $t_j(q_i, q_j, h_{ij})$ to A_j.

$t = 0$ — θ_i is realized, and A_i decides whether to participate or not.

$t = 1$

$t = 2$

Communication phase: For each t, agents chooses $m_{ijt} \in \mathcal{M}_{ijt}(h_{it})$

$t = T$ — Agent i selects q_i.

Payments are made according to the contracts.

2.2.2 Communication Protocol

The set of communication protocols consistent with decentralized contracting do not allow any direct communication between the employee j and the owner P. Each of them communicates only with the manager i. Moreover, P does not monitor communication between the manager and the employee, neither does the employee monitor communica-

the manager may receive a cost report from the employee, combine this with her own cost information to produce a summary cost report to P. Following this P may set an output target, with the manager subsequently allocating production responsibility between herself and the employee.

tion between the manager and P.[5]

Let $\mathcal{P}_D^l$ denote the set of communication protocols at level $l = 1, 2$ of the hierarchy. The Principal selects a protocol $p_1 \in \mathcal{P}_D^1$ at the upper layer $l = 1$. This determines the communicational responsibilities of the manager *vis-a-vis* the Principal, and constrains the protocols that the manager can choose from for the bottom layer $l = 2$.[6] Given p_1, the subset of protocols that i can choose for the lower level network is a subset of $\mathcal{P}_D^2$, represented by a correspondence $\mathcal{P}^2(p_1) : \mathcal{P}_D^1 \rightrightarrows \mathcal{P}_D^2$. Hence $\mathcal{P}_D$, the set of communication protocols feasible in the decentralized contracting regime, is represented by $\mathcal{P}_D^1$ the set of protocols for the upper tier, along with the correspondence $\mathcal{P}^2(.)$.

Note that $\mathcal{P}_D \subset \mathcal{P}_C$, i.e., any protocol feasible under decentralized contracting is also feasible under centralized contracting, while the converse is not true. In centralized contracting, P has the option of selecting the same hierarchical communication protocol as in decentralized contracting. However, decentralized contracting must necessarily involve a hierarchical protocol, whereas centralized contracting is not so constrained.

Since $\mathcal{P}_D \subset \mathcal{P}_C$, we do not need any fresh notation for communication protocols in decentralized contracting, apart from noting that they form a strict subset of communication protocols in centralized contracting.

2.2.3 Contracts and Production Decisions

Just as in centralized contracting, in decentralized contracting P can monitor the inputs supplied by either agent, as well as transfer payments made by the manager to her subordinates.

The contract between P and the manager is therefore represented by the transfer rule $t_i = t_i(q_1, q_2, h_{Pi})$, where h_{Pi} denotes (h_P, h_i^U) the history of messages exchanged between P and i on the upper level network. Here h_P denotes the messages sent and received by P, and h_i^U denotes messages sent (or received) by i to (or from) P.

The subcontracts offered by the manager to the employee specifies transfers t_j as a function of q_1, q_2 and messages exchanged on the lower level network $h_{12} \equiv (h_i^L, h_j^L)$. Here h_i^L and h_j^L denotes messages sent or received by i and j respectively among one another.

[5] We use the term 'monitor' as a shorthand for 'observe' and/or 'ex post verify'.

[6] For instance, it should allow the manager enough time to be able to execute his communicational responsibilities on both networks. If P wants the manager to report on some cost estimate for delivering V, for which prior communication with j is necessary, the protocol in the lower level network should ensure that the manager communicates with j before the time to report to P arrives.

Production decisions q_i, q_j are made at $t = T$ by i, j respectively, based on their personal information at that point. The manager decides $\hat{q}_i(\theta_i, h_i)$ where $h_i \equiv (h_i^U, h_i^L)$, and the employee decides $\hat{q}_j(\theta_j, h_j)$. Production decisions may be centralized or decentralized, as in the centralized contracting regime. The same formal definitions of (completely, partially) decentralized and centralized production decisions apply here as in the centralized contracting regime.[7]

3 Allocations Attainable under Centralized and Decentralized Contracting

Given a particular set of contracts offered by P in centralized contracting and a given communication protocol $p \in \mathcal{P}_C$ (including communication strategy c_P^*), we have a well-defined Bayesian game of incomplete information.

An *allocation attainable under centralized contracting* is a state-contingent production and transfer rule $q_a(\theta_1, \theta_2), t_a(\theta_1, \theta_2), a = 1, 2$ such that there exists a communication protocol $p \in \mathcal{P}_C$, centralized contracts $t_a(q_1, q_2, h_a), a = 1, 2$, and an associated tuple of communication and production strategies $\tilde{c}(.) \equiv \{c_i(\theta_i), c_j(\theta_j), c_P\}$ and $\tilde{q}(.) \equiv \{\hat{q}_i(\theta_i, h_i), \hat{q}_j(\theta_j, h_j)\}$ which constitutes a Perfect Bayesian Equilibrium (PBE) of the corresponding subgame, such that for any state $\theta \equiv (\theta_1, \theta_2)$ and any $a = 1, 2$:

$$
\begin{aligned}
q_a(\theta) &= \hat{q}_a(\theta_l, h_a(\tilde{c}(\theta))) && (1) \\
t_a(\theta) &= t_a(\tilde{q}(\theta), h_a(\tilde{c}(\theta))) && (2)
\end{aligned}
$$

Under decentralized contracting, a different Bayesian game is induced by choice of a contract for the manager and 'upper layer' communication protocol p_1 by P. Agent i, the manager, must select contracts t_j for agent j, a communication protocol $p_2 \in \mathcal{P}^2(p_1)$, and a communication strategy $c_i(\theta_i)$ to be executed by i during the communication phase. Production decisions and the strategies of agent j are similar to what they are under centralized contracting.

[7] In our formulation, subcontracts and communication protocol for the lower level network are designed by the manager at the *ex ante* stage. If they were designed instead at the *interim* stage, employees would need time to read the subcontract offered, which would cut into the time available for coordinating production plans. In that case, the set of possible subcontracts offered and accepted at the interim stage will belong to a finite set. Our formulation is equivalent to this: one can think of the subcontract offered *ex ante* as a finite menu of subcontracts offered at the interim stage, with subsequent communication between the agents between $t = 0$ and T serving to select one from the menu.

An *allocation attainable under decentralized contracting* is a state-contingent production and transfer rule $q_a(\theta), t_a(\theta), a = 1, 2$ such that there exists a contract $\hat{t}_i(q_1, q_2, h_{Pi})$ and communication protocol $p_1 \in \mathcal{P}_D^1$ (with communication plan $\hat{c}_P$) selected by P for the top tier, and a Perfect Bayesian Equilibrium (PBE) of the associated subcontracting subgame consisting of a subcontracts offered by i: $\hat{t}_j(q_1, q_2, h_{12})$, a communication protocol $p_2 \in \mathcal{P}^2(p_1)$, a tuple of communication strategies $\hat{c}(.) \equiv \{\hat{c}_i(\theta_i), \hat{c}_j(\theta_j), \hat{c}_P\}$ and production strategies $\hat{q}(.) \equiv \{\hat{q}_i(\theta_i, h_i), \hat{q}_j(\theta_j, h_j)\}$, such that for any state $\theta \equiv (\theta_i, \theta_j)$:

$$\begin{aligned} q_a(\theta) &= \hat{q}_a(\theta_a, h_a(\hat{c}(\theta))), a = 1, 2 \\ t_i(\theta) &= \hat{t}_i(q_1(\theta), q_2(\theta), h_{Pi}(\hat{c}(\theta))) - t_j(\theta) \\ t_j(\theta) &= \hat{t}_j(q_1(\theta), q_2(\theta), h_{12}(\hat{c}(\theta))) \end{aligned}$$

The following result is a trivial consequence of the fact that decentralized contracting involves a restricted set of communication protocols relative to centralized contracting.

Proposition 1 *Any allocation attainable under decentralized contracting can also be attained under centralized contracting.*

Proof. Consider an allocation $q_a(\theta), t_a(\theta), a = i, j$ attained by decentralized contracting with protocols p_1, p_2 at the two layers, transfer rules $\hat{t}_i, \hat{t}_j, \hat{t}_H$, communication and production strategies $\hat{c}, \hat{q}_1, \hat{q}_2$. Recall that the communication protocol $p \equiv (p_1, p_2)$ is feasible in centralized contracting. Recall also the assumption that P can verify all messages sent and received by all agents in centralized contracting. Therefore h_{12} is verifiable by P in centralized contracting. So the Principal can select the protocol p, and the following contracts:

$$\begin{aligned} t_j(q_1, q_2, h_{12}) &= \hat{t}_j(q_1, q_2, h_{12}) \\ t_i(q_1, q_2, h_{12}, h_{Pi}) &= \hat{t}_i(q_1, q_2, \hat{t}_j(q_1, q_2, h_{12}), h_{Pi}) - \hat{t}_j(q_1, q_2, h_{12}) \end{aligned}$$

Then from $t = 0$ the continuation game involving choice of communication and production strategies by the agents is the same as under decentralized contracting, so the same strategies constitute a PBE of this game. ∎

The argument of Proposition 1 resembles that underlying the Revelation Principle: under identical communication and contracting constraints, centralized contracts can be designed by the Principal to duplicate any mechanism based on decentralized contracts.

It confirms the notion that delegation of subcontracting offers no advantages over centralized contracting, once we are careful to incorporate the communication inherent in the act of selecting and offering a subcontract by the 'managing' agent. In particular, this setting does not allow the flexibility that constituted the key advantage of delegation in MMR (1992).

The key assumption in MMR was that the manager can offer a subcontract which is conditioned on the realization of θ_i; this cannot be duplicated with a centralized contract because that would require the manager to communicate θ_i to P. If, however, the subcontract is conditioned on θ_i, it must be offered at the interim stage, after $t = 0$. In that case the time taken by i to write the contract, and thereafter by j to read it, needs to incorporated within the time available for communication prior to production decisions. In other words, the act of offering or accepting a subcontract represent forms of communication which also entail delays. Once these delays are explicitly incorporated, the subcontract can no longer be fine-tuned to the realization of θ_i which is real-valued: this would involve subcontracts that are excessively detailed, rendering them infeasible within the time constraint for making production decisions. Only a finite number of possible subcontracts can be offered. With centralized contracting it is then feasible for the Principal to offer agent j the same menu of contracts as under decentralized contracting.

4 Characterizing Optimal Allocations in Centralized Contracting

Proposition 1 pertains to allocation of control over contracting, but says nothing about control over production, or the design of communication. Having designed contracts, P needs to decide whether to retain control rights over production as well. For instance, following exchange of messages with the agents, should the Principal set production targets? Or should the Principal let the agents decide what to produce, under the influence of an incentive mechanism where transfers depend on outputs and exchanged messages?

In order to address this, we need to make some progress in characterizing optimal allocations subject to the communication restrictions.

One restriction on output allocations is obvious: the output of any given agent can depend on information about the type of the other agent only in a coarse manner, on the basis of exchanged messages. The finiteness of the message spaces implies that an agent cannot signal its entire private information to others. This is represented in the following notion of communication-feasibility.

Definition 1 *The output allocation* $(q_1(\theta_1,\theta_2), q_2(\theta_1,\theta_2))$ *is communication feasible if and only if there exists* $p \in \mathcal{P}$, $c(\theta) = (c_i(\theta_i), c_j(\theta_j), c_P) \in C$ *and* $\hat{q}_i(\theta_i, h_i)$ *so that for all* (θ_i, θ_j)*:*

$$q_i(\theta_i, \theta_j) = \hat{q}_i(\theta_i, h_i(c(\theta)))$$

Of course, i's production q_i can depend finely on θ_i its own type, provided this decision is (completely) decentralized to i. Whereas if the Principal makes production decisions then both q_1 and q_2 will depend coarsely on θ_1, θ_2. The key question concerns which system is in the Principal's *ex ante* interest.

The next step is to note a 'rectangle property' that holds for any feasible communication protocol.

Lemma 1 [Rectangle property]: *Consider any communication protocol* $p \in \mathcal{P}$*. Then for any* $h_{it} \in H_{it}$*:*

$$\{c \in C \mid h_{it}(c) = h_{it}\}$$

is a rectangle set in the sense that if $h_{it}(c_i, c_{-i}) = h_{it}(c_i^{'}, c_{-i}^{'}) = h_{it}$ *for* $(c_i, c_{-i}) \neq (c_i^{'}, c_{-i}^{'})$*, then*

$$h_{it}(c_i^{'}, c_{-i}) = h_{it}(c_i, c_{-i}^{'}) = h_{it}$$

Proof:

The proof is by induction. Note that $h_{i0}(c) = \phi$ for any c, so it is true at $t = 0$. Suppose the result is true for all dates upto $t-1$, we shall show it is true at t.

Note that

$$h_{it}(c_i, c_{-i}) = h_{it}(c_i^{'}, c_{-i}^{'}) = h_{it} \tag{3}$$

implies

$$h_{i\tau}(c_i, c_{-i}) = h_{i\tau}(c_i^{'}, c_{-i}^{'}) = h_{i\tau} \tag{4}$$

for any $\tau \in \{0, 1, .., t-1\}$. Since the result is true until $t-1$, we also have

$$h_{i\tau}(c_i^{'}, c_{-i}) = h_{i\tau}(c_i, c_{-i}^{'}) = h_{i\tau} \tag{5}$$

for all $\tau \leq t-1$. So under any of the configurations of communication plans (c_i, c_{-i}), $(c_i^{'}, c_{-i}^{'})$, $(c_i^{'}, c_{-i})$ or $(c_i, c_{-i}^{'})$, member i experience the same message history $h_{i,t-1}$ until $t-1$. Then i has the same message space at t, and (3) implies that i sends the same messages to others at t, under either c_i or $c_i^{'}$.

(3) also implies that under either c_{-i} or $c_{-i}^{'}$, others send the same messages to i at all dates until $t-1$, following receipt on the (common) messages sent by i until $t-1$ under these different configurations. The result now follows from the fact that messages sent by others to i depend on the communication plan of i only via the messages they receive from i. So i must also receive the same messages at t under any of these different configurations of communication plans. ■

Lemma 1 has the following implication. Consider any history h_{it} observed by i until t. Then (given knowledge of the communication plan c_P of the Principal), the set of possible configurations of communication plans of the two agents that could have generated h_{it} can be expressed as the (Cartesian) product of $\tilde{\mathcal{C}}_1(h_{it})$ and $\tilde{\mathcal{C}}_2(h_{it})$, where $\tilde{\mathcal{C}}_a(h_{it})$ is a subset of $\mathcal{C}_a$, a=1,2. So defining

$$\Theta_a(h_{it}) \equiv \{\theta_a | c_a(\theta_a) \in \tilde{\mathcal{C}}_a(h_{it})\}$$

it follows that the set of types (θ_1, θ_2) that could have generated the history h_{it} can be expressed as the Cartesian product of subsets $\Theta_1(h_{it}), \Theta_2(h_{it})$. We note this formally below.

Lemma 2 *Given any configuration of communication strategies $(c_1(\theta_1), c_2(\theta_2), c_P)$, and given any history h_{it} that could be generated by these strategies, the set of possible types that could have generated this history can be expressed as*

$$\{(\theta_1, \theta_2) \mid h_{it}(c(\theta_1, \theta_2)) = h_{it}\} = \Theta_i(h_{it}) \times \Theta_j(h_{it}). \tag{6}$$

Hence upon observing the history h_{it}, **all** *types $\theta_i \in \Theta_i(h_{it})$ of agent i will update their prior beliefs about θ_j in the same way, i.e., by conditioning on the event that $\theta_j \in \Theta_j(h_{it})$.*

Another implication of the rectangle property will prove useful later. To simplify exposition we shall suppress c_P, the communication plan of the Principal, since this is specified as part of the mechanism and can be taken as given by the agents.

Lemma 3 *Without loss of generality, the Principal can restrict attention to communication protocols with the property that for every possible communication plan $c_a \in \mathcal{C}_a$ for agent $a \in \{1, 2\}$, there exists type θ_a such that $c_a = c_a(\theta_a)$.*

In other words, attention can be restricted to protocols with no off-equilibrium or unused communication plans. This is because unused communication plans can be deleted, without affecting the set of histories generated by the chosen communication strategies. This will simplify the representation of incentive constraints subsequently: the Principal needs only to deter deviations by any type $\theta_i \in \Theta_i(h_{it})$ following history h_{it}, to mimicking the communication plan chosen by some other $\theta_i' \in \Theta_i(h_{it})$ who has pooled so far with θ_i.

This is noted in the following result, which appears also in Fadel and Segal (2007, Proposition 3).

Proposition 2 *An allocation $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ is attainable under centralized contracting if and only if:*

(i) $q_i(\theta_i, \theta_j)$ is communication feasible: there exists communication protocol $p \in \mathcal{P}$ with production strategies $\hat{q}_i(\theta_i, h_i)$, and communication strategies $c_i(\theta_i), i = 1, 2$, such that for all (θ_i, θ_j):

$$q_i(\theta_i, \theta_j) = \hat{q}_i(\theta_i, h_i(c_i(\theta_i), c_j(\theta_j)))$$

and every communication plan is used by some type:

$$\mathcal{C}_i(p) = \{c_i(\theta_i) \mid \theta_i \in \Theta_i\}.$$

(ii) Defining the set of possible histories

$$H_{it} \equiv \{h_{it}(c(\theta)) \mid \theta \in \Theta_1 \times \Theta_2\}$$

that could be generated thereby, the following incentive constraint holds for any $t \in \{0, 1, 2, ...T\}$, any $h_{it} \in H_{it}$ and any θ_i, $\theta_i' \in \Theta_i(h_{it})$:

$$\begin{aligned} & E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\ \geq \quad & E_{\theta_j}[t_i(\theta_i', \theta_j) - \theta_i q_i(\theta_i', \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \end{aligned}$$

(iii) $E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i \hat{q}_i(\theta_i, \theta_j)] \geq 0$ *for any* θ_i

Proof

Necessity is obvious. To prove sufficiency, given $p \in \mathcal{P}$, $c(\theta)$ and $\hat{q}_i(\theta_i, h_i)$ which satisfies (i), let us define $\hat{t}_i(q_i, h_i)$ by

$$\hat{t}_i(q_i, h_i) \equiv E_{(\theta_i,\theta_j)}[t_i(\theta_i, \theta_j) \mid q_i(\theta_i, \theta_j) = q_i, h_i(c(\theta)) = h_i]$$

Then for $\theta_i \in \Theta_i(h_{it})$,

$$\begin{aligned} & E_{\theta_j}[\hat{t}_i(q_i(\theta_i, \theta_j), h_i(c(\theta_i, \theta_j))) \mid \theta_j \in \Theta_j(h_{it})] \\ = \; & E_{\theta_j}[t_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \end{aligned}$$

For $h_i \in H_i \equiv H_{iT}$, define

$$Q_i(h_i) \equiv \{\hat{q}_i(\theta_i, h_i) \mid \theta_i \in \Theta_i(h_i)\}.$$

Now consider the mechanism with protocol p and where agent i is paid $\hat{t}(q_i, h_i)$ only if $q_i \in Q_i(h_i)$ and messages sent by i at every date are consistent with the communication plan associated with some type θ_i; otherwise the agent is paid nothing. In this mechanism, suppose that agent i with type θ_i uses the communication plan $c_i(\theta_i)$ until date $t-1$, and h_{it} is realized at t. Then possible deviations by i at date t can be restricted to switching to the communication plan of by some other $\theta_i^{'} \in \Theta_i(h_{it})$ from date t onwards. Using Lemma 2, the resulting expected payoff of i is given by

$$E_{\theta_j}[t_i(\theta_i^{'}, \theta_j) - \theta_i q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})].$$

Condition (ii) ensures no such deviation is profitable for the agent. And condition (iii) ensures participation of i at $t = 0$. Hence this mechanism implements $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ as a PBE allocation. ■

As Fadel and Segal (2007) explain, this result indicates a trade-off between communication efficiency and incentive constraints. To render feasible any production allocation that depends more finely on the realized types of agents, communication protocols with more stages or larger message spaces are needed. These tend to impose more incentive constraints: enlarging the number of stages or range of message options at any stage is associated with a corresponding enlargement of the number of incentive constraints that the allocation must respect. For instance, along an iterative process of communication agents learn something about the types of other agents; this should not distort their subsequent communication or output decision choices.

We now arrive at the main result of this section, a characterization of output allocations that are attainable under centralized contracting (in combination with some transfer rules), and the associated maximum *ex ante* profit for the Principal. It is a generalization of the characterization underlying the Revenue Equivalence Theorem in auction theory (Myerson (1981)).

Proposition 3 *The output allocation $q_i(\theta_i, \theta_j)$ is attainable under centralized contracting (in combination with some transfer rule) if and only if*

(i) $q_i(\theta_i, \theta_j)$ is communication feasible.

(ii) If the associated communication protocol and strategies that implement $q_i(\theta_i, \theta_j)$ generate the set of message histories H_{it} for each agent i at date t, then $E_{\theta_j}[q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]$ is non-increasing in θ_i on $\Theta_i(h_{it})$ for any $h_{it} \in H_{it}$ and any t.

The maximum expected payoff for P which implements $q_i(\theta_i, \theta_j)$ is

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i)q_i(\theta_i, \theta_j) - v_j(\theta_j)q_j(\theta_i, \theta_j)] \tag{7}$$

The proof of Proposition 3 is long and detailed, and is presented in the Appendix. The necessity of conditions (i) and (ii) follows straightforwardly from the necessity of the corresponding conditions (i) and (ii) in Proposition 2. The sufficiency argument presents a number of complications. One has to construct a rule for transfers that will preserve the dynamic incentive constraints ((ii) in Proposition 2) at every date and following every message history. Moreover, the set of types $\Theta(h_{it})$ pooling until message history h_{it} need not constitute an interval. The monotonicity property (ii) for output decisions holds only 'within' $\Theta(h_{it})$, which may span two distinct intervals. The monotonicity property may therefore not hold for type ranges lying between the two intervals, which complicates the conventional argument for construction of transfers that incentivize the output allocation.

To overcome these problems, previous versions of this paper imposed strong distributional conditions (such as exponentially distributed types) to ensure that the additional dynamic incentive constraints do not bind. The construction used in the proof of Proposition 3 however works quite generally, allowing us to dispense with any special distributional conditions (apart from the monotone hazard rate condition standard in this class of models). It is based on a construction of a related output allocation which coincides with the actual allocation on equilibrium-path histories, and is globally non-increasing (with respect to types whose communication strategies could be mimicked on or off the equilibrium path). The construction is involved as it has to ensure that this property holds at every date and following every history. The transfer rule is designed to implement this related output allocation subject to incentive and participation constraints at minimum cost, as in the conventional analysis of private value auction problems. The global monotonicity (in combination with the monotone hazard rate property of the cost distribution) ensures the incentive compatibility of the rule, while the fact that the rule agrees with the actual output allocation on the equilibrium path ensures that it enables the Principal to realize her virtual expected profit.

Proposition 3 implies the mechanism design problem can be simplified as follows.

Proposition 4 *The mechanism design problem can be represented as choice of a communication protocol, communication strategies and output decision strategies to maximize the Principal's 'virtual' profit (7) subject to communication feasibility alone.*

In the case of unlimited communication, this reduces to the familiar property that an optimal output allocation can be computed on the basis of unconstrained maximization of expected virtual profits.

The proof of Proposition 4 is straightforward and therefore omitted. It relies on the argument that the solution to the problem of maximizing expected virtual profit subject to communication feasibility alone (i.e., ignoring the monotonicity constraint (ii) in the statement of Proposition 3) must have the following property. At every date and following every possible message history, the continuation communication strategy of any agent must maximize the expected virtual profit (conditional on the information revealed by the message history so far), given the communication and production strategy of others. In particular, it should not be possible for the agent to increase (conditional expected) virtual profit by switching to the continuation communicational strategy used by some other type with whom the agent has pooled so far. A standard 'revealed preference' argument then implies (given the monotone hazard rate property for distribution of types) that the monotonicity constraint (ii) in Proposition 3 must be satisfied. Hence this constraint cannot bind, and can be dropped from the statement of the problem.

Proposition 4 implies a simple and convenient separation between costs imposed by incentive considerations, and those imposed by communicational constraints. The former is represented by the replacement of production costs of the agents by their incentive-rent-inclusive virtual costs in the objective function of the Principal, in exactly the same way as in a world with costless, unlimited communication. The costs imposed by communicational constraints are represented by the restriction of the feasible set of output allocations, which must now vary more coarsely with the type realizations of the agents. This can be viewed as the natural extension of Marschak-Radner characterization of optimal team decision problems to a setting with incentive problems. In particular, the same computational techniques can be used to solve these problems both with and without incentive problems: only the form of the objective function needs to be modified to replace actual costs by virtual costs. The 'desired' communicational strategies can be rendered incentive compatible at zero additional cost.

Van Zandt (2006) and Fadel and Segal (2007) discuss the question of 'communication cost of selfishness', which relates to a different notion of separation between incentive and communicational complexity issues. In their context they take an arbitrary social choice function (allocation in our notation) and examine whether the communicational complexity of implementing it is increased by the presence of incentive constraints. If not, communicational and incentive aspects of the problem can be separated in the sense that optimal communication protocols can be designed independently of incentive considerations. In our context we fix communicational complexity and select an allocation to maximize the Principal's expected profits (the exact representation of which depends on whether or not incentive problems are present). We do not know if there is a connection between the two separation properties.

5 Optimality of Decentralized Production Decisions

The preceding characterization of the mechanism design problem allows us to prove that an optimal mechanism must completely decentralize production decisions to agents, rather than have someone else set targets. The intuitive reason is that with incorporation of agents' information rents in the objective function, the incentives of agents and the Principal are sufficiently aligned that production decisions can be delegated to agents in order to achieve greater 'flexibility' with respect to their cost realizations. A given agent i's production q_i is restricted to depend on cost realizations of the other agent j in a coarse manner owing to restrictions on communication. But there is no restriction on the way that q_i depends on agent i's *own* cost realization θ_i. Since virtual costs $v_i(\theta_i)$ are strictly increasing in θ_i, an optimal mechanism entails output decisions which are strictly decreasing in θ_i, conditional on any given message history. Since what others know about i's type is based only on messages they have exchanged with i, which have not communicated θ_i entirely, the decision over q_i must be left to i rather than anyone else.

Proposition 5 *In any optimal mechanism, production decisions must be completely de-*

centralized.

The proof is relegated to the appendix as it involves some technical details. These arise in the first step of the argument, which shows that there exist type intervals for each agent over which communication strategies and message histories are pooled. The second step of the argument then applies to any such interval of pooled types.

6 Decentralization of Communication

Proposition 4 also has useful implications for the ranking of different communication protocols. Given any set of communication strategies in a given protocol, in state (θ_i, θ_j) agent i learns that θ_j lies in the set $\Theta_j(h_i(c_i^*(\theta_i), c_j^*(\theta_j)))$, which generates an information partition for agent i over agent j's type.

Say that a protocol $\mathcal{C}_1$ is *more informative* than another $\mathcal{C}_2$ if for any set of communication strategies in the former, there exists a set of communication strategies in the latter which yields (at date T) an information partition to each agent over the type of the other agent which is more informative in the Blackwell sense in (almost) all states of the world.

It then follows that a more informative communication protocol permits a wider choice of communication feasible output allocations. Hence the Principal prefers more informative protocols. She has no interest in blocking the flow of communication among agents.

This result in turn has interesting implications for ranking of centralized and decentralized communication protocols. This is developed in detail in a previous version of this paper; we recount the main idea informally.

Say that a protocol is *completely centralized* if i and j never send or receive messages from one another directly; all communication is between agents and the Principal. Examples include tatonnement mechanisms and revelation mechanisms (in a setting without communication limits). In contrast a protocol is *completely decentralized* if neither agent communicates with the Principal at any date. In such a protocol, the Principal is not involved at all in the communication network; agents communicate directly with one another.

Suppose that the size of message sets at any date for any agent is determined by the (maximum) time it takes that agent to read and write messages, in a context where messages are represented in binary form and each agent takes a fixed amount of time to read or write one bit of information. Suppose also that there is at least one agent (i, say) who can read and write messages at least as quickly as the Principal. Then given any completely centralized protocol, there exists a completely decentralized protocol which is more informative. This can be shown by constructing a set of communication strategies in the completely decentralized protocol which mimics the communication in the completely centralized protocol. Specifically, agent i can mimic the communication strategy of the Principal in the centralized protocol *vis-a-vis* agent j. Since i does not need to communicate with himself as he knows his own state, this strategy frees up some time for agent i. This can be used by i to send some additional messages to j, in a way that generates a strict improvement in virtual profit. This argument illustrates a drawback of completely centralized protocols, in which the Principal becomes a bottleneck as all communication between agents must be channeled through her. Direct communication between agents permit greater exchange of information, leading to more flexible production decisions.

7 Summary and Concluding Comments

This paper developed a theory of mechanism design for a production team in a context where agents and Principal have limited capacity to communicate with one another. The main finding is that the Marschak-Radner view that the value of decentralizing production decisions derives (at least partly) from limitations on ability of agents to communicate, extends to settings with an incentive problem. As is well-known, in a context of unlimited communication and commitment ability of the Principal, attention can be focused on revelation mechanisms every aspect of which — contracting, production decisions and communication — are centralized. This flies in the face of pervasiveness of delegation of decision-making from owners of firms to managers, or customers to primary contractors or trading intermediaries. Previous attempts to adapt mechanism design theory to contexts of limited communication in order to explain the value of decentralized mechanisms were based on a number of *ad hoc* assumptions. These included unnatural restrictions on centralized contracts, and on communication systems.

The approach of this paper avoided imposing such restrictions on the class of mechanisms, apart from restricting (possibly dynamic) communication plans to a finite set. A partial analogue of the 'Revelation Principle' was obtained: attention can be restricted to centralized contracting mechanisms, though production and communication systems cannot be centralized in general. This helps clarify the nature of the precise argument for decentralization, i.e., that it pertains to production decisions and communication, rather than contracting rights.

Assuming messages received by agents are *ex post* verifiable, a general characterization of attainable allocations in terms of incentive and communication constraints was obtained. We showed that the incentive constraints with respect to communication do not bind, as incentive schemes can be designed to align the objectives of the Principal and the agents. Then the only constraints represented by limited communication pertain to limited coordination of production assignments: the more information can be brought to bear on production decisions, the better for the Principal. Hence it is essential to let each agent decide his or her own production, after exchanging messages with others: centralized decision-making about production in the form of 'targets' or 'orders from above' are suboptimal.

A natural task for future research is to examine the robustness of these results to a number of simplifying assumptions made: viz. verifiability of messages, absence of errors or noise in communication channels.

Other more general questions concerning organizational design can also be examined. Under what conditions does the presence of a third party facilitate communication and coordination among production agents? This would provide insight into the role of managers who do not participate in any production activities, whose only role is to process information communicated by production agents and help formulate production plans. In the model presented here, such third party 'coordinators' would have no room for strategic behavior, owing to the assumption of absence of collusion. If the model were extended to accommodate collusion, it would lead to a theory of hierarchies where intermediaries not directly involved in production play a coordinating role and behave strategically.

Another question pertains to the effects of changing communication technology on organizational design. Comparative statics of such a model with respect to information technology could generate predictions that could be tested against empirical patterns of how these have been changing in recent times (a brief overview of which is provided in Mookherjee (2006)).

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Appendix: Proofs

Proof of Proposition 4: Necessity of conditions (i) and (ii) are straightforward. So we prove sufficiency of these two conditions for an output allocation to be attainable under centralized contracting in combination with a transfer rule, which generates an expected profit of

$$E[V(q_i(\theta_i,\theta_j),q_j(\theta_i,\theta_j)) - v_i(\theta_i)q_i(\theta_i,\theta_j) - v_j(\theta_j)q_j(\theta_i,\theta_j)]$$

For the proof, the following notation will be useful.

(i) $h_{it} \succ h_{is}$ if $t \geq s$ and $\Theta_i(h_{it}) \times \Theta_j(h_{it}) \subset \Theta_i(h_{is}) \times \Theta_j(h_{is})$

(ii)$H_{it}(\theta_i, h_{is}) \equiv \{h_{it} \mid h_{it} \succ h_{is}, \theta_i \in \Theta_i(h_{it})\}$ for (θ_i, h_{is}) such that $\theta_i \in \Theta_i(h_{is})$.

(iii) $h_{it}(\theta_i,\theta_j) \equiv \{h_{it} \in H_{it} \mid (\theta_i,\theta_j) \in \Theta_i(h_{it}) \times \Theta_j(h_{it})\}$.

In (i), the history h_{it} at date t is a successor of history h_{is} at date s: it results from further exchange of messages between dates $s+1$ and t after h_{is} has occurred. The set $H_{it}(\theta_i, h_{is})$ is the set of all possible histories that type θ_i could observe at t, following history h_{is} observed at date s. And $h_{it}(\theta)$ is the history observed by i at t in state θ.

The following Lemma will prove useful.

Lemma 4 *Choose arbitrary $t \in \{1,..,T\}$, $h_{i,t-1} \in H_{i,t-1}$ and $\theta_i \in \Theta_i(h_{i,t-1})$. Then for any $h_{it}, h_{it}^{'} \in H_{it}(\theta_i, h_{i,t-1})$,*

$$\Theta_i(h_{it}) = \Theta_i(h_{it}^{'}).$$

The argument is the following. Messages sent by i at t depend only on θ_i and $h_{i,t-1}$. Hence conditional on θ_i and $h_{i,t-1}$, the succeeding histories $h_{it}, h_{it}^{'}$ can differ only because of differing messages *received* by i at t, in turn owing to different realizations of θ_j. Hence the set of types θ_i that observe the history $h_{i,t-1}$ and send the same messages as θ_i equals both $\Theta_i(h_{it})$ and $\Theta_i(h_{it}^{'})$.

Lemma 4 implies that the set $\{h_{it} \mid h_{it} \succ h_{i,t-1}\}$ of states succeeding $h_{i,t-1}$ can be partitioned into a collection of subsets of h_{it} with the property that $\Theta_i(h_{it})$ is equal among all h_{it} included in the same subset. Let these subsets be numbered from 1 to $L(h_{i,t-1})$ where $L(h_{i,t-1})$ is the total number of these subsets, and let $H_{it}(k, h_{i,t-1})$ denote the subset corresponding to $k \in \{1,..,L(h_{i,t-1})\}$. Then

$$\Theta_i(h_{it}) = \Theta_i(h_{it}^{'})$$

(which can also be expressed as $\Theta_i(k, h_{i,t-1})$) for any h_{it}, $h'_{it} \in H_{it}(k, h_{i,t-1})$. Moreover

$$\cup_{h_{it} \in H_{it}(k,h_{i,t-1})} \Theta_j(h_{it}) = \Theta_j(h_{i,t-1})$$

for any k and

$$\cup_{k=1}^{L(h_{i,t-1})} H_{it}(k, h_{i,t-1}) = \{h_{it} \mid h_{it} \succ h_{i,t-1}\}.$$

We are now in a position to set out the key steps of the proof of sufficiency of conditions (i) and (ii) of the proposition.

Claim 1

For $q_i(\theta_i, \theta_j)$ which satisfies conditions (i) and (ii) in the proposition, there exists $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ defined on $[\underline{\theta}_i, \bar{\theta}_i] \times \cup_{\tau=0}^{T} H_{i\tau}$ which satisfies the following conditions (a), (b) and (c).

(a) For any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$, $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ is non-increasing in $\tilde{\theta}_i$ on $[\underline{\theta}_i, \bar{\theta}_i]$.

(b) For any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$ and any $\tilde{\theta}_i \in \Theta_i(h_{it})$,

$$\tilde{q}_i(\tilde{\theta}_i, h_{it}) = E_{\theta_j}[q_i(\tilde{\theta}_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]$$

(c) For any $h_{i,t-1} \in \cup_{\tau=0}^{T} H_{i\tau}$ and any $k \in \{1, .., L(h_{i,t-1})\}$,

$$\Sigma_{h'_{it} \in H_{it}(k,h_{i,t-1})} \frac{\Pr(\Theta_j(h'_{it}))}{\Pr(\Theta_j(h_{i,t-1}))} \tilde{q}_i(\tilde{\theta}_i, h'_{it}) = \tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$$

Claim 1 states that there exists an 'auxiliary' output rule $\tilde{q}_i$ as a function of type $\tilde{\theta}_i$ and message history which is globally non-increasing in type (property (a)) following any history h_{it}, and nevertheless equals the expected value (conditional on h_{it}) of output in the allocation rule that is being sought to be implemented (property (b)). Property (ii) in the statement of the Proposition only allows the latter to be non-increasing over the set of types that arrive at that history h_{it} on the equilibrium path. This auxiliary output rule will be used later in the construction of transfer payments that efficiently incentivize the desired output allocation.

In order to establish Claim 1, the following Lemma is needed.

Lemma 5 *For any $B \subset R_+$ which may not be connected, let A be an interval satisfying $B \subset A$. Suppose that $F_i(a)$ for $i = 1, ..., N$ and $G(a)$ are functions defined on A, each of which has the following properties:*

- *$F_i(a)$ is non-increasing in a on B for any i.*

- *$\Sigma_i p_i F_i(a) = G(a)$ for any $a \in B$ and for some p_i so that $p_i > 0$ and $\Sigma_i p_i = 1$.*

- *$G(a)$ is non-increasing in a on A.*

Then we can construct $\bar{F}_i(a)$ defined on A for any i so that

- *$\bar{F}_i(a) = F_i(a)$ on $a \in B$ for any i.*

- *$\Sigma_i p_i \bar{F}_i(a) = G(a)$ for any $a \in A$ and for the same p_i*

- *$\bar{F}_i(a)$ is non-increasing in a on A for any i.*

This lemma says that we can construct functions $\bar{F}_i(a)$ so that the properties of functions $F_i(a)$ on B are also maintained on the interval A which covers B.

Proof of Lemma 5

If this statement is true for $N = 2$, we can easily show that this also holds for any $N \geq 2$. Suppose that this is true for $N = 2$.

$$\Sigma_{i=1}^{N} p_i F_i(a) = p_1 F_1(a) + (p_2 + ... + p_N) F^{-1}(a)$$

with

$$F^{-1}(a) = \Sigma_{i \neq 1} \frac{p_i}{p_2 + ... + p_N} F_i(a).$$

Applying this statement for $N = 2$, we can construct $\bar{F}_1(a)$ and $\bar{F}^{-1}(a)$ which keeps the same property on A as on B. Next using the constructed $\bar{F}^{-1}(a)$ instead of $G(a)$, we can apply the statement for $N = 2$ again to construct desirable $\bar{F}_2(a)$ and $\bar{F}^{-2}(a)$ on A based on $F_2(a)$ and $F^{-2}(a)$ which satisfy

$$\frac{p_2}{p_2 + ... + p_N} F_2(a) + [1 - \frac{p_2}{p_2 + ... + p_N}] F^{-2}(a) = F^{-1}(a).$$

on B. We can use this method recursively to construct $\bar{F}_i(a)$ for all i.

Next let us show that the statement is true for $N = 2$. For $a \in A \backslash B$, define $\underline{a}(a)$ and $\bar{a}(a)$, if they exists, so that

$$\underline{a}(a) \equiv \sup\{a' \in B \mid a' < a\}$$

and

$$\bar{a}(a) \equiv \inf\{a^{'} \in B \mid a^{'} > a\}.$$

It is obvious that at least one of either $\underline{a}(a)$ or $\bar{a}(a)$ exists for any $a \in A \backslash B$.

Let's specify $\bar{F}_1(a)$ and $\bar{F}_2(a)$ so that $\bar{F}_1(a) = F_1(a)$ and $\bar{F}_2(a) = F_2(a)$ for $a \in B$, and for $a \in A \backslash B$ as follows.

(i) For $a \in A \backslash B$ so that only $\underline{a}(a)$ exists,

$$\bar{F}_1(a) = F_1(\underline{a}(a))$$

$$\bar{F}_2(a) = \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}$$

(ii)For $a \in A \backslash B$ so that both $\underline{a}(a)$ and $\bar{a}(a)$ exist,

$$\bar{F}_1(a) = \min\{F_1(\underline{a}(a)), \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}\}$$

$$\bar{F}_2(a) = \max\{F_2(\bar{a}(a)), \frac{G(a) - p_1 F_1(\underline{a}(a))}{p_2}\}$$

(iii)For $a \in A \backslash B$ so that only $\bar{a}(a)$ exists,

$$\bar{F}_1(a) = \frac{G(a) - p_2 F_2(\bar{a}(a))}{p_1}$$

$$\bar{F}_2(a) = F_2(\bar{a}(a))$$

It is easy to check that $\bar{F}_i(a)$ is non-increasing in a on A for $i = 1, 2$ and

$$p_1 \bar{F}_1(a) + p_2 \bar{F}_2(a) = G(a)$$

for $a \in A$. This completes the proof of the lemma. ■

Proof of Claim 1:

Choose arbitrary $t \in \{1, ..., T\}$ and $h_{i,t-1} \in H_{i,t-1}$, and $k \in \{1, 2, ..., L(h_{i,t-1})\}$. Lemma 5 implies that for $\tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$ which satisfies (a) and (b) for this $h_{i,t-1}$, we can construct a function $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ for any $h_{it} \in H_{it}(k, h_{i,t-1})$ so that (a), (b) and (c) are satisfied. This result is obtained upon applying the Lemma with

$$B = \Theta_i(k, h_{i,t-1})$$

$$
\begin{aligned}
A &= [\underline{\theta}_i, \bar{\theta}_i] \\
a &= \tilde{\theta}_i \\
G(\tilde{\theta}_i) &= \tilde{q}_i(\tilde{\theta}_i, h_{i,t-1}) \\
F_{h_{it}}(\tilde{\theta}_i) &= \tilde{q}_i(\tilde{\theta}_i, h_{it}) \\
p_{h_{it}} &= \frac{\Pr(\Theta_j(h_{it}))}{\Pr(\Theta_j(h_{i,t-1}))} \\
\{1, ..., N\} &= H_{it}(k, h_{i,t-1}).
\end{aligned}
$$

This means that for $\tilde{q}_i(\tilde{\theta}_i, h_{i,t-1})$ which satisfies (a) and (b) for any $h_{i,t-1} \in H_{i,t-1}$, we can construct $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which satisfies (a)-(c) for any $h_{it} \in H_{it}$.

With $h_{i0} = \phi$, since $\tilde{q}_i(\tilde{\theta}, h_{i0}) = E_{\theta_j}[q_i(\theta_i, \theta_j)]$ satisfies (a) and (b), $\tilde{q}_i(\tilde{\theta}_i, h_{i1})$ is constructed so that (a)-(c) are satisfied for any $h_{i1} \in H_{i1}$. Recursively $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ can be constructed for any $h_{it} \in \cup_{\tau=0}^{T} H_{i\tau}$ so that (a)-(c) are satisfied. ■

Claim 2

For $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ constructed in Claim 1,

$$E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] = \tilde{q}_i(\tilde{\theta}_i, h_{it})$$

for any $\theta_i \in \Theta_i(h_{it})$.

Proof of Claim 2

From the construction of $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which satisfies (c), for any $\theta_i \in \Theta_i(h_{it})$,

$$
\begin{aligned}
& \tilde{q}_i(\tilde{\theta}_i, h_{it}) = \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+1}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+1}) \\
= \; & \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+1}))}{\Pr(\Theta_j(h_{it}))} \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it+1})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it+1}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) \\
= \; & \Sigma_{h_{it+1} \in H_{it+1}(\theta_i, h_{it})} \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it+1})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) \\
= \; & \Sigma_{h_{it+2} \in H_{it+2}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{it+2}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{it+2}) = \cdots \\
= \; & \Sigma_{h_{iT} \in H_{iT}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT})
\end{aligned}
$$

The fourth equality comes from

$$H_{it+2}(\theta_i, h_{it}) = \{H_{it+2}(\theta_i, h_{it+1}) \mid h_{it+1} \in H_{it+1}(\theta_i, h_{it})\}.$$

Since

$$H_{iT}(\theta_i, h_{it}) = \{h_{iT}(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})\},$$

$$\begin{aligned}
& \Sigma_{h_{iT} \in H_{iT}(\theta_i, h_{it})} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT}) \\
= \quad & \Sigma_{h_{iT} \in \{h_{iT}(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})\}} \frac{\Pr(\Theta_j(h_{iT}))}{\Pr(\Theta_j(h_{it}))} \tilde{q}_i(\tilde{\theta}_i, h_{iT}) \\
= \quad & E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})]
\end{aligned}$$

This completes the proof of the Claim. ■

We are now in a position to complete the proof of sufficiency. Based on $\tilde{q}_i(\tilde{\theta}_i, h_{it})$ which was constructed in Claim 1, consider the following mechanism: $(t_i(\theta_i, \theta_j), q_i(\theta_i, \theta_j))$ with

$$t_i(\theta_i, \theta_j) = \theta_i q_i(\theta_i, \theta_j) + \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) d\tilde{\theta}_i$$

Then for any t, any $h_{it} \in H_{it}$ and any $\theta_i, \theta_i^{'} \in \Theta_i(h_{it})$,

$$\begin{aligned}
& E_{\theta_j}[t_i(\theta_i^{'}, \theta_j) - \theta_i q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] \\
= \quad & (\theta_i^{'} - \theta_i) E_{\theta_j}[q_i(\theta_i^{'}, \theta_j) \mid \theta_j \in \Theta_j(h_{it})] + \int_{\theta_i^{'}}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i^{'}, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] d\tilde{\theta}_i \\
= \quad & (\theta_i^{'} - \theta_i) \tilde{q}_i(\theta_i^{'}, h_{it}) + \int_{\theta_i^{'}}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{it}) d\tilde{\theta}_i \\
\leq \quad & \int_{\theta_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{it}) d\tilde{\theta}_i \\
= \quad & \int_{\theta_i}^{\bar{\theta}_i} E_{\theta_j}[q_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{it})] d\tilde{\theta}_i \\
= \quad & E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \mid \theta_j \in \Theta_j(h_{it})]
\end{aligned}$$

The second equality comes from (b) in Claim 1 and the statement of Claim 2 and the third inequality comes from (a) in Claim 1. The fourth equality comes from the statement of Claim 2. This means that the incentive constraint is satisfied at all stages of communication.

At the interim stage with $t = 0$,

$$\begin{aligned}
& E_{\theta_j}[t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j)] \\
= & \int_{\underline{\theta}_i}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j))] d\tilde{\theta}_i \\
= & \int_{\underline{\theta}_i}^{\bar{\theta}_i} E_{\theta_j}[\tilde{q}_i(\tilde{\theta}_i, h_{iT}(\theta_i, \theta_j)) \mid \theta_j \in \Theta_j(h_{i0})] d\tilde{\theta}_i \\
= & \int_{\underline{\theta}_i}^{\bar{\theta}_i} \tilde{q}_i(\tilde{\theta}_i, h_{i0}) d\tilde{\theta}_i \\
= & \int_{\underline{\theta}_i}^{\bar{\theta}_i} E_{\theta_j}[q_i(\tilde{\theta}_i, \theta_j)] d\tilde{\theta}_i
\end{aligned}$$

The third equality comes from the statement of Step 2, and the fourth one is from (b) in Claim 1. This implies that the interim participation constraint is satisfied and P's ex-ante payoff is equal to

$$E[V(q_i(\theta_i, \theta_j), q_j(\theta_i, \theta_j)) - v_i(\theta_i) q_i(\theta_i, \theta_j) - v_j(\theta_j) q_j(\theta_i, \theta_j)].$$

which is the maximum possible payoff under the constraint that θ_i is private information of i. This completes the proof of the proposition. ■

Proof of Proposition 5:

The proof relies on the following Lemma.

Lemma 6 *In an optimal mechanism, each agent i's communication strategy $c_i^*(\theta_i)$ is almost everywhere locally constant.*

Proof.

Step 1

In what follows fix the optimal communication plan for P and suppress it in the notation, while focusing on optimal choice of communication plan by each agent. With $h_i = h_i(c_i, c_j)$, the optimal production $\hat{q}_i^*(\theta_i, h_i)$ and communication plan $c_i^*(\theta_i)$ for agent i satisfy

$$\begin{aligned}
\hat{q}_i^*(\theta_i, h_i) \in & \arg\max_{q_i} E_{\theta_j}[V(q_i, \hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))) \mid h_i(c_i, c_j^*(\theta_j)) = h_i] \\
- & v_i(\theta_i) q_i
\end{aligned} \tag{8}$$

and

$$\begin{aligned} c_i^*(\theta_i) &\in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i) \equiv E_{\theta_j}[V(\hat{q}_i^*(\theta_i, h_i(c_i, c_j^*(\theta_j)), \hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))) \\ &- v_i(\theta_i)\hat{q}_i^*(\theta_i, h_i(c_i, c_j^*(\theta_j))) - v_j(\theta_j)\hat{q}_j^*(\theta_j, h_j(c_i, c_j^*(\theta_j)))] \end{aligned}$$

In (8) the optimal production decision conditional on a given message history h_i does not depend on i's communication plan c_i, since h_i includes all messages sent by i and the latter are a sufficient statistic for inferences made by i about the output decisions to be made by j.

Step 2: *There exists an optimal communication strategy $c_i^*(\theta_i)$ for each agent i such that*

$$c_i^*(\theta_i) \in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i)$$

and $c_i^*(\theta_i)$ is almost everywhere locally constant.

Before we set out the argument for Step 2, note that since $v_i(\theta_i)$ is continuous, the Maximum Theorem implies that $\pi(\theta_i, c_i)$ is a continuous function of θ_i, for any c_i.

Suppose Step 2 is false. Then there exists a non-degenerate interval $(\theta_i^*, \theta_i^{**})$ over which the optimal communication strategy cannot be selected to be a constant strategy for any subinterval. In other words, for any θ_i in this interval, if $\hat{c}_i(\theta_i) \in \arg\max_{c_i \in \mathcal{C}_i} \pi(\theta_i, c_i)$, then in any neighborhood of θ_i there exists $\theta_i^{'}$, and an alternative communication plan $c_i' \in \mathcal{C}_i$ such that

$$\pi(\theta_i, \hat{c}_i(\theta_i)) \geq \pi(\theta_i, c_i^{'})$$

and

$$\pi(\theta_i^{'}, c_i^{'}) > \pi(\theta_i^{'}, \hat{c}_i(\theta_i)).$$

Now choose arbitrary $\theta_i^0 \in (\theta_i^*, \theta_i^{**})$. $B(\theta_i^0)$ and $C_i(\theta_i^0)$ are constructed as follows:

- $C_i(\theta_i^0)$ is defined as $\{c_i \in C_i \mid \pi(\theta_i^0, \hat{c}_i(\theta_i^0)) = \pi(\theta_i^0, c_i)\}$. This is the set of communication plans that maximize $\pi(\theta_i^0, .)$.

- In the case that $\bar{C}_i(\theta_i^0) \equiv C_i \backslash C_i(\theta_i^0)$ is not empty: Since $\pi(\theta_i, c_i)$ is continuous for θ_i, for $c_i \in \bar{C}_i(\theta_i^0)$, there exists neighborhood $B(\theta_i^0, c_i)$ of θ_i^0 so that $\pi(\theta_i^{'}, \hat{c}_i(\theta_i^0)) > \pi(\theta_i^{'}, c_i)$ for any $\theta_i^{'} \in B(\theta_i^0, c_i)$. Since there are a finite number of elements in $\bar{C}_i(\theta_i^0)$, $\Pr(\cap_{c_i \in \bar{C}_i(\theta_i^0)} B(\theta_i^0, c_i)) > 0$. Define $B(\theta_i^0) \equiv \cap_{c_i \in \bar{C}_i(\theta_i^0)} B(\theta_i^0, c_i)$. Then for any $\theta_i^{'} \in B(\theta_i^0)$, $\pi(\theta_i^{'}, c_i^{'}) < \pi(\theta_i^{'}, \hat{c}_i(\theta_i^0))$ for any $c_i^{'} \in \bar{C}_i(\theta_i^0)$.

- In the case that $\bar{C}_i(\theta_i^0) \equiv C_i \backslash C_i(\theta_i^0)$ is empty: $B(\theta_i^0)$ is chosen as an arbitrary neighborhood of θ_i^0.

By hypothesis, there exists $\theta_i^1 \in B(\theta_i^0)$ and $c_i^{'}$ so that

$$\pi(\theta_i^1, c_i^{'}) > \pi(\theta_i^1, \hat{c}_i(\theta_i^0)).$$

Hence $c_i^{'}$ does not belong to $\bar{C}_i(\theta_i^0)$. This implies that $c_i^{'}$ belongs to $C_i(\theta_i^0)$.

Next construct $C_i(\theta_i^1)$ and $B(\theta_i^1)$ from θ_i^1 using the same procedure as in the construction of $C_i(\theta_i^0)$ and $B(\theta_i^0)$ from θ_i^0.

We claim that $C_i(\theta_i^1) \subseteq C_i(\theta_i^0)$. Suppose c_i does not belong to $C_i(\theta_i^0)$. Then $c_i \in \bar{C}_i(\theta_i^0)$. Since $\theta_i^1 \in B(\theta_i^0)$, this implies $\pi(\theta_i^1, c_i) < \pi(\theta_i^1, \hat{c}_i(\theta_i^0))$. Since $\hat{c}_i(\theta_i^0)) < \pi(\theta_i^1, c_i^{'})$, it follows that c_i does not belong to $C_i(\theta_i^1)$.

Note also that $C_i(\theta_i^1)$ does not include $\hat{c}_i(\theta_i^0)$. Hence the number of elements in $C_i(\theta_i^1)$ is strictly smaller than $C_i(\theta_i^0)$.

In a manner similar to the choice of θ_i^1 given θ_i^0, we can choose $\theta_i^2 \in B(\theta_i^1)$ and construct $C_i(\theta_i^2)$ and $B(\theta_i^2)$. This procedure can be repeated until the number of elements in $C_i(\theta_i^k)$ becomes one. Then since $\hat{c}_i(\theta_i)$ is constant for $\theta_i \in B(\theta_i^k)$, we obtain a contradiction, thus completing the proof of Step 2 and Lemma 6.

Return now to the proof of Proposition 5. Lemma 6 implies that there exist $\bar{c}_i \in \mathcal{C}_i$ and non-degenerate interval $[\theta_i^{'}, \theta_i^{''}] \subset \{\theta_i \mid c_i^*(\theta_i) = \bar{c}_i\}$. $q_i^*(\theta_i, \theta_j) = \hat{q}_i^*(\theta_i, h_i(\bar{c}_i, c_j^*(\theta_j)))$ is strictly decreasing in θ_i on $[\theta_i^{'}, \theta_i^{''}]$, since this solves

$$\max_{q_i} E_{\theta_j}[V(q_i, \hat{q}_j^*(\theta_j, h_j(\bar{c}_i, c_j^*(\theta_j)))) \mid h_i(\bar{c}_i, c_j^*(\theta_j),) = h_i] - v_i(\theta_i)q_i.$$

On the other hand, $h_j = h_j(\bar{c}_i, c_j^*(\theta_j))$ and $h_P = h_P(\bar{c}_i, c_j^*(\theta_j))$ are independent of θ_i on $[\theta_i^{'}, \theta_i^{''}]$. This implies that q_i^* is not measurable with respect to h_j and h_P. ∎