Saving Seats for Strategic Customers

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We consider a service provider in a market with two segments. Members of the first request a reservation ahead of service and will not patronize the firm without one. Members of the second walk in and demand service immediately. These customers have a fixed cost of reaching the firm and may behave strategically. In equilibrium, they randomize between walking in and staying home. The service provider must decide how much of a limited capacity to make available to advance customers. When the advance demand segment offers a higher per customer margin, the firm may opt to decline some reservation requests in order to bolster walk-in demand. When walk-in customers are more valuable, we have a variation of Littlewood (1972). Where Littlewood would always save some capacity for valuable late arrivals, here it is possible that the optimal policy saves *no* capacity for walk-ins. Thus, it may be better to ignore rather than pamper walk-in customers. This outcome is robust to changes in the model.

Key words: Revenue management; service management; reservations.

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1. Introduction

Managing a flow of customers with varying profitability and arrival patterns is an important challenge in many service businesses. In industries ranging from airlines to restaurants, managers must decide how much capacity to make available to segments that arrive early and how much to save for later arrivals. This fundamental problem has many theoretical intricacies that have inspired a large body of research. Much of this work, however, has ignored that capacity management decisions do not exist in a vacuum. How a firm doles out its capacity affects how customers show up. The firm must consequently consider how its allocation of capacity affects the demand for its services.

The restaurant industry exemplifies the issues involved. Restaurants face the question of whether or not to take reservations. That is, they must choose whether to court customers who require a guaranteed seat or to count on customers who prefer the flexibility of just walking in. At a high level, this is not an either-or choice. A restaurant can offer some or all of its seats to reservation seekers and then fill the remaining seats with walk-in customers. Indeed, if one is given profit margins and fixed demand distributions for each segment, models exist in the literature to help solve this problem (e.g., Littlewood (1972), Bertsimas and Shioda (2003)). But the demand distributions are not necessarily fixed, particularly for walk-in customers. If a walk-in customer incurs a cost to ask for a seat (e.g., a cab ride across town), she is unlikely to make the effort unless the chance of getting a seat is sufficiently high. Such considerations have kept some restaurants from offering reservations at all (Bruni (2006)) or at least limiting the number of reservations they will take (Arnett (2005)).

In this paper, we consider a service provider with a fixed capacity facing two segments, advance customers and walk-in customers. Both segments require service at the same point in time but differ in when they request service. Advance customers – as the name suggests – contact the firm to reserve capacity ahead of the service time. Walk-in customers in contrast arrive at the time of service and require immediate attention. In the context of a restaurant, the time of service would be, say, Friday evening. Advance customers are those who make reservations early in the week while walk ins are those who show up unannounced on Friday. Walk-in customers incur a cost to visit the firm and request service and thus a face a lottery; they must sink the cost to ask for service without any guarantee that capacity is available. Whether a given walk-in customer is willing to take that risk depends on her perceived chance of being served. Thus there is a dependency between how much capacity the firm makes available to advance customers and how many walk ins request service.

We consider two cases. In the first, advance demand is more profitable but uncertain while the number of walk-in customers in the market is fixed. Here, it may be optimal to set aside capacity for walk-in customers. If a large amount of capacity is made available to advance customers, walk-in traffic may be insufficient to compensate for poor advance-demand realizations. The firm consequently commits to potentially turning away some valuable early service requests in order to assure a high level of walk-in traffic. The firm is not guaranteed to use all of its capacity. The optimal policy targets a probability of unused capacity that increases with the profitability of the advance segment and the walk-in segment's cost to request service.

Saving capacity for strategic walk-in customers is intuitively appealing. Remarkably, this result does not necessarily carry over to our second setting in which walk-in demand is more profitable but uncertain. Here, the service provider only restricts advance sales when the margin on advance customers and the walk-in segment's fixed cost are low. Otherwise, the firm makes all capacity available for advance customers. That is, it may be better to ignore the more valuable segment than to save capacity for them.

Not saving additional capacity for more profitable but strategic walk-in customers is an intriguing finding; it is also a robust outcome. Our basic model assumes that advance demand is deterministic but this is not necessary for the result. If advance demand is uncertain, the firm may not make all capacity available for advance demand but may still save less capacity for walk ins than it would if they were not strategic. Our basic model also assume that walk-in customers are homogeneous and all share the same net utility for the service. Heterogeneous valuations for the service alters the absolute amount of capacity saved for strategic walk ins but again strategic walk-in customers may result in less capacity being held back from the advance market.

Whether walk in or advance demand is more profitable, saving capacity for walk-in demand is only an issue when capacity is limited. Given ample capacity, walk ins face little risk of being denied service and strategic behavior is irrelevant. The question then is whether the service provider would pick a capacity that results in saving little capacity for walk ins. We show that strategic walk-in customers affect both the firm's capacity and reservation policy. As the walk-in segment's cost for requesting service increases, it may be optimal to increase the amount of capacity installed while saving additional capacity for walk ins. This holds regardless of which segment is more profitable. However, once the walk-in segment's cost increases sufficiently, total capacity and the amount of capacity set aside for walk-ins falls. Indeed, given a high enough cost, the firm chooses a capacity and reservation policy that keeps any walk-in customers from patronizing the firm.

Our work is related to several branches of literature. The first obviously is revenue management for which Talluri and VanRyzin (2004) provides a comprehensive survey. In particular, our second setting is a variant of Littlewood (1972) who assumes late-arriving customers are more profitable but not strategic. His distribution of late demand is independent of the number of seats available while ours is a function of how many seats are set aside for walk ins. In Littlewood (1972), it is always optimal to save some seats for late arrivals but here it may be best not to save any seats.

Other work that has considered balancing advance and walk-in sales includes Kimes (2004) and Bertsimas and Shioda (2003). The former examine how a restaurateur can maximize revenue per available seat hour and discusses the relative advantages of offering reservations or relying on walk ins. She does not examine how offering reservations impacts walk-in demand levels. In Bertsimas and Shioda (2003), the restaurant's decisions include whether to accept a reservation request and what wait time to quote a walk-in party. They assume reservation and walk-in demand are independent. We have a single service period but allow for an interplay between reservation and walk-in demand.

When our advance customers are more valuable, the walk-in segment serves as a salvage opportunity for capacity unclaimed by advance customers. The key feature is that demand in the salvage market is affected by the amount of capacity open to advance customers. This setting is related to Cachon and Kök (2007). They consider a newsvendor model in which the per-unit salvage value depends on the amount of unsold stock. In our model, the salvage value is fixed but the probability that excess inventory is sold depends on the stocking decision. When our walk-in customers are more valuable, the demand distribution depends on the amount of capacity that is available. This similar to Dana and Petruzzi (2001). They consider a retailer whose customers have random outside options. Whether an individual customer patronizes the retailer depends on that customer's option and the firm's inventory level. Our consumer model is simpler but has a similar implication: Consumer demand depends on the inventory available.

Others have considered how the presence strategic customers impacts operations. Netessine and Tang (2009) provides a recent survey on this subject. Much of this work has focused on managing inventory in retail environments. Su and Zhang (2008) examine supply chain performance under a variety of contract terms when strategic customers weigh buying early at full price against the possibility of buying discounted goods later. Cachon and Swinney (2009) take on similar issues but focus on quick response and other supply chain improvement initiatives.

In service settings, Lariviere and Van Mieghem (2004) consider congestion-averse consumers choosing arrival times over a horizon and show that as the horizon and number of customers grow the resulting arrival pattern converges to a Poisson process. Our model has only a single sale period. In Alexandrov and Lariviere (2007), a firm sells to a single segment of strategic customers which can be served either through advance reservations or through walk ins. If reservations are employed, some customers may fail to keep their appointment resulting in lost sales for the firm. Despite that, the firm may still offer reservations. Our model emphasizes the role of multiple segments. In particular, there are customers who will not patronize the service provider if reservations are offered.

Below, we present the basics of the model. Sections 3 and 4 consider, respectively, having early demand and late demand be more profitable. We demonstrate the robustness of saving fewer seats for more profitable walk-in customers in Section 5. In section 6, the firm sets both its capacity and reservation level. Section 7 concludes. Proofs are in the Appendix.

2. Model fundamentals

We consider a monopolist in a market with two segments. All customers desire service at the same point in time. The segments differ in several ways including when they request service. The first segment, *advance customers*, contacts the firm before the time service is required and requests reservations. An advance customer only patronizes the firm if she receives a reservation. The second segment, *walk-in customers*, requests service immediately. Let N_a denote the number of advance customers and N_w the number of walk-in customers. We assume all customers require the same amount of the firm's capacity to be served.

The segments differ in the margins they provide the firm. The margin is π_a for reservation customers and π_w for walk-in customers. More generally, we can think of the margin earned from a given customer as random but drawn from segment specific distributions with means π_a and π_w. We take the margins as fixed and beyond the firm's control. Thus our model does not fit settings like airlines and hotels where the amount customers pay may depend on when they request service. It is appropriate for services such as restaurants and beauty salons. In these industries, all customers are presented with the same price list but what they ultimately pay depends on choices finalized after arriving at the firm. Differing margins across segments then imply that customers from different segment make systematically different choices. We can also allow $\pi_a = \pi_w$ if we introduce the possibility that advance customers may fail to keep their reservations. See the discussion at the end of Section 3.

Finally, the segments differ in their value for service and their cost of requesting service. Advance customers have a value $U > 0$ for receiving service and incur no cost for requesting a reservation. Consequently, every advance customer in the market requests a reservation. Walk-in customers value dining at the restaurant at $V > 0$ but must incur a cost T to ask for a seat. $V > T > 0$. We will refer to T as the travel cost and to V/T as the customer's net utility. If a walk-in customer opts to stay home, her utility is zero.

The firm can serve K customers. For the moment K is fixed so the service provider's sole decision is its reservation level R, i.e., how much capacity to make available to advance customer. $0 \leq R \leq K$. K is common knowledge and R is observable to the customers. Reservation holders always keep their reservations and are guaranteed seats. (In keeping with the restaurant example, we will measure capacity in terms of seats.) Hence, given a realized number of advance customers, N_a, the firm's sales to the advance segment are $\min\{N_a, R\}$. Advance customers have priority over walk-in customers and the firm cannot overbook. Thus, giving out a reservation may prohibit the firm from seating a walk-in customer. If the number of advance requests exceeds R, the reservations are rationed randomly and each customer is equally likely to receive one. Given N_a advance requests, the service provider will have $K-\min\{N_a, R\}$ seats available for walk-in customers. If demand exceeds available capacity, each walk-in customer is equally likely to be served.

Customers are assumed atomistic, i.e., each is small relative to the size of the market. As we will see below, walk-in customers may follow a mixed strategy. Having atomistic customers then implies that while we cannot predict perfectly what an individual will do, we can accurately predict the aggregate outcome.

3. Uncertain but more profitable advance demand

Here we assume that $\pi_a > \pi_w$. The number of walk-in customers N_w is deterministic and greater than K. The number of advance customers N_a is random with continuous distribution $F_a(n)$ on support $\left[\underline{N}_a, \overline{N}_a\right]$. $\overline{N}_a > K > \underline{N}_a \geq 0$. $f_a(n)$ denotes the density of F_a, and $\bar{F}_a(n) = 1 - F_a(n)$. The distribution $F_a(n)$ is commonly known, but only the firm observes realized advance demand.

Given a realized advance demand level of n, the firm's revenue from advance customers when it makes R seats available for reservations is $\pi_a \min\{n, R\}$. Its expected revenue from advance customers given R is then $\Pi_a(R) = \pi_a S_a(R)$ where $S_a(R) = \int_{\underline{N}_a}^{R} n f_a(n)\,dn + R\bar{F}_a(R)$. $\Pi_a(R)$ is increasing in R. Hence, the firm would set R to its maximum value of K if it were certain to have more than $K - \underline{N}_a$ walk ins. It would always fully utilize capacity and earn

$$\Pi_a(K) = \pi_a S_a(K) + \pi_w(K - S_a(K)). \tag{1}$$

Unfortunately, the firm cannot take sufficient walk-in demand as a given. Any given walk-in customer faces a lottery. If she "spends" the travel cost T, she may win V or nothing. Winning requires getting a seat, and hence whether one walks in depends upon the chance of being seated. Let γ denote a walk-in customer's probability of getting a seat. Her expected utility is then $\gamma V - T$, and she walks in if $\gamma \geq T/V$. The value of γ, of course, depends the reservation policy of the firm, the distribution of advance demand, and the behavior of walk-in customers.

Lemma 1. *Suppose the firm sets a reservation level of R and walk-in customers follow a symmetric equilibrium. Let $\nu(R)$ denote the number of walk-in customers who request service given R.*

1. *If* $N_w \leq \frac{V}{T}(K - S_a(R))$ *all walk-in customers request service, i.e.,* $\nu(R) = N_w$*, and the chance of being served exceeds* T/V.

2. *Otherwise, each walk-in customer visits the firm with probability*

$$\lambda(R) = \frac{V(K - S_a(R))}{TN_w}. \tag{2}$$

$\nu(R) = \lambda(R) N_w = \frac{V}{T}(K - S_a(R))$ *and* $\nu'(R) < 0$.

The walk-in segment cannot observe the exact number of available of seats, and the equilibrium is consequently based on the expected number of available seats $K - S_a(R)$. When there are on average many seats or walk-in customers have very high net utilities (i.e., V/T is large), strategic interaction between walk-in customers is inconsequential form the firm's perspective since everyone walks in. Once seats are sufficiently limited, walk-in customers begin to ration themselves and only $\nu(R) < N_w$ actually walk-in.

Assuming atomistic customers implies that the realized number of customers walking in to the firm will be exactly $\nu(R)$. Consequently, if advance demand, n, is less than $K - \nu(R)$, walk-in demand will be insufficient to utilize capacity fully. Otherwise, only $K - n$ walk-ins can be served. Letting $\Pi_w(R)$ be revenue from walk-in customers given the reservation level and $\nu(R) < N_w$, this yields

$$\begin{aligned}\Pi_w(R) &= \pi_w\left(\nu(R)F_a(K-\nu(R)) + \int_{K-\nu(R)}^{R}(K-n)f_a(n)dn + \bar{F}_a(R)(K-R)\right)\\ &= \pi_w(K - S_a(R)) - \pi_w\int_{\underline{N}_a}^{K-\nu(R)}(K-\nu(R)-n)f_a(n)\,dn. \end{aligned}\tag{3}$$

Recall that $S_a(R)$ is expected sales to advance customers given a reservation level R. The first term of (3) is then expected sales to walk ins ignoring the strategic behavior of walk-in customers. The second term thus represents the firm's loss due to strategic customers.

The firm consequently faces a trade off. Raising the number of seats available via reservations increases the expected number of (more profitable) advance customers that the firm serves. However, because $\nu(R)$ is decreasing, a higher reservation level also decreases walk-in demand.

Proposition 1. *Suppose $N_w > \frac{V}{T}(K - S_a(K))$. Let R^* denote the optimal reservation level.*

1. *If $\pi_a \geq \pi_w \left(1 + \frac{V}{T} F_a (K - \nu(K))\right)$, $R^* = K$.*
2. *If $\pi_a < \pi_w \left(1 + \frac{V}{T} F_a (K - \nu(K))\right)$, $R^* < K$ and is found from*

$$F_a (K - \nu(R^*)) = \frac{T(\pi_a - \pi_w)}{V \pi_w}. \tag{4}$$

If $N_w < \frac{V}{T}(K - S_a(K))$, Lemma 1 gives that all walk-in customers attempt to get seats even if advance customers can claim all of capacity. The naive solution of making all capacity available via reservation is then obviously optimal. Proposition 1 shows that such a decision may also be optimal even if walk-in customers self-ration as long as the margin on advance customers is sufficiently high. When π_a is low, it is optimal to hold back some capacity. Effectively, the firm commits to turning away advance customers before reaching its capacity in order to assure walk-in customers a good chance of getting a seat. It does this despite the fact that walk-ins are less valuable. Limiting sales to advance customers, however, assures a relatively large amount of walk-in traffic which provides a safety net if realized advance demand is low.

It is straightforward to show that $\nu(R) > K - R$ for all $R \leq K$. Consequently, if the firm gives out all available R reservations, there will be sufficient walk-in demand to fill up any remaining capacity. This does not mean that the service provider always has a full house. $F_a (K - \nu(R^*))$ is the probability that the firm has some unused capacity. When there is little difference in the segments' margins (i.e., $\pi_a - \pi_w$ is small), the firm severely restricts the number of reservations it gives out in order to reduce the chance of not fully utilizing its capacity. This suggests an alternative interpretation for the first part of the proposition: if π_a is sufficiently large, the firm cannot achieve its desired probability of idle capacity even if it makes its entire capacity available to advance customers.

Note that the optimal reservation level is increasing in K, implying that it is easier to get a reservation at a firm with a larger capacity. This, however, does not come at the expense of walk-in customers. The optimality condition (4) requires that the difference between capacity and the number of customers actually walking in must be constant as K and R^* are adjusted. Hence, the increased reservation level (and subsequent increase in sales to advance customers) does not completely offset the increase in capacity and the number of walk-in patrons requesting service increases. We will examine how the firm would choose K in Section 6 below.

These results can be easily adjusted to accommodate advance customers who do not necessarily keep their reservations. Assume each reservation holder fails to keep her reservation with probability ϕ for $0 \leq \phi < 1$. There are two possibilities to consider. First, suppose that the firm immediately realizes that a reservation holder is a no show, and it is able to re-offer her seat. The expected number of seats available for walk-in given R is

$$K - \int_{\underline{N}_a}^{R} (1-\phi)\, n f_a(n)\, dn - (1-\phi)\, R\bar{F}_a(R) \geq K - S_a(R).$$

Thus walk-in customers expect more capacity to be available for a given reservation level and hence more will actually request service. The firm can therefore afford to make more capacity available to advance customers even though they are less reliable. Alternatively, suppose that the service provider does not realize that a reservation holder is a no show until the end of the service period when it is too late to re-offer her seat to a walk-in customer. The number of expected available seats for walk-in customers remains $K - S_a(R)$ so their behavior given R is unchanged. However, the expected margin for each reservation given out falls to $(1-\phi)\,\pi_a$. The firm thus has an incentive to (weakly) increase the number of seats saved for walk-in customers. This assumes $(1-\phi)\pi_a > \pi_w$. If the reverse holds, we have a setting in which walk-in customers are more valuable, a case which we consider next.

4. Uncertain but more profitable walk-in demand

We now reverse the assumptions of the previous section. Fix the size of the advance segment at $N_a > K$, and assume $\pi_a < \pi_w$. The number of walk-in customers N_w is random with continuous distribution $F_w(n)$ on support $\left[\underline{N}_w, \overline{N}_w\right]$. $\overline{N}_w > K > \underline{N}_w \geq 0$. $f_w(n)$ denotes the density and $\bar{F}_w(n) = 1 - F_w(n)$. The distribution of N_w is common knowledge, but a walk-in customer does not observe the realized value of N_w before deciding whether to request service. Let μ_w denoted the expected value of N_w.

R again denotes the amount of capacity made available to advance customer. In this setting, however, all R units are certain to be given out while capacity saved for walk-in customers may go unused. This is true even when walk-in customers do not behave strategically (equivalently, when $T=0$). In this case, the firm's profit is

$$\hat{\Pi}_L(R) = \pi_a R + \pi_w S_w(K-R), \tag{5}$$

where $S_w(x) = \int_{\underline{N}_w}^{x} n f_w(n)\,dn + x\bar{F}_w(x)$. This is the classic problem of Littlewood (1972). If $F_w(K) \leq \frac{\pi_w - \pi_a}{\pi_w}$, the optimal reservation level R_L is zero. Otherwise, R_L solves

$$F_w(K-R_L) = \frac{\pi_w - \pi_a}{\pi_w}.$$

Of course, once walk-in customers incur a cost to request service, the Littlewood quantity R_L may not be optimal. Walking in is still risky, and how many walk-in customers actually attempt to patronize the firm depends on the chance of getting a seat.

Lemma 2. *Suppose that the service provider sets a reservation level of R and that walk-in customers follow a symmetric equilibrium. Let $\hat{Z}$ solve $S_w\left(\hat{Z}\right) = \frac{\mu_w T}{V}$.*

1. *If $\hat{Z} \leq K-R$, all walk-in customers request service, and the chance of being served exceeds T/V.*

2. *Otherwise, each walk-in customer requests service with probability*

$$\hat{\lambda}(R) = \frac{K-R}{\hat{Z}},$$

and the chance of getting a seat is T/V.

When the firm saves a large amount of capacity for walk ins, the distribution of customer who actually walk in is simply $F_w(n)$. Once the amount of saved capacity $K-R$ drops below the critical level $\hat{Z}$, the number of walk-in patrons who request service falls to $\hat{\lambda}(R)\,N_w$. Hence, as the firm gives out more reservations, the number of walk-ins patronizing the firm becomes stochastically smaller. To understand the role $\hat{Z}$ plays in the equilibrium, let $\Delta_w(x)$ denote the fill rate when the demand distribution is $F_w(n)$ and the stocking level is x. By construction, $\Delta_w\left(\hat{Z}\right) = T/V$, the equilibrium fill rate if walk-in customers self-ration. Thus when the number seats saved for walk ins $K - R$ equals $\hat{Z}$ and all walk-in customers request service, each individual walk-in customer has an expected utility of zero. Alternatively, when the number of saved seats falls below $\hat{Z}$, the distribution of walk-in demand is rescaled to $F_w\left(n/\hat{\lambda}(R)\right)$ and the corresponding fill rate for a stocking level of x is $\Delta_w\left(x/\hat{\lambda}(R)\right)$. Consequently, the stocking level needed for the equilibrium fill rate would be $\hat{\lambda}(R)\hat{Z}$. The walk-in customers, of course, cannot adjust the number of saved seats $K - R$; that is the firm's prerogative. However, they can move to an equilibrium that scales the available seats to so that $\frac{K-R}{\hat{\lambda}(R)} = \hat{Z}$.

Turning to the service provider's problem, if not all walk-ins request service, its profit from the walk-in segment is

$$\Pi_w(R) = \pi_w(K-R)\left[\int_{\underline{N}_w}^{\hat{Z}} \frac{nf_w(n)}{\hat{Z}}dn + \bar{F}_w\left(\hat{Z}\right)\right] = \pi_w(K-R)\,S_w\left(\hat{Z}\right)/\hat{Z}.$$

Profits from walk-in customers are linear in R because the number of customers actually walking in is scaled by $\hat{\lambda}(R)$, which is linear in the reservation level. $S_w\left(\hat{Z}\right)/\hat{Z}$ is always less than one. It can be interpreted as the fraction of units sold when $\hat{Z}$ units are stocked. Here, it is the fraction of capacity saved for walk-in customers that is on an average utilized.

The firm's total profit as a function of the reservation level $\hat{\Pi}(R)$ is then

$$\hat{\Pi}(R) = \begin{cases} \hat{\Pi}_L(R) & \text{if } R \le K - \hat{Z} \\ \pi_a R + \Pi_w(R) & \text{if } R > K - \hat{Z} \end{cases}. \tag{6}$$

As shown in Figure 1, $\hat{\Pi}(R)$ is kinked. (In the figure, $\hat{\Pi}(R)$ is plotted as a solid line while $\hat{\Pi}_L(R)$ is plotted as a dashed line.) For reservation levels below $K - \hat{Z}$, walk-in customer do not self-ration; $\hat{\Pi}(R)$ follows the Littlewood objective and is strictly concave. Beyond $K - \hat{Z}$, not all walk ins visit the firm; $\hat{\Pi}(R)$ is linear and can slope up or down. The optimal reservation level then depends on two factors, the location of the Littlewood quantity R_L relative to the kink in $\hat{\Pi}(R)$ and the slope beyond the kink.

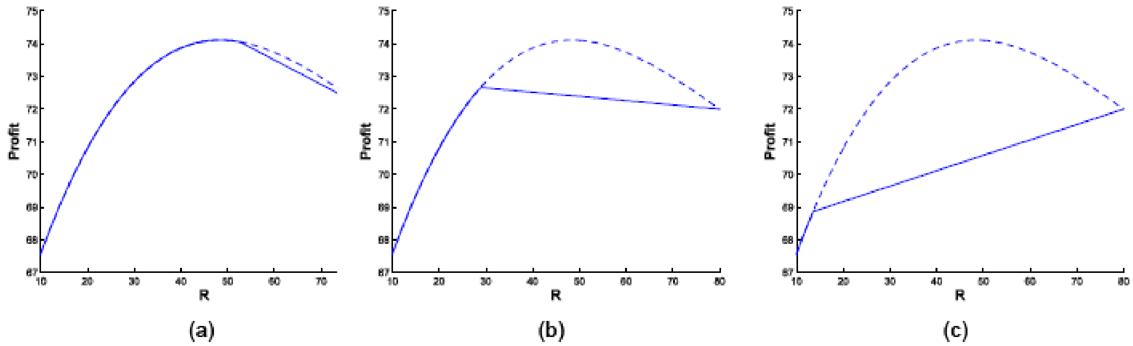


Figure 1 Profit as a function of R with both strategic (the solid line) and non-strategic customers (the dashed line). For all examples $F_w(n) = (n/100)^2$ for $0 \leq n \leq 100, K = 80, \pi_a = 0.9, \pi_w = 1$, and $V = 1$. For (a), $T = 0.5$. For (b), $T = 0.7$. For (c), $T = 0.85$.

PROPOSITION 2. *Let $\hat{R}$ denote the optimal reservation level.*

1. *If $R_L \leq K - \hat{Z}$, $\hat{R} = R_L$.*
2. *If $R_L > K - \hat{Z}$,*

$$\hat{R} = \begin{cases} (K - \hat{Z})^+ & \text{if } \pi_a/\pi_w \leq S_w\left(\hat{Z}\right)/\hat{Z} \\ K & \text{if } \pi_a/\pi_w > S_w\left(\hat{Z}\right)/\hat{Z}, \end{cases}$$

where $(K - \hat{Z})^+ = \max\left\{K - \hat{Z}, 0\right\}$.

When advance demand was uncertain but more profitable, the firm could disregard strategic walk-in behavior of as long as walk-in demand was sufficiently robust. Otherwise, reservations were limited. Part of that logic carries over. If all walk-in customers come in over a large range of saved capacity, the firm effectively lives in Littlewood's world and can simply choose R_L. See Figure 1(a). This requires that $\hat{Z}$ and R_L be relatively small. That, in turn, implies that walk-in customers have a high net utility (since $\hat{Z}$ is decreasing in V/T) and that walk-in customers are significantly more profitable than advance customers (since R_L is decreasing in $\pi_w - \pi_a$).

Once walk-in customers are rationing themselves, the firm does not necessarily save extra capacity for them. The firm protects additional capacity for walk ins when they are significantly more profitable than advance customers. A unit of capacity saved for a walk-in garners π_w when it sells but the average return is only $\pi_w S_w\left(\hat{Z}\right)/\hat{Z}$. As the the margin on advance customers increases, the optimal decision eventually tips to committing all capacity to advance customers. Thus although Littlewood (1972) always holds back capacity for the more profitable segment, it may be best to forego doing any business with this segment if they behave strategically.

5. On the robustness of saving fewer seats

The previous section had two surprising results. First, the service provider may save fewer seats for high-value walk-in customers when they behave strategically than when they are not strategic. Second, the optimal reservation policy has a bang-bang structure; the firm either sets aside enough seats so that all walk in customers request service or saves no seats for them. In this section we examine the robustness of those conclusions to changes in model assumption. We first consider having advance demand also be uncertain and then introduce heterogeneity in how customers value the service. We show that either extension may make the bang-bang allocation of capacity go away. However, the first insight remains: The firm may still prefer to save fewer seats for strategic customers.

5.1. Uncertain reservation demand

To this point our analysis of high-value walk-in demand has assumed that advance demand is both ample and certain. Here we relax that assumption and suppose that the number of advance customers is equal to capacity, K, with probability $0<\rho<1$, and is 0 with probability $1-\rho$. Intuitively, walk-in customers should expect more seats to be available for any given reservation level and thus a greater number should be willing to request service. This reasoning is borne out in the following lemma that characterizes the equilibrium behavior of walk-in customers.

Lemma 3. *Suppose the service sets a reservation level of R and walk-in customers follows a symmetric equilibrium. Let $\lambda^\rho(R)$ solves*

$$\rho S_w\left(\frac{K-R}{\lambda}\right)+(1-\rho)S_w\left(\frac{K}{\lambda}\right)=\frac{T\mu_w}{V},\ \textit{for any}\ 0\le R\le K. \tag{7}$$

1. *$\lambda^\rho(R)$ is decreasing in R. And, there exists R^0_ρ such that $\lambda^\rho(R^0_\rho)=1$*

2. *$\lambda^\rho(R) \geq \hat{\lambda}(R)$ for any $0 \leq R \leq K$. Therefore, $R^0_\rho \geq K-\hat{Z}$.*

3. *If $R \leq R^0_\rho$, all walk-in customers request a seat, and the chance of getting a seat exceeds T/V.*

4. *Otherwise, each walk-in customer visits the firm with probability $\lambda^\rho(R)$, and the chance of getting a seat is T/V.*

The basic structure of the equilibrium is similar to what prevails when advance demand is deterministic. There may exist a range of reservations levels over which all walk-in customers request service, but once R increases sufficiently walk ins begin rationing themselves. There is, however, an important difference. When advance demand was deterministic the equilibrium probability of walking in was proportional to the number of available seats. That is no longer true. Consequently, the expected return per saved seat is not fixed at $\pi_w S_w\left(\hat{Z}\right)/\hat{Z}$ and the service provider's profit is no longer linear over the range of reservation level on which the walk-in customer ration themselves.

From the lemma, the firm's profit from the walk-in customers for a reservation level $R \geq R^0_\rho$ is

$$\begin{aligned}
\Pi^\rho_{walk}(R) &= \rho\pi_w\left[\int_{\underline{N}}^{\frac{K-R}{\lambda^\rho(R)}} \lambda^\rho(R) n f_w(n)dn + \int_{\frac{K-R}{\lambda^\rho(R)}}^{\overline{N}} (K-R) n f_w(n)dn\right] \\
&\quad +(1-\rho)\pi_w\left[\int_{\underline{N}}^{\frac{K}{\lambda^\rho(R)}} \lambda^\rho(R) n f_w(n)dn + \int_{\frac{K}{\lambda^\rho(R)}}^{\overline{N}} (K-R) n f_w(n)dn\right] \\
&= \pi_w \frac{T\mu_w}{V}\lambda^\rho(R),
\end{aligned}$$

where the equality follows from Equation 7. Hence, the firm's total profit as a function of the reservation level is

$$\hat{\Pi}_\rho(R) = \begin{cases} \rho\hat{\Pi}_L(R) + (1-\rho)\pi_w S_w(K) & \text{if } R \leq R^0_\rho \\ \pi_a\rho R + \pi_w \frac{T\mu_w}{V}\lambda^\rho(R) & \text{if } R > R^0_\rho. \end{cases}$$

As $\rho \to 1$, $\hat{\Pi}_\rho(R)$ converges to $\hat{\Pi}(R)$, the firm's profit function in Section 4. Moreover, like $\hat{\Pi}(R)$, $\hat{\Pi}_\rho(R)$ is kinked. It is strictly concave for reservation levels below R^0_ρ. However, unlike $\hat{\Pi}(R)$, $\hat{\Pi}_\rho(R)$ is not linear for $R \geq R^0_\rho$ and may have an interior maximum over $\left(R^0_\rho, K\right)$. Let $\tilde{R}_\rho = \arg\max_{R^0_\rho \leq R \leq K}\{\pi_a\rho R + \pi_w \frac{T\mu_w}{V}\lambda^\rho(R)\}$ and note that the Littlewood quantity is independent of the distribution of advance demand. The optimal reservation level then is either the Littlewood's reservation level R_L, or $\tilde{R}_\rho$, the reservation level that maximizes the firm's profit on the range of reservations over which walk-in customers ration themselves.

Proposition 3. *Let $\hat{R}_\rho$ denote the optimal reservation level.*

1. *If $R_L < R_\rho^0$,*

$$\hat{R}_\rho = \begin{cases} R_L & \text{if } \hat{\Pi}_\rho(R_L) > \hat{\Pi}_\rho(\tilde{R}_\rho)) \\ \tilde{R}_\rho & \text{if } \hat{\Pi}_\rho(R_L) \leq \hat{\Pi}_\rho(\tilde{R}_\rho)) \end{cases}$$

2. *If $R_L \geq R_\rho^0$, $\hat{R}_\rho = \tilde{R}_\rho$.*

Similar to Proposition 2, Proposition 3 states that the firm may save fewer seats for walk-in customers than Littlewood's suggestion, $K - R_L$, since $\tilde{R}_\rho$ may be greater than R_L. However, that does not necessarily imply a reservations-only policy. As we will now show through a numerical example, the firm may choose a reservation level that exceeds the Littlewood quantity but still saves seats for walk ins.

5.1.1. Example: Pareto Distribution Here, we assume that walk-in demand follows a Pareto distribution with the scale parameter $s > 0$ and the shape parameter 2, i.e. $F_w(n) = 1 - \frac{s^2}{n^2}$. To simplify notation and minimize technicalities, we fix $\pi_a = 1$ and $V = 1$. We also restrict the travel cost, T, to values between $\frac{1}{2}$ and $\frac{2K-s}{2K}$ and the probability of high demand ρ to values greater than $2(1-T)$. We then have

$$\begin{aligned} R_L &= K - s\sqrt{\pi_w}, \\ R_\rho^0 &= \frac{K}{K-(1-\rho)\hat{Z}}(K - \hat{Z}) = K - \frac{\rho K s}{2(1-T)K - (1-\rho)s}, \\ \tilde{R}_\rho &= \begin{cases} R_\rho^0 & \text{if } R^{**} < R_\rho^0 \\ R^{**} & \text{if } R_\rho^0 \leq R^{**} \leq K \\ K & \text{if } R^{**} > K. \end{cases} \end{aligned}$$

where $R^{**} = \frac{K}{1-\rho}\left(1 - \sqrt{4\pi_w T(1-T)}\right)$.

In this setting, if $R_\rho^0 > R_L$ (i.e., walk-in customers only ration themselves at reservation levels greater than Littlewood's reservation level), the marginal return of saving an extra seat for walk-in customers at R_ρ^0 exceeds the expected return from advance customers. Thus, the firm disregards reservation levels above R_ρ^0, and chooses R_L as the optimal reservation level.

The Littlewood quantity R_L will fall below R_ρ^0 when either there is a high probability of having no advanced demand at all, i.e. ρ is small, or the walk-in customers provide a significant premium, i.e. π_w is high. On the other hand, if the walk-in customers ration themselves for a wide-range of reservation level, the firm will disregard the reservation levels where all walk-in customers patronize the firm, and choose $\tilde{R}_\rho$ as its optimal decision. It is still possible that the service provider opts to just keep walk-ins from self-rationing (i.e., chooses R_ρ^0) or opts to give out all its capacity via reservations. The former is likely when ρ is small and π_w is high; the latter when the reverse holds. For intermediate values of ρ, the firm picks an intermediate quantity that can be above or below the Littlewood quantity. This is illustrated in the following proposition and Figure 2.

Proposition 4. *Let $\hat{R}_\rho$ denote the optimal reservation level.*

1. *If (ρ, π_w) is in Region-5, i.e. $R_L < R_\rho^0$, then $\hat{R}_\rho = R_L$.*
2. *If $R_L \geq R_\rho^0$, then*

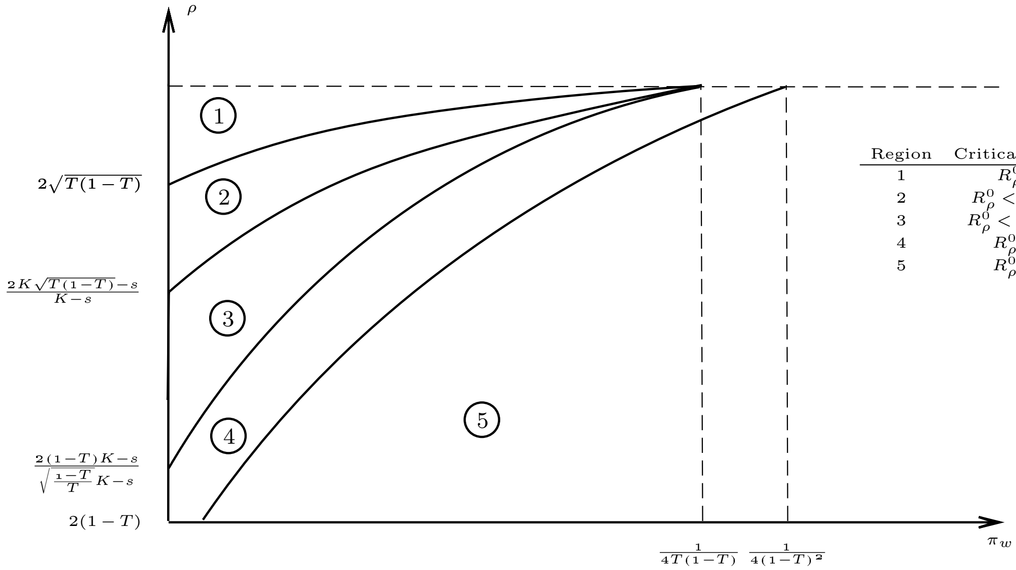
$$\hat{R}_\rho = \begin{cases} K & \textit{if } (\rho, \pi_w) \textit{ is in Region-1} \\ R^{**} & \textit{if } (\rho, \pi_w) \textit{ is in either Region-2 or Region-3} \\ R_\rho^0 & \textit{if } (\rho, \pi_w) \textit{ is in Region-4} \end{cases}$$

3. *If (ρ, π_w) is in either Region-1 or Region-2, then $\hat{R}_\rho > R_L$.*

We have thus shown that the introduction of uncertain advance demand may eliminate the bang-bang nature of the service provider's reservation policy. However, it may still be optimal save fewer seats for more valuable walk-in customers if they act strategically.

5.2. Heterogeneous customer valuations

We now return to assuming that reservation demand is certain and ample in order to examine how results depend on having homogeneous customer valuations. Instead of assuming that all walk-in customers have the same value V for the service, we now suppose each walk-in customer has a value of $V(\epsilon) = T + \varepsilon$ for dining out where ε is a random variable with distribution $\Phi(\varepsilon)$ on the



Region	Critical Reservation Levels
1	$R_\rho^0 < R_L,\ \tilde{R}_\rho = K$
2	$R_\rho^0 < R_L,\ R_L < \tilde{R}_\rho < K$
3	$R_\rho^0 < R_L,\ R_\rho^0 < \tilde{R}_\rho < R_L$
4	$R_\rho^0 < R_L,\ \tilde{R}_\rho = R_\rho^0$
5	$R_\rho^0 > R_L,\ \tilde{R}_\rho = R_\rho^0$

Figure 2 Critical reservation levels R_L, R_ρ^0, $\tilde{R}_\rho$ for a given (ρ, π_w)

support $(0, \bar{\varepsilon})$ for $0 < \bar{\varepsilon} \le \infty$. We assume that $\Phi(\varepsilon)$ is continuous. Let $\overline{\Phi}(\varepsilon) = 1 - \Phi(\varepsilon)$. As before, all customers have a positive net utility for the service if they are guaranteed access. However, because $\overline{N}_w > K$, one cannot guarantee that all walk ins will be served. Consequently, not all walk-in customers will request service. In particular if a customer with realized shock $\hat{\varepsilon}$ is indifferent between requesting service and staying home, all customers for whom $\varepsilon < \hat{\varepsilon}$ stay home while those for whom $\varepsilon \ge \hat{\varepsilon}$ walk in. This intuition is formalized in the following lemma.

Lemma 4. *Suppose the service provider sets a reservation level of R and walk-in customers follows a symmetric equilibrium. Let $\hat{\varepsilon}(R)$ solves*

$$S_w\left(\frac{K-R}{\overline{\Phi}(\varepsilon)}\right) = \frac{T\mu_w}{T+\varepsilon}, \text{ for any } 0 \le R \le K. \tag{8}$$

1. *If $\varepsilon \ge \hat{\varepsilon}(R)$, a walk-in customers requests a seat, and the chance of getting a seat is $\frac{T}{T+\hat{\varepsilon}(R)}$. Otherwise, she stays at home.*
2. *$\hat{\varepsilon}(R)$ is non-decreasing in the reservation level R.*
3. *$\hat{\varepsilon}(R) > 0$ for any R.*

The firm's profit function can be written as

$$\begin{aligned}\hat{\Pi}_{\varepsilon}(R) &= \pi_a R + \pi_w \overline{\Phi}(\hat{\varepsilon}(R)) S_w \left(\frac{K-R}{\overline{\Phi}(\hat{\varepsilon}(R))}\right) \\ &= \pi_a R + \pi_w (K-R) S_w \left(\hat{Z}(R)\right) / \hat{Z}(R),\end{aligned} \tag{9}$$

where $\hat{Z}(R) = \frac{K-R}{\overline{\Phi}(\hat{\varepsilon}(R))}$. In comparing (9) with (6), two points are worth making. First, because not every walk-in customer will request service, $\hat{\Pi}_{\varepsilon}(R)$ lies below the corresponding Littlewood objective $\hat{\Pi}_L(R)$. (See Figure 3.) There is never a range of reservation levels over which the firm can safely ignore the strategic behavior of customers, and this behavior always costs the firm money. Second, the fraction of saved seats that are on average utilized $S_w\left(\hat{Z}(R)\right)/\hat{Z}(R)$ is no longer constant. Indeed, the equilibrium between the walk-in customers results in $\hat{Z}(R)$ being decreasing in R so the fraction of saved seats that are utilized must increase. The expected return per saved must consequently climb.

Of course, the fact that the profit function lies below the Littlewood function does not imply that the optimal reservation level is less than the Littlewood quantity. However, there is a simple sufficient condition for when the firm saves fewer seats for strategic walk ins than for non-strategic walk ins. Specifically, the firm gives out more reservations than Littlewood would recommend if

$$\frac{\pi_a}{\pi_w} > \frac{S_w\left(\hat{Z}(R_L)\right)}{\hat{Z}(R_L)}.$$

This condition is analogous to that in Proposition 2 except that we are looking at just R_L. It will not generally hold for all R and hence we lose the bang-bang allocation of capacity between the segments. But we do still have that it may be optimal to save fewer seats for strategic walk ins. This is seen in Figure 3. In this numerical example, $R_L = 63.28$, and $\hat{R}_{\varepsilon} = 79.2$.

The following proposition presents the above discussion formally.

Proposition 5. *Let $\hat{R}_{\varepsilon}$ denote the optimal reservation level.*

1. $\hat{\Pi}_{\varepsilon}(R) \leq \hat{\Pi}_L(R)$.
2. *If* $\frac{\pi_a}{\pi_w} > \frac{S_w\left(\hat{Z}(R_L)\right)}{\hat{Z}(R_L)}$, *then* $R_L < \hat{R}_{\varepsilon}$.

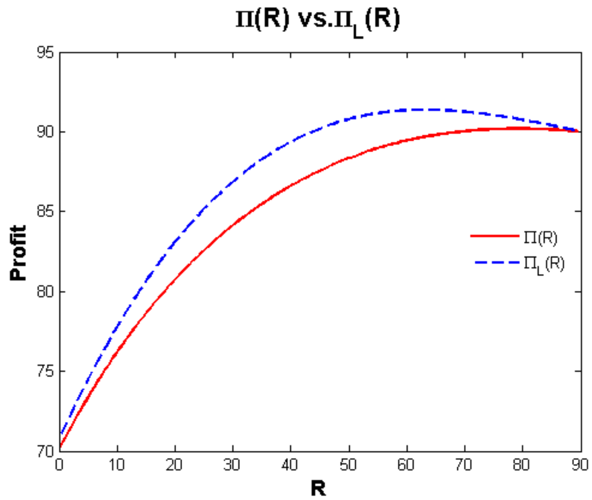


Figure 3 Firm profit with and without strategic customers. $\Phi(\varepsilon)=\varepsilon/10$ for $0\leq\varepsilon\leq 10$, $F_w(n)=(n/100)^2$ for $0\leq n\leq 100$, $K=90$, $\pi_r=1$, $\pi_w=14/13$, and $T=10$

6. Setting capacity

To this point we have shown that the strategic behavior of walk-in customers may induce the service provider to alter its reservation policy. Whether and to what extent the firm alters its policy depends on its capacity level. Regardless of whether walk-in customers are worth more or less than advance customers, the firm can essentially ignore the strategic behavior of the walk-in segment if it has enough capacity. That leads to the question of whether the service provider would ever choose a capacity level at which it must account for its customers behaving strategically.

Here we examine this issue numerically. We return to the setting in which walk-in customers are homogeneous with valuation V. We suppose that the marginal cost of building capacity is $c>0$ and that both π_a and π_w are greater than c. Thus both segments are profitable, and the lower value segment is not just a means to salvage excess capacity. We also allow demand from segment j to be random with continuous distribution $F_j(n)$ on support $[\underline{N}_j,\overline{N}_j]$ for $j=a,w$. This is necessary to have an interesting problem. If one segment's demand were deterministic as we assumed earlier, the firm could build adequate capacity for that segment and deal with the other segment separately.

6.1. Non-Strategic Walk-in Customers

To establish a baseline, we first consider the firm's optimal capacity choice and reservation policy when walk-in customers do not behave strategically (i.e., when $T = 0$). For a fixed capacity K, revenue from advance customers given a reservation level R is $\pi_a S_a(R)$. Given realized advance sales n, there are $K - \min\{R, n\}$ seats available for walk ins which yields an expected revenue of $\pi_w S_w(K - \min\{R, n\})$. The service provider's total expected revenue is then:

$$\Pi(R, K) = \pi_a S_a(R) + \pi_w \int_{\underline{N}_a}^{R} S_w(K - n) f_a(n)\, dn + \pi_w S_w(K - R) \bar{F}_a(R) - cK.$$

The above revenue is increasing in R if $\pi_a > \pi_w$. For a given K, the firm would always set its reservation level to K. When walk-in customers offer a higher margin, the provider's expected revenue given K is strictly concave in R, and is maximized a the Littlewood quantity R_L.

PROPOSITION 6. *Let $R^*(K)$ denote the optimal reservation level for a given K when walk-in customers do not behave strategically. Let K_N^* be the corresponding optimal capacity choice.*

1. *If $\pi_a > \pi_w$, $R^*(K) = K$ and K_N^* solves $\left(\pi_w \int_{\underline{N}_a}^{K} \bar{F}_w(K - n) f_a(n)\, dn\right) + \pi_a \bar{F}_a(K) = c$.*
2. *If $\pi_a < \pi_w$, $R^*(K) = R_L$ and K_N^* solves $\left(\pi_w \int_{\underline{N}_a}^{R_L} \bar{F}_w(K - n) f_a(n)\, dn\right) + \pi_a \bar{F}_a(R_L) = c$.*

6.2. Strategic Walk-in Customers

We now consider walk-in customers with a positive travel cost $T > 0$ who thus behave strategically. For concreteness, we suppose that the number of customers in each segment is uniformly distributed on $\left(0, \overline{N}_j\right)$ for $j = a, w$. We vary the travel cost (and thus manipulate the customers' net utility) and examine how this impacts the service provider's capacity and reservation level.

We first consider when advance customers are more valuable (See Figure 4(a)). As one would expect, for low values of T, the walk-in segment does not behave strategically. K_N^* and walk-ins net utility are sufficiently large that walk-in customers all request service. The firm's capacity and reservation level are fixed at K_N^*. However, as T increases, walk-in customers would self-ration if the firm stuck with a capacity of K_N^*. The firm responds by increasing its capacity while making all of that capacity available to advance customers. As the travel cost rises further, the firm continues to increase capacity but also limits reservations – guaranteeing some seats will be available for walk-in customers. These steps induce all walk-in customers to request service. That is, while the firm never chooses a capacity that leads some walk-in customers not to request service, it does alter its capacity and reservation policy from the non-strategic case. Note that even though the firm has more capacity than the non-strategic case, it may limit reservations enough that it serves on average fewer advance customers.

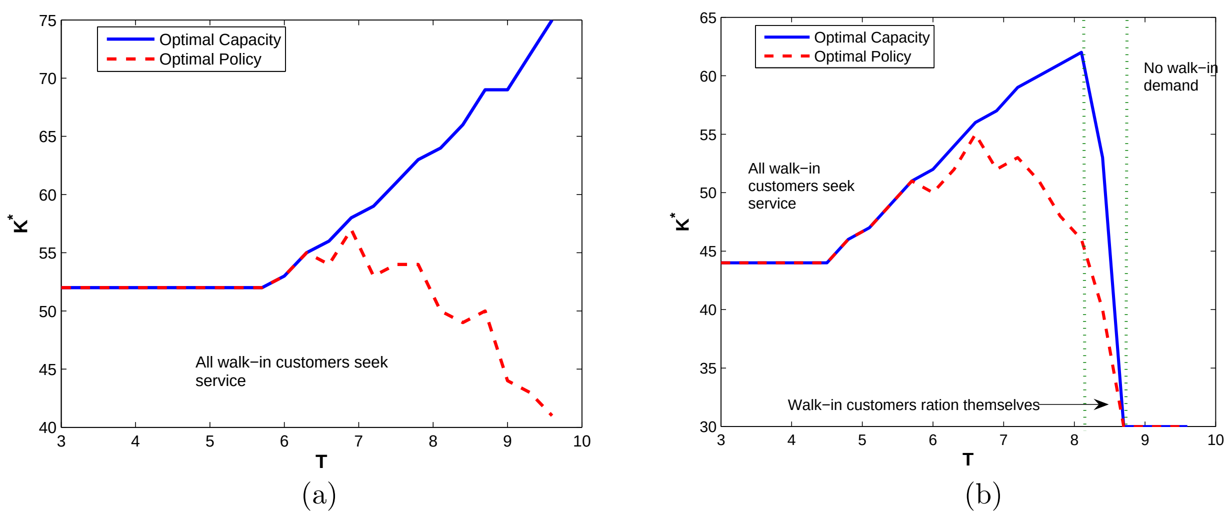


Figure 4 Optimal capacity (the solid line) and optimal reservation policy (the dashed line) as a function of T. For all examples $F_w(n) = \frac{n}{50}$ for $0 \leq n \leq 50$, $F_a(n) = \frac{n}{60}$ for $0 \leq n \leq 60$, $\pi_w = 1$, and $V = 10$. In (a), $\pi_a = 1.3$, and $c = 0.6$. In (b), $\pi_a = 1.8$, and $c = 0.9$.

Our first example has a relatively cheap capacity cost; the low value walk-in customers provide the firm with a 67% mark up over the cost of capacity. Our second example (See Figure 4(b)) has a higher capacity cost so walk-ins offer only a 11% mark up over the cost of capacity. Here, for low values of T, the story is similar to what we had before. The firm first follows its strategy for non-strategic customers and then expands capacity while (possibly) limiting reservations. However, for high values of the travel cost, the service provider gives up on inducing all walk-in customers to request service. It shrinks the available capacity and although it still limits the number of reservations, the firm saves fewer seats for walk-in customers. In this range, walk-in customers self-ration, randomizing between requesting service and staying home. As the travel cost increases further, the firm no longer does any business with the walk-ins. It sets its capacity as if it faced only advance customers with no salvage opportunity and makes all of its capacity available to them. In equilibrium, this is in fact the case as the walk-ins expect so few available seats that none request service.

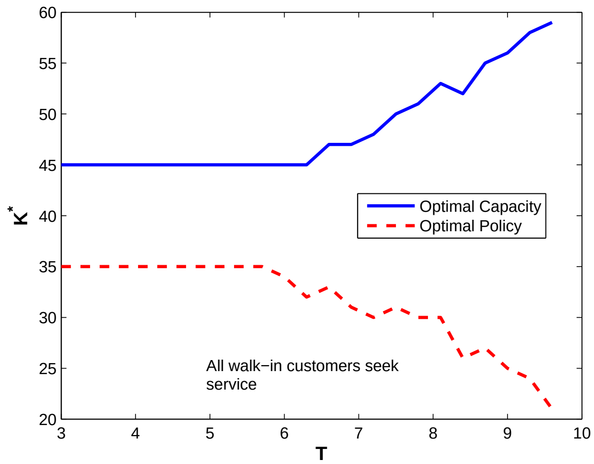
Figure 5 presents a complementary analysis for the setting when $\pi_w > \pi_a$. Figure 5(a) presents the case of cheap capacity. There is again a range in which travel costs are low enough that customer strategic behavior is not an issue; the firm uses K_N^* and R_L. However, as T increases, the firm alters both its capacity and reservation policy. Adding capacity and restricting reservations keeps all walk-in customers coming in. The story again changes when capacity costs increase (See Figure 5(b)). As in the previous case, when travel costs are sufficiently high, the firm stops trying to accommodate walk ins. Capacity is reduced and a greater fraction of capacity is made available to advance customers. Eventually the firm stops dealing with the walk-in segment.

These examples have shown that it is optimal for the firm to account for strategic interactions among the walk-in customers when setting its capacity. When capacity is cheap and the net utility of the walk-in segment is relatively high, this means building additional capacity and possibly reducing the reservation level. When the capacity is expensive and the net utility is low, less is done to court walk-in customer. Some (and eventually all) of these customers stay home.

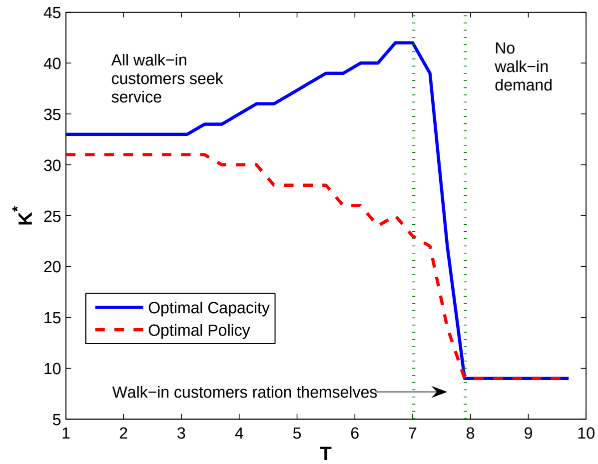
It is also worth noting that strategic walk-ins cost the service provider as soon as it deviates from what it would do without strategic customers. It at best sees the same demand but incurs a greater capacity cost. However, the firm always does better than selling only to the advance segment.

7. Conclusion

We have presented a simple model of a service provider selling to two segments. The first attempts to reserve capacity in advance of service. The other demands service immediately. The key assumptions are that the second, walk-in segment incurs a cost to request service but cannot verify how



(a)



(b)

Figure 5 Optimal capacity (the solid line) and optimal reservation policy (the dashed line) as a function of T. For all examples $F_w(n) = \frac{n}{50}$ for $0 \le n \le 50$, $F_a(n) = \frac{n}{60}$ for $0 \le n \le 60$, $\pi_w = 1$, and $V = 10$. In (a), $\pi_a = 0.8$, and $c = 0.6$. In (b), $\pi_a = 0.95$, and $c = 0.8$.

much capacity is available relative to market demand. Our model thus addresses settings such as restaurants or barber shops for which walk-in customers must be physically present to request service but cannot verify whether the firm has available capacity without visiting the firm. These assumptions make walking in a risky proposition; a customer may expend the cost of getting to the service provider but be unable to get a seat. A customer's decision to walk-in then depends on how much capacity the firm holds back and the actions of other walk-in customers.

We consider two cases. In the first, reservation customers are more profitable but the size of that segment is uncertain. The second is a variant of a classical revenue management problem (Littlewood (1972)); the late arriving segment is of uncertain size but more profitable. In either setting, when sufficiently few seats are likely available, walk-in customers randomize between walking in and staying home. As fewer and fewer seats are available, a smaller and smaller share of walk-in customers actually attempts to patronize the restaurant.

This problem is mitigated when capacity is large. In either setting, a firm with sufficient capacity can ignore any strategic behavior. For the first case, this implies making all capacity available for reservations and counting on walk-in traffic to fill any empty seats. In the second, it implies solving a newsvendor problem to determine how much capacity to hold back.

The service provider deviates from these policies when walk-in customers limit their patronage. When advance demand is more profitable but random, the optimal response is to lower the reservation level. The firm commits to potentially turning away some high-value advance customers in order to bolster walk-in demand. The firm may have second thoughts on this policy if reservation demand is strong, but it pays off if advance demand is low by assuring greater walk-in traffic.

The policy of saving extra seats for walk ins is intuitively appealing and may carry over to the setting with more profitable but random walk-in demand. But it is not always the best policy. It is only optimal when the margin on advance customers and the travel cost of walk-in customers are sufficiently low. When these are high, it is optimal to deal only with the advance-demand segment.

We show that this outcome of saving fewer seats for high-value walk-in customers is robust to changes in model assumptions. In particular it remains a possibility when reservation demand is random or when walk-in customers have heterogenous valuations. Further, we have demonstrated that strategic walk-in customers impacts the capacity choice of the firm and in particular the firm may choose a capacity that induces some walk-in customers to not visit the firm.

We have imposed a number of simplifying assumptions. For example, it is assumed that asking for a reservation is costless. If there is a fixed cost τ to requesting a reservation, a reservation customer must weigh the cost of asking with the likelihood of success. (We have in mind a non-monetary, inconvenience cost akin to the walk-in segment's travel cost.) When reservation demand is random, the analysis would parallel Lemma 2. If τ is sufficiently low relative to the segment's value of dining out U, everyone will still request a reservation and our results are unchanged. When the cost of requesting a reservation is high, advance customers would play a mixed strategy and not all advance customers would request a reservation.

If advance demand is deterministic, the analysis would mimic Lemma 1 but with a fixed number of seats R instead of an expected number of seats $K - S_a(R)$. Advance customers would randomize between requesting a reservation and staying home. Assuming staying home has a value of zero, they ask for a reservation with probability $\min\left\{1, \frac{UR}{\tau N_a}\right\}$ and the number of reservation requests would exceed R. As all reservations will be given out, our analysis is unchanged. Comparing this outcome with that of Section 3 shows that uncertainty in the number of available seats is a key driver of the results in Proposition 1. If walk-in customers could always verify the number of available seats, they would generate enough demand to fill them all.

There are some obvious ways in which our model can be extended. First, competition is a natural generalization. We conjecture that how the firms alter their reservation policies from the monopoly case would depend on their relative capacities as well as how customers value the two firms.

Second, one can more fully explore the consequences of disregarding strategic behavior. This is most pertinent when walk ins are the more profitable segment. Ignoring strategic behavior, the service provider would solve a newsvendor model but may then observe a pattern of demand that is stochastically smaller than he had anticipated. In a repeated environment, one would have system similar to Cooper et al. (2006) in which a decision maker repeatedly recalibrates a mis-specified model ignoring how his action impacts the observed data. If we assume that the firm starts with a correct estimate of walk-in demand, it would begin with the Littlewood quantity R_L but then pick higher and higher reservation levels as walk-in customer increasingly ration themselves. In the limit, all seats would be available via reservations. This may, in fact, be the optimal action but it could also be diametrically opposite to the best decision.

Appendix A: Proofs in Section 3

A.1. Proof of Lemma 1

Following Dana and Petruzzi (2001), the chance of getting a seat is given by the expected fill rate. If walk ins randomize with probability $\lambda(R)$, the chance any one customer gets a seat is $\gamma(R) = \frac{K - S_a(R)}{\lambda(R) N_w}$. If $N_w < \frac{V}{T}(K - S_a(R))$ and $\lambda(R) = 1$, $\gamma(R) > T/V$ and every customer has a positive expected utility from walking in; $\lambda(R) = 1$ is an equilibrium. If $N_w > \frac{V}{T}(K - S_a(R))$, $\gamma(R) \leq T/V$ if $\lambda(R) = 1$, and it cannot be an equilibrium for all customers to enter. For customers to randomize between walking in and staying home, we must have $\gamma(R) = T/V$, which yields 2. $\nu'(R) < 0$ follows from $S_a'(R) > 0$.

A.2. Proof of Proposition 1

The firm's profit given R is

$$\Pi(R) = \pi_a S_a(R) + \pi_w (K - S_a(R)) - \pi_w \int_{\underline{M}}^{K-\nu(R)} (K - \nu(R) - n) f_a(n)\, dm.$$

We then have

$$\Pi'(R) = S_a'(R)\left[\pi_a - \pi_w - \pi_w \frac{V}{T} F_a(K - \nu(R))\right], \tag{10}$$

Equation (10) yields (4). It is straightforward to show that second order conditions are satisfied. Note that $R^* \leq K$. Hence, if $F_a(K - \nu(K)) < \frac{\pi_a - \pi_w}{\pi_w}\left(\frac{T}{V}\right)$, it is optimal to have $R^* = K$.

Appendix B: Proofs in Section 4

B.1. Proof of Lemma 2

Suppose a given walk-in customer actually visits the restaurant with probability $\hat{\lambda}(R)$. Given $K - R$ available seats, expected walk-in sales are then $\hat{\lambda}(R) S_w\left(\frac{K-R}{\hat{\lambda}(R)}\right)$ and mean walk-in demand is $\hat{\lambda}(R)\mu_w$. The resulting fill rate is then

$$\frac{\hat{\lambda}(R) S_w\left(\frac{K-R}{\hat{\lambda}(R)}\right)}{\hat{\lambda}(R)\mu_w} = \frac{S_w\left(\frac{K-R}{\hat{\lambda}(R)}\right)}{\mu_w}.$$

Following Dana and Petruzzi (2001) (Deneckere and Peck (1995) have similar results), the equilibrium fill rate must equal T/V. Let $S_w(\hat{Z}) = \frac{T\mu_w}{V}$. If $K - R \geq \hat{Z}$, the chance of getting a seat exceeds T/V even if all the walk-in customers request reservation since $S_w(R)$ is increasing in R, and thus all walk-in customers request one. If the number of available seats is less than $\hat{Z}$, walk-in customers randomize between walking in and staying home. For indifference, we need $\frac{K-R}{\hat{\lambda}(R)} = \hat{Z}$.

B.2. Proof of Proposition 2

For $R_L \leq K - \hat{Z}$, the firm's profit is maximized by R_L. Beyond R_L profit is either decreasing or maximized at $\pi_a K$ which is less than $\hat{\Pi}(R_L)$ by the optimality of R_L. When $R_L > K - \hat{Z}$, the firm's profit is increasing for $R < K - \hat{Z}$. For R between $K - \hat{Z}$ and K, $\hat{\Pi}'(R) = \pi_a - \pi_w S_w\left(\hat{Z}\right)/\hat{Z}$, which yields the second part of the proposition.

Appendix C: Proofs in Section 5

C.1. Proof of Lemma 3

1. Define $\Psi(R,\lambda) = \rho S_w\left(\frac{K-R}{\lambda}\right) + (1-\rho)S_w\left(\frac{K}{\lambda}\right)$. Then, we have that $\frac{\partial \Psi(R,\lambda)}{\partial R} < 0$ and $\frac{\partial \Psi(R,\lambda)}{\partial \lambda} < 0$. Thus, by the implicit function theorem, $\lambda^\rho(R)$ is decreasing in R. Moreover, let R^0_ρ solve $\rho S_w(K - R) + \rho S_w(K) = \frac{T\mu_w}{V}$. Then, by construction $\lambda^\rho(R^0_\rho) = 1$. We want to note that $R^0_\rho < 0$ if $S_w(K) < \frac{T\mu_w}{V}$ and $R^0_\rho > K$ if $(1-\rho)S_w(K) > \frac{T\mu_w}{V}$.

2. Using the equilibrium conditions, we have that $S_w\left(\frac{K-R}{\hat{\lambda}(R)}\right) \geq S_w\left(\frac{K-R}{\lambda^\rho(R)}\right)$. And this implies that $\lambda^\rho(R) \geq \hat{\lambda}(R)$ since $S_w(R)$ is increasing in R.

3. and 4. The proof of the characteristic of the equilibrium is very similar to the proof of Lemma 2.

C.2. Proof of Proposition 4

Using the explicit values of the critical reservation levels, we have that

- $R^0 \geq R_L \Leftrightarrow \rho \leq \frac{\left(2(1-T)K - s\right)\sqrt{\pi_w}}{K - s\sqrt{\pi_w}}$.
- $R^{**} \geq R^0 \Leftrightarrow \rho \geq \frac{\left(2(1-T)K - s\right)\sqrt{\pi_w}}{\sqrt{\frac{1-T}{T}}K - s\sqrt{\pi_w}}$.
- $R^{**} \geq R_L \Leftrightarrow \rho \geq \frac{\left(2K\sqrt{T(1-T)} - s\right)\sqrt{\pi_w}}{K - s\sqrt{\pi_w}}$.
- $R^{**} \geq K \Leftrightarrow \rho \geq 2\sqrt{\pi_w T(1-T)}$.

Define $h_1(\pi_w) = \frac{\left(2(1-T)K - s\right)\sqrt{\pi_w}}{K - s\sqrt{\pi_w}}$, $h_2(\pi_w) = \frac{\left(2(1-T)K - s\right)\sqrt{\pi_w}}{\sqrt{\frac{1-T}{T}}K - s\sqrt{\pi_w}}$, $h_3(\pi_w) = \frac{\left(2K\sqrt{T(1-T)} - s\right)\sqrt{\pi_w}}{K - s\sqrt{\pi_w}}$, and $h_4(\pi_w) = 2\sqrt{\pi_w T(1-T)}$. Given that $1/2 < T < \frac{2K-s}{2K}$, we have that all of these functions are increasing in π_w. Moreover, we have that $h_1(\pi_w) \leq h_2(\pi_w)$ for all $\pi_w \geq 1$, and $h_2(\pi_w) \leq h_3(\pi_w) \leq h_4(\pi_w)$ for all $1 \leq \pi_w \leq \frac{1}{4T(1-T)}$. Finally we consider sufficiently small values of s to satisfy that $\frac{2(1-T)K-s}{\sqrt{\frac{1-T}{T}}K - s} > 2(1-T)$. We draw Figure 3 using these observations. Once we draw Figure 3, Proposition 4 follows immediately. (*Note: We draw the functions as concave in the figure but we do not need concavity for this proposition. To sustain concavity we need an additional condition that* $T < \frac{2K-3s}{2K}$*.*)

C.3. Proof of Lemma 4

1. By construction, the chance of getting a seat is $\frac{T}{T+\hat{\varepsilon}(R)}$. Then, it is immediate that only the walk-in customer with value greater than $\hat{\varepsilon}(R)$ will request a service.

2. Define $\psi(R,\varepsilon) = S_w\left(\frac{K-R}{\overline{\Phi}(\varepsilon)}\right)(T+\varepsilon)$. Then, we have that $\frac{\partial\psi(R,\varepsilon)}{\partial R} < 0$ and $\frac{\partial\psi(R,\varepsilon)}{\partial\varepsilon} > 0$. Thus, by the implicit function theorem, $\hat{\varepsilon}(R)$ is non-decreasing in the reservation level R.

3. Suppose $\hat{\varepsilon}(0) \leq 0$. Then, we have that $S_w(K) \geq \mu_w$ which is a contradiction since $\overline{N_w} > K$. Therefore, $0 < \hat{\varepsilon}(0) \leq \hat{\varepsilon}(R)$ for any R.

C.4. Proof of Proposition 5

1. Since $S_w(R)$ is concave in R, we have that $S_w(R) \geq \alpha S_w(R/\alpha)$ for $\alpha \leq 1$. Therefore,

$$\hat{\Pi}_\varepsilon(R) = \pi_r R + \pi_w \overline{\Phi}(\hat{\varepsilon}(R)) S_w\left(\frac{K-R}{\overline{\Phi}(\hat{\varepsilon}(R))}\right) \leq \pi_r R + \pi_w S_w(K-R) = \hat{\Pi}_L(R).$$

2. Let $\xi(R) = \frac{S_w(Z(R))}{Z(R)}$. Since $\hat{\varepsilon}(R))$ is increasing in R, $S_w(Z(R))$ is decreasing in R by the equilibrium condition 8. Therefore, $Z(R)$ is decreasing in R since $S_w(R)$ is increasing in R. We also have that $S_w(R)/R$ is decreasing in R. Then, by combining these two, we have that $\xi(R)$ is increasing in R.

One can rewrite the profit function as follows

$$\hat{\Pi}_\varepsilon(R) = \pi_r R + \pi_w \xi(R)(K-R),$$

and by taking the derivative of that, we have

$$\frac{d\hat{\Pi}_\varepsilon(R)}{dR} = [\pi_r - \pi_w \xi(r)] + \pi_w \frac{d\xi(R)}{dR}(K-R).$$

Note that if $\pi_r > \pi_w \xi(R_L)$, we have that $\pi_r > \pi_w \xi(R_L)$ for any $R < R_L$ since $\xi(R)$ is increasing in R. Then, it is clear that $\frac{d\hat{\Pi}_\varepsilon(R)}{dR} > 0$ for any $R < R_L$, and thus $R_L < \hat{R}_\varepsilon$.

Appendix D: Proofs in Section 6

D.1. Proof of Proposition 6

1. Let $\Pi(K)$ be the profit of the firm given the capacity K. Using the optimal reservation policy of the firm for any K, we have that

$$\Pi(K) = \int_0^K [\pi_a n + \pi_w S_w(K-n)] f_a(n) dn + \pi_a K \bar{F}_a(K) - cK,$$

when $\pi_a > \pi_w$. Taking the derivative of the above profit function, we have that

$$\Pi'(K) = \left(\int_0^K \pi_w \bar{F}_w(K-n) f_a(n) dn \right) + \pi_a \bar{F}_a(K) - c.$$

Hence, the results follows by equating the above FOC to zero.

2. In this case, again using the optimal reservation policy of the firm for any K, we have that

$$\Pi(K) = \int_0^{R_N^*(K)} [\pi_a n + \pi_w S_w(K-n)] f_a(n) dn + (\pi_a R_N^*(K) + \pi_w S_w(K - R_N^*(K))) \bar{F}_a(R_N^*(K)) - cK.$$

Note that for any K such that $F_w(K) \leq \frac{\pi_w - \pi_a}{\pi_w}$, the above profit function can be written as follows:

$$\Pi'(K) = \pi_w S_w(K) - cK.$$

Taking the derivative of this profit function, we have that

$$\Pi'(K) = \pi_w \bar{F}_w(K) - c \geq \pi_a - c > 0,$$

where the first inequality holds since $F_w(K) \leq \frac{\pi_w - \pi_a}{\pi_w}$. Therefore, the firm never chooses a K such that $F_w(K) \leq \frac{\pi_w - \pi_a}{\pi_w}$.

On the other hand, when $F_w(K) > \frac{\pi_w - \pi_a}{\pi_w}$, the derivative of the firm's profit function is:

$$\Pi'(K) = \left(\int_0^{R_N^*(K)} \pi_w \bar{F}_w(K-n) f_a(n) dn \right) + \pi_a \bar{F}_a(R_N^*(K)) - c,$$

since $\frac{dR_N^*(K)}{dK} = 1$. Then, the result follows by equating the above FOC to zero.

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References

Alexandrov A. and M. A. Lariviere. 2007. Are Reservations Recommended? working paper, The Center for Operations and Supply Chain Management, Northwestern University.

Arnett, A. 2005. Reservations Are Just a Few Mouse Clicks Away. *Boston Globe*. May 18.

Bertsimas, D. and R. Shioda. 2003. Restaurant Revenue Management. *Operations Research*. **51** 472 - 486.

Bruni, F. 2006. Reservations About Reservations. *New York Times* web site. dinersjournal.blogs.nytimes.com/2006/04/26/reservations-about-reservations.

Cachon, G. P. and A.G. Kök. 2007. Implementation of the Newsvendor Model with Clearance Pricing: How to (and how not to) Estimate a Salvage Value. *Manufacturing & Service Operations Management*. **9** 276-290.

Cachon G. P. and R. Swinney. 2009. Purchasing, pricing, and quick response in the presence of strategic consumers. *Management Science*, **55** 497-511.

Cooper, W. L., T. Homem-de-Mello, and A. J. Kleywegt. 2006. Models of the Spiral-down Effect in Revenue Management. *Operations Research*. **54** 968-987.

Dana, J. D. and N. C. Petruzzi. 2001. Note: The Newsvendor Model with Endogenous Demand. *Management Science*. **47** 1488-1497.

Deneckere, R. and J. Peck. 1995. Competition Over Price and Service Rate When Demand is Stochastic: A Strategic Analysis. *The RAND Journal of Economics*. **26** 148-162.

Kimes, S. E. 2004. Restaurant Revenue Management. working paper, The Center for Hospitality Research, Cornell University.

Lariviere, M. and J. van Mieghem. 2004. Strategically seeking service: How competition can generate poisson arrivals. *Manufacturing & Service Operations Management*, **6** 23-40.

Littlewood, K. 1972. Forecasting and Control of Passenger Bookings. *Proceeding of the Twelfth Annual AGIFORS Symposium*. 95-117.

Netessine, S. and C. Tang. 2009. *Consumer-Driven Demand and Operations Management Models*, Springer.

Shaked, M. and J. G. Shantikumar. 1994. *Stochastic Orders*, Springer.

Su, X. and F. Zhang. 2008. Strategic customer behavior, commitment, and supply chain performance. *Management Science*. **54** 1759-1773.

Talluri, K. T. and G. J. van Ryzin. 2004. *The Theory and Practice of Revenue Management*, Boston: Kluwer Academic Publishers.