Inefficient Diversification*

Kostas Bimpikis†　　Alireza Tahbaz-Salehi‡

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Abstract

This paper argues that in environments that exhibit non-convexities, risk diversification by financial institutions may be socially inefficient. To this end, we study a stylized, micro-founded model in which individual banks have an incentive to hold diversified portfolios in order to minimize the likelihood of bank runs. Yet, at the same time, diversification may increase the aggregate risk faced by the banks' depositors, creating a negative externality. The increase in systemic risk in such an environment is due to the fact that even though diversification decreases the probability of each bank's failure, it may increase the probability of joint failures, which may be socially inefficient when the depositors are risk-averse. The presence of such externalities suggests that financial innovations that enable banks to engineer more diversified portfolios may have non-trivial welfare implications.

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†Graduate School of Business, Stanford University.
‡Columbia Business School, Columbia University.

1 Introduction

The proliferation of new complex financial instruments can be considered as one of the most significant events in finance over the past two decades. According to the conventional wisdom, financial innovations in the form of credit default swaps and similar products have enabled financial institutions to diversify risk more effectively and as a result, have increased the efficiency of the system as a whole. In fact, the potential benefits of such instruments served as one of most important rationales for deregulation in the decade prior to The Financial Crisis of 2007–2009.[1] On the other hand, and more recently, it has also been argued that these instruments may have contributed to the fragility of the financial system by reducing the diversity among different institutions. For example, as argued by Andrew Haldane, the Executive Director of Financial Stability in the Bank of England, "diversification strategies by individual firms generated a lack of diversity across the system as a whole," leading to a financial system that exhibited "both greater complexity and less diversity," at the detriment of the system's stability (Haldane (2009, p. 8)).

In this paper, we argue that the presence of non-convexities in the economy (e.g., due to the possibility of costly bank runs), creates a wedge between the investment incentives of financial institutions and their depositors' welfare, thus, implying that risk diversification may indeed be inefficient from a social welfare point of view.

We study a stylized economy consisting of two competitive banking sectors, owned by risk-neutral bankers, and a mass of risk-averse consumers, who are subject to idiosyncratic preference shocks as in the canonical model of Diamond and Dybvig (1983). In order to ensure themselves against these shocks, the consumers deposit their funds in the banks, who would then invest in a common pool of assets on their behalf in exchange for standard demand deposit contracts. We further assume that even though the final returns on the banks' investments are observable, their investment decisions are not contractible.

Given that the banks are subject to runs when the returns on their investments are below a certain level, bankers have an incentive to choose diversified portfolios, as diversification decreases the probability of a run on each of the banks. Yet, such diversified portfolios may be socially inefficient: more diversification implies that the returns on the banks' portfolios would become more correlated, and hence, the probability of simultaneous bank runs would increase. We show that if the depositors are risk-averse, the welfare loss due to joint failures may outweigh the gains in reducing the probability of individual bank runs, implying an inefficient equilibrium.

Excessive equilibrium diversification in our model relies on a number of key ingredients. First and foremost, the possibility of costly bank runs creates non-convexities in the mapping from the banks' investment returns to the payments to the depositors upon withdrawal. In the absence of such non-convexities, the incentives of the banks and their depositors are always fully aligned, and thus, the equilibrium and efficient levels of diversification would coincide. The second ingredient is the assumption that the banks invest in a common pool of assets, which implies that they cannot

[1] See, for example, Greenspan (1997) and Financial Crisis Inquiry Commission (2011).

construct diversified portfolios without increasing the correlation between their returns. Clearly, if each bank has access to a different set of assets with independent returns, diversification does not lead to correlated portfolios. The third ingredient is the risk-aversion of the depositors. If the depositors are risk-neutral, their incentives (as far as the extent of the diversification is concerned) would be aligned with those of the bankers. Yet, risk-aversion guarantees that, all else equal, the depositors would be worse off as the probability of simultaneous runs increases. Finally, the fact that the banks' investment decisions are not contractible implies that bankers do not internalize the adverse effects of their diversification on depositors.

We remark that in our model, there are no externalities among the banks. Rather, it is the negative externality of the banks' decisions on the depositors that is the source of inefficiency. In particular, with diversified portfolios, a negative return on only one of the assets may lead to simultaneous runs on both banks; an outcome that would have been avoided with no diversification. Thus, effectively, by choosing more diversified (and hence, more similar) portfolios, the banks reduce the set of contingencies in which the depositors are (at least, partially) paid above the liquidation value.

Our analysis highlights that regulatory mechanisms that focus on each bank's risk in isolation may not be sufficient for mitigating risks at a systemic level. Rather, an effective regulatory policy may need to take the *endogenous* correlations between different banks' portfolios into account.[2] Our results also suggest that financial innovations that enable banks to engineer more diversified portfolios, may indeed lead to lower social welfare, as the probability that several financial institutions default together during periods of financial distress may increase. This observation thus suggests that the well-known benefits of financial innovations (such as more efficient levels of risk-sharing) may be inseparable from a "curse of financial engineering" manifested in the form of higher systemic risk.

Related Literature Several papers, such as Acharya (2009), Ibragimov, Jaffee, and Walden (2011) and Wagner (2010, 2011) study the possible adverse effects of diversification on increasing systemic risk. Like the current paper, this literature emphasizes the fact that joint failures of financial institutions may create a higher social cost compared to that of individual failures. The key common assumption in these papers is that a systemic market crash creates direct externalities between different intermediaries, leading to welfare losses. For example, Ibragimov et al. (2011) study a model in which the rationale for diversification is weakened in the presence of heavy-tailed risks and high correlations between risks within an asset class. The key assumption in their model is that joint failures of intermediaries would lead to slower recovery of the financial system, hence creating larger social costs. In contrast to this literature, no such inter-bank externalities exist in our model. Rather, the welfare loss is due to the fact that the banks do not internalize the impact of their joint investment decisions on the depositors.

Recently, Wagner (2010) shows that diversification may not be desirable if liquidation is more

[2] We emphasize that similar observations have been made by others. See, for example, Acharya, Pedersen, Philippon, and Richardson (2010), Adrian and Brunnermeier (2011) and Brunnermeier, Gorton, and Krishnamurthy (2011).

costly in a systemic crisis in which multiple banks have to liquidate assets at the same time. In particular, he shows that if the banks' liquidation costs depend on the extent of liquidation by other banks — for example, due to pecuniary externalities — then the negative externality that banks impose on one another guarantees that banks' portfolio choices would be inefficient. In a related paper, Wagner (2011) explores the diversification-diversity tradeoff in the presence of pecuniary externalities, and shows that the risk of joint liquidation creates an incentive for investors to forgo the benefits of diversification and hence, optimally choose heterogenous, not fully diversified portfolios. In contrast to these works, our model exhibits no such pecuniary externalities as liquidation costs do not depend on other banks' portfolios, returns, or liquidation decisions. Rather, it is the non-convexity of the mapping from the assets' returns to the payments to the depositors that is driving our results. Furthermore, in contrast to Wagner's, banks in our model find it optimal to hold fully diversified portfolios to minimize the probability of a run.

The paper is also related to recent works by Stiglitz (2010a,b), who, by the means of a reduced-form model, argues that full integration of global financial markets can exacerbate contagion, whereas capital controls can be welfare enhancing. We build a micro-founded model which properly accounts for the benefits and costs of diversification in a banking context and show that non-convexities create a wedge in the incentives of banks and depositors. As a result, under fairly general conditions, the equilibrium level of diversification does not coincide with the efficient level.

Our paper is also related to Acharya and Yorulmazer (2007, 2008), who study a model in which banks have an *ex ante* incentive to herd and increase the likelihood of joint failures in order to induce a bailout by the government. Somewhat relatedly, Farhi and Tirole (2012) argue that untargeted policy instruments used by a central bank during the times of financial distress, such as lowering the Fed Funds rate, would incentivize the banks to take on too much correlated risk. A different strand of literature, such as Kubler and Schmedders (2012) and Simsek (2013), studies the potential negative implications of financial innovations on portfolio risk and asset price volatility when agents have heterogenous beliefs. Also relevant is Buffa (2013), who develops a structural model of credit risk in which asset value dynamics are endogenously determined by optimal portfolio allocation. He shows that in the presence of systemic externalities, strategic considerations lead financial institutions to adopt polarized and stochastic risk exposures, but without sacrificing full diversification. Finally, our paper is related to the broader literature on systemic risk and financial contagion, such as Shaffer (1994), Allen and Gale (2000), Goldstein and Pauzner (2004), Dasgupta (2004), and more recently, Acharya et al. (2010) and Allen, Babus, and Carletti (2012), among others.

Outline of the Paper The rest of the paper is organized as follows. We first present a simple reduced-form variant of the model in order to illustrate the key economic forces at play. Section 3 contains the full-fledged micro-founded model. Our main results are presented in Section 4. Section 5 concludes. All proofs and some other mathematical details can be found in the Appendices.

2 Reduced-Form Model

This section contains a simple reduced-form version of the model, which we use to illustrate the key economic forces at play. The complete micro-founded model is presented and analyzed in the subsequent sections.

Consider an economy consisting of two risk-neutral financial institutions (henceforth, banks for short), indexed by a and b, and a risk-averse representative consumer who has deposited a unit of endowment in each of the banks. Each bank can invest the deposits in a distinct, "non-overlapping" set of finitely many assets with independent returns. The banks are subject to default (for example, due to a run) if the returns on their investments are below some exogenously given threshold d, in which case they make zero profits. On the other hand, if the realized return of the bank's investment is larger than d, it obtains a constant payoff of $R > 0$. Thus, bank $i \in \{a, b\}$ chooses a portfolio that minimizes its default probability, p_i.[3]

The payoff to the representative depositor also depends on the returns of the banks' portfolios. In particular, if the realized returns of a bank is larger than d, the depositor receives a return of c, whereas she only gets $\alpha < c$ in case of default.[4] Thus, the expected utility of the representative depositor can be written as

$$V_n = u(2c) + (p_a + p_b)\big[u(c+\alpha) - u(2c)\big] + p_a p_b\big[u(2c) + u(2\alpha) - 2u(c+\alpha)\big],$$

which is strictly decreasing in both p_a and p_b. This immediately implies that when the sets of assets in which the banks invest in do not overlap, the incentives of the banks and the depositor are fully aligned: all parties prefer investments that reduce the probability of *individual* defaults. In particular, if diversified portfolios decrease the probability of an individual failure, diversification is desirable both from the banks' and the social welfare points of view.

This picture, however, would be dramatically different if there is some overlap in the sets of assets that the two banks can invest in. Such overlapping investments imply that the returns on the banks' investments may no longer be independent. Clearly, the optimal portfolio from the banks' point of view remains identical to the non-overlapping assets case, as bank i still prefers investments that minimize p_i. However, the representative depositor's expected utility would be equal to

$$V_o = p_{ab} \cdot u(2\alpha) + (p_a + p_b - 2p_{ab})u(c+\alpha) + (1 - p_a - p_b + p_{ab})u(2c),$$

where p_{ab} is the probability that both banks default simultaneously. The above expression suggests that the consumer's utility not only depends on the probability of individual failures, p_a and p_b, but also on the probability of a systemic failure in which both banks default *simultaneously*. Hence, even if diversification reduces the probability of individual failures (and hence, maximizing each bank's profits), the induced correlation in banks' returns may reduce the depositor's utility (by increasing p_{ab}) and possibly even social welfare. Yet, such an effect is not internalized by the banks, as

[3]The assumption that each bank can invest in only finitely many assets with independent returns implies that it cannot completely diversify the risk away.

[4]In the full-fledged model presented in Section 3 we provide micro-foundations for these assumptions.

the incentives of the banks and the depositor are no longer fully aligned. This negative externality implies that, in contrast to the non-overlapping case, in the presence of common assets the welfare implications of diversification are non-trivial.

We remark that regardless of the extent of the overlap between the two sets of assets, the expected utility of a risk-neutral depositor is equal to

$$V_n = V_o = 2c - (p_a + p_b)(c - \alpha),$$

which is strictly decreasing in p_a and p_b. Thus, when the depositor is risk-neutral, her interests are fully aligned with those of the banks, even if the sets of assets in which banks can invest in are overlapping: under risk-neutrality, all parties only care about reducing the probability of individual failures. This is not surprising in light of the fact that the expected utility of such a depositor no longer depends on the correlation between the transfers she receives from the two banks.

The above simple example thus points towards a more general observation: the interplay between (i) the non-convexities in the mapping from the assets' returns to the agent's payoff; and (ii) the agents' risk-aversion creates a wedge between the incentives of different agents, and may lead to inefficiently high levels of risk diversification. In the rest of the paper, we develop and analyze a micro-founded model and show that under fairly general conditions, the intuitions presented in this section carry through.

3 Micro-Founded Model and Equilibrium

3.1 Depositors

Consider a single-good economy that lasts for three dates $t = 0, 1, 2$. The economy is populated by a unit mass of depositors (also referred to as consumers) with 2 units of endowment of the good at $t = 0$ and no endowment at other dates. The individuals are *ex ante* identical, but are subject to idiosyncratic preference shocks, which affect their demand for the consumption good at future dates. In particular, the consumers have standard Diamond-Dybvig preferences: a fraction π of them are *impatient* (also referred to as short-lived and denoted by s) and value consumption only at $t = 1$, whereas the rest are *patient* (also referred to as long-lived and denoted by ℓ) and value consumption at both future dates; that is,

$$U^\theta(c_1, c_2) = \begin{cases} u(c_1) & \text{if } \theta = s \\ u(c_1 + c_2) & \text{if } \theta = \ell, \end{cases}$$

where $u(\cdot)$ is continuously differentiable, increasing and strictly concave. Each depositor learns her type at the beginning of date $t = 1$, which remains a private information of the individual throughout. Finally, we assume that, except for depositing their endowments in the banks, the depositors have no means of transferring their wealth from $t = 0$ to future dates.

3.2 Banks

In addition to the consumers, there are two competitive banking "sectors" in the economy, indexed by a and b, whose role is to make investments on behalf of the consumers. Each bank is owned by a risk-neutral banker who maximizes expected profits. Banks in each sector have k units of endowment which they need to pay as an entry cost. The deposits of the consumers are invested by the banks in two different assets. Even though the returns on the banks' investments are observable, we assume that they are non-contractible.

Given that the type of the consumers and the banks' portfolio returns are non-contractible, a *demand deposit contract* between a bank in sector $i \in \{a, b\}$ and the depositors is a pair $c_i = (c_{i1}, c_{i2})$, capturing the claim of depositors who withdraw in periods $t = 1$ and $t = 2$, respectively. In the event that a bank cannot fulfill its promise to repay all withdrawing depositors based on the face value of the demand deposit contracts, the banks' assets are liquidated and distributed equally among the withdrawing depositors.

After paying the entry cost k and raising capital from the depositors in exchange for demand deposit contracts, the banks invest in two distinct illiquid assets, indexed 1 and 2. Asset $j \in \{1, 2\}$ has a random return of z_j at period $t = 1$ and a fixed return of R after it matures at $t = 2$. The returns of the assets at $t = 1$ are independent and identically distributed with mean μ and a smooth probability density function $f(\cdot)$ which has full support over $[0, \infty)$.

We denote the level of investment of bank i in asset 1 by $\gamma_i \in [0, 1]$. Thus, γ_i captures the extent of diversification in bank i's portfolio, with $\gamma_i = 1/2$ corresponding to a fully diversified portfolio. We assume that banks can liquidate their portfolio at $t = 1$, but this liquidation is costly: if assets are liquidated prematurely at $t = 1$, they would have a fixed return of $\alpha < R$, which is a small positive number. For simplicity, throughout the paper, we focus on the limiting case that $\alpha \to 0$, which implies that in the case of a bank run, the bank needs to liquidate all its holdings of both assets.

3.3 Competitive Equilibrium

The strategy of a bank in sector i consists of a decision to enter, a demand deposit contract $c_i = (c_{i1}, c_{i2})$ offered to the depositors upon entry, and an investment decision $\gamma_i(c_i)$, if contract c_i is accepted by the depositors. On the other hand, a depositor chooses which pair of contracts to accept, how to deposit its endowment in the two banks, and whether to withdraw its deposits at date $t = 1$ after observing the realization of its type and the short-term returns of the banks' portfolios.[5]

Let $\Pi_i(c_i, \gamma_i)$ denote the expected profits of a bank in sector i per unit of deposit as a function of the face value of the contract it offers and its investment decision — provided that it is accepted by the depositors — when the patient depositors withdraw optimally. Thus, the total profits of such a bank are equal to $\omega_i \Pi_i(c_i, \gamma_i)$, where ω_i is the total amount of deposits in bank i.[6] The optimal

[5]Given that withdrawing at $t = 1$ is a strictly dominant action for impatient depositors, only patient depositors may wait until $t = 2$ to withdraw their deposits.

[6]Hence, by assumption, $\omega_a + \omega_b = 2$.

investment decision of the bank is thus given by

$$\gamma_i^*(c_i) \in \arg\max_{\gamma_i \in [0,1]} \Pi_i(c_i, \gamma_i). \tag{1}$$

We also denote the transfer, per unit of deposit, from bank i to the depositors of type $\theta \in \{s, \ell\}$ when a contract of face value c_i is accepted and the bank chooses γ_i by $T_i^\theta(c_i, \gamma_i)$.[7] Therefore, the expected utility of the depositors when they accept contracts c_a and c_b and banks choose γ_a and γ_b is equal to

$$\widehat{V}(c_a, c_b, \omega_a, \omega_b, \gamma_a, \gamma_b) = \pi \mathbb{E} u \left(\sum_{i \in \{a,b\}} \omega_i T_i^s(c_i, \gamma_i) \right) + (1-\pi) \mathbb{E} u \left(\sum_{i \in \{a,b\}} \omega_i T_i^\ell(c_i, \gamma_i) \right).$$

We can thus define the *ex ante* indirect utility of a depositor who accepts the pair of contracts c_a and c_b as

$$V(c_a, c_b, \omega_a, \omega_b) = \widehat{V}\left(c_a, c_b, \omega_a, \omega_b, \gamma_a^*(c_a), \gamma_b^*(c_b)\right).$$

Note that by (1), the investment decisions of the banks are independent of how the depositors allocate their funds between them. Given the above, we now define the competitive equilibrium of the economy.

Definition 1. A *competitive equilibrium* consists of a pair of demand deposit contracts $c_a, c_b \in \mathbb{R}_+^2$ offered by the banks, investment decisions $\gamma_a, \gamma_b : \mathbb{R}_+^2 \to [0,1]$, contract choice, and deposit and withdrawal decisions by the depositors such that

(i) For any $\hat{c}_a$, $\hat{c}_b$, $\hat{\gamma}_a$ and $\hat{\gamma}_b$, the withdrawal decision of each depositor is optimal.

(ii) For any contract $\hat{c}_i$, bank i invests optimally, i.e, $\gamma_i(\hat{c}_i) \in \arg\max_{\gamma_i} \Pi_i(\hat{c}_i, \gamma_i)$.

(iii) Banks in sector i offer contracts that maximize their profits.

(iv) Depositors accept the pair of contracts that offers them the highest utility, and deposit their endowments within the two banks optimally.

A few remarks are in order. First, note that the above notion of equilibrium requires that depositors withdraw their deposits from the banks optimally in *all* subgames that follow the acceptance of a contract. Furthermore, the contracts that are offered have to satisfy incentive-compatibility, as the depositors understand that each bank would choose the portfolio that maximizes its expected profits. Note that a contract c_i is accepted in equilibrium only if it satisfies the Rothschild and Stiglitz (1976) condition of robustness to the introduction of additional profitable contracts. Finally, we remark that, as in Diamond and Dybvig (1983), the banks in our model are subject to runs that arise due to self-fulfilling expectations or sunspots. Throughout the rest of the paper, however, we restrict our attention to what Allen and Gale (2007) refer to as "essential" banks runs and rule out the possibility of such coordination failures by assumption.

[7] Note that the transfer from the banks to the depositors is also a function of the returns of the assets z_1 and z_2. However, for notational simplicity we drop this dependence.

Definition 2. An equilibrium is *symmetric* if banks in both sectors offer identical demand deposit contracts and choose an identical level of diversification; i.e., if $(c_{a1}, c_{a2}) = (c_{b1}, c_{b2})$ and $\gamma_a = 1 - \gamma_b$.

Note that in any symmetric equilibrium, the depositors find it weakly dominant to allocate their funds equally between the two banks; that is, $\omega_a = \omega_b = 1$. In particular, if for a given contract, the banks find it optimal to not fully diversify their portfolio (i.e., $\gamma_a = 1 - \gamma_b \neq 1/2$), then a given depositor has a strict incentive to allocate her funds equally between the two banks, as it would enable her to diversify risk. On the other hand, if the banks choose $\gamma_a = \gamma_b = 1/2$, then no depositor can increase her expected payoff by depositing her funds unequally between the two banks. Thus, for the rest of the paper, we can restrict our attention to the case in which $\omega_a = \omega_b = 1$.

4 Diversification and Systemic Risk

In this section, we show that the equilibrium is not necessarily efficient. In particular, we show that under fairly broad conditions, the banks choose an overly diversified portfolio as far as social welfare is concerned. Hence, by forcing the banks to hold less diversified portfolios, a regulator can achieve an outcome that Pareto dominates the equilibrium allocation.

Before presenting our main results, however, it is helpful to compare the micro-founded model presented in the previous section with the reduced-form model of Section 2. The characterization provided in Appendix A shows that when the face value of the contracts offered by the banks satisfy

$$c_{i1} \leq c_{i2} < R/(1-\pi),$$

then there is a run on bank i if and only if $x_i < \pi c_{i1}$, where $x_i = \gamma_i z_1 + (1-\gamma_i) z_2$ is the short-term return of the bank's portfolio. This is due to the fact that when the short-term return of bank i's portfolio is below πc_{i1}, the bank needs to liquidate its assets in order to meet its obligations to the impatient depositors. However, this implies that it would not be able to meet its obligations to the patient depositors at $t = 2$, inducing a bank run. Conversely, when $x_i > \pi c_{i1}$, the bank does not need to liquidate its assets to pay the impatient depositors. Assumption $R > (1-\pi)c_{i2}$ then implies that the patient depositors have no incentive to withdraw at $t = 1$, ruling out a bank run.

Thus, the transfer from bank i to the depositors as a function of its short-term returns is given by

$$T_i^{\theta}(c_i, \gamma_i) = \begin{cases} x_i & \text{if } x_i < \pi c_{i1} \\ c_{i1} & \text{if } x_i > \pi c_{i1}, \theta = s \\ c_{i2} & \text{if } x_i > \pi c_{i1}, \theta = \ell. \end{cases}$$

Furthermore, the expected profit of the bank is equal to

$$\Pi_i(c_i, \gamma_i) = \mathbb{E}\Big[\big(R + x_i - \pi c_{i1} - (1-\pi)c_{i2}\big)\mathbf{1}_{\{x_i > \pi c_{i1}\}}\Big].$$

Figures 1(a) and 1(b) below depict the transfers to the depositors and bank i's profits in terms of i's short-term returns, respectively.

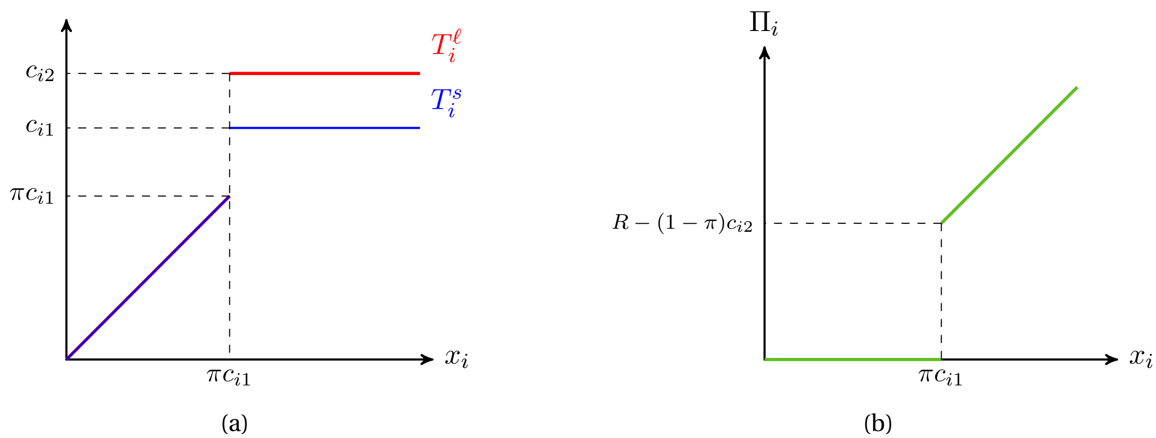

Figure 1. (a) Transfers to the patient (red) and impatient (blue) depositors; (b) bank's profits.

Figure 1 illustrates the similarities and distinctions between our micro-founded model and the reduced-form model of Section 2. First, note that as in the reduced-form model, there exists some threshold — equal to πc_{i1} — at which the payoffs to the depositors and the banks exhibit discontinuities. As already mentioned, such discontinuities are consequences of the fact that when the short-term returns on a bank's investment are small, withdrawal by the impatient depositors forces the bank to liquidate its assets prematurely. Anticipating this, the patient depositors would also decide to withdraw early, leading to a bank run. Hence, there exists a threshold under which the payoff to the bank and all depositors would be equal to zero and x_i, respectively. However, unlike the reduced-form model, the run threshold and the payoff levels are endogenously determined in equilibrium, as they depend on the face value of the contract offered by the bank. Finally, note that the payoffs to the depositors and banks are not necessarily constant on both sides of the threshold; rather, T_i^θ and Π_i are linearly increasing in x_i below and above the threshold, respectively. This implies that unlike the reduced-form model, banks' profits are not necessarily decreasing in the default probabilities.

4.1 Diversification in Equilibrium

Our first result provides sufficient conditions under which the banks choose a fully diversified portfolio in equilibrium.

Theorem 1. *Suppose that there exists a constant $\bar{z} > 0$ such that $f(\cdot)$ is non-decreasing over $[0, \bar{z}]$. Then, there exist a constant $\overline{R}$ and functions $\underline{\kappa}(\cdot)$ and $\overline{\kappa}(\cdot)$ such that for all $R > \overline{R}$ and all $\underline{\kappa}(R) < k < \overline{\kappa}(R)$ banks choose a fully diversified portfolio.*

Thus, for high values of R and k (in addition to some mild technical conditions), the banks would invest equally in both assets, as a fully diversified portfolio maximizes their profits. The intuition behind the above result can be understood by drawing parallels with the reduced-form model of

Section 2: a high enough rate of return R means that the banks find runs very costly, and hence, have an incentive to invest in such a way that the likelihood of a bank run is minimized. A high entry cost k, on the other hand, guarantees that the face value of the equilibrium contract and hence, the run threshold are small enough so that the banks can reduce the probability of a run via diversification.[8]

In view of the banks' desire for diversification, financial innovations that enable them to engineer more diversified portfolios have a direct positive effect on the depositors' welfare. In particular, competition guarantees that banks make zero profits in equilibrium and all the surplus generated by such innovations are fully transferred to the depositors via contracts with better terms.[9] Thus, in line with the conventional wisdom, this result illustrates how financial instruments that enable inter-bank risk-sharing would positively affect the social welfare.

Finally, we remark that Theorem 1 merely provides a set of sufficient (but not necessary) conditions for full diversification in equilibrium. As we show in Subsection 4.4, banks may choose a fully diversified portfolio even if the conditions of the theorem are not satisfied.

4.2 Diversification Externality

Theorem 1 establishes conditions under which banks invest equally in both assets in equilibrium. Our next theorem, which is the main result of the paper, shows that such an equilibrium is necessarily inefficient.

Theorem 2. *Under the assumptions of Theorem 1, the equilibrium is inefficient.*

Thus, even though banks choose to hold fully diversified portfolios, this outcome is not optimal from the social welfare point of view. Perfect correlation between the banks' short-term returns, which is a side effect of full diversification, increases systemic risk to the extent that it outweighs the benefits of diversification. The intuition behind this result is straightforward: with fully diversified portfolios, a negative return on only *one* of the assets may lead to a systemic crisis with simultaneous runs on *both* banks, whereas such an outcome would have been avoided with less diversification. Thus, effectively, by choosing more diversified (and hence, more similar) portfolios, the banks reduce the set of contingencies in which the depositors are (at least, partially) paid above the liquidation value.

In addition to the possibility of costly premature liquidations of the assets, the inefficiency established in Theorem 2 relies on three key ingredients. First and foremost, the result is driven by the

[8] If k is too small, the face value of the demand deposit contracts and hence, the threshold below which a run would occur would be large. In that case, banks may find it optimal to take excessive risk and hold a non-diversified portfolio. On the other hand, assumption $k < \overline{\kappa}(R) = R + \mu$ guarantees that the banks find it optimal to enter. See Appendix A for the detailed characterization of the equilibrium.

[9] If a bank in sector i can make positive profits by offering a contract $c_i = (c_{i1}, c_{i2})$, another bank in the same sector can "undercut" it by offering a contract with slightly better terms, say $\tilde{c}_i = (c_{i1}, c_{i2} + \delta)$ for some arbitrary small $\delta > 0$, and attract all the depositors. Note that the contract offered under this deviation remains incentive-compatible as the banks have a dominant action in all the subgames that follow the acceptance of the contract; they would choose a fully diversified portfolio regardless. However, such an incentive-compatible undercutting may not be possible if the banks do not have a dominant action in the investment subgames. If so, banks may make strictly positive profits in equilibrium, despite competition. For more on this, see, for example, Bennardo and Chiappori (2003).

presence of an overlap between the sets of assets the banks can invest in. Clearly, it is the presence of such overlaps which creates the "side effect" of correlated returns when banks decide to diversify: they cannot construct diversified portfolios without becoming more similar to one another. In the next subsection, we show that in the absence of such overlaps, in contrast to Theorem 2, diversification is always socially optimal.

The second key ingredient is the risk-aversion of the depositors. Had the depositors been risk-neutral, their expected utility function, $\widehat{V}$, would have been separable in transfers T_a^θ and T_b^θ from the banks, and hence, *ceteris paribus*, a change in the likelihood of simultaneous runs would have had no effect on social welfare. On the other hand, as the proof of the theorem shows, the more risk-averse the depositors are, the higher the social welfare gains of moving away from full diversification would be.

The inefficiency of equilibrium established in Theorem 2 also relies on the assumption that the investment decisions of the banks, i.e., γ_a and γ_b, are not contractible. This assumption implies that, once the contracts offered by the banks are accepted by the depositors, the banks do not internalize the adverse affect of diversification on the depositors when they choose their portfolios. This externality thus leads to an inefficient equilibrium. If the banks had the power to commit to specific investment decisions, then the equilibrium and efficient levels of diversification would have coincided.[10] Finally, we remark that the above result holds despite the fact that there are no inter-bank externalities.

Theorem 2 has a number of implications. First, it suggests that a regulator can strictly increase the social welfare by restricting the set of portfolios the banks can choose from. Given that there is excessive diversification in equilibrium, forcing the banks to hold less diversified portfolios would be socially beneficial. Furthermore, our result suggests that the introduction of financial instruments that facilitate more diversification and inter-bank risk-sharing has non-trivial welfare implications. In particular, it points towards the possible existence of a "curse of financial engineering": as banks use financial instruments to increase their profits and reduce their default probabilities, they may also, at the same time, increase the likelihood of a systemic crisis in which several financial institutions default simultaneously, with potentially severe social costs.

We remark that, even though Theorem 2 provides conditions under which there is too much diversification in equilibrium, the misalignment between the profit incentives of the banks and the depositors' welfare can also manifest itself through the existence of equilibria with too little diversification. In particular, as we show in Appendix A, when the long-term return R and the entry cost k are such that the contracts offered in equilibrium by the banks satisfy $R < (1-\pi)c_{i1}$, the banks choose a non-diversified portfolio (as their expected profit functions are convex in the short-term returns of their portfolios). The fact that c_{i1} is large, then implies that the run threshold for an individual bank is relatively high, and thus depositors' welfare would increase with more diversification.

[10]Note that full diversification would have been inefficient even if the banks had the power to commit to specific investment decision. However, the equilibrium would have no longer entailed full diversification, and the banks would have chosen the socially efficient level of diversification instead.

This is due to the fact that more diversification decreases the probability of bank defaults.

4.3 Non-Overlapping Assets

Our next result shows that if the pairs of assets in which the banks can invest in do not overlap, full diversification would be an efficient equilibrium.

Theorem 3. *Suppose that the banks have access to non-overlapping pairs of assets. Then, under the assumptions of Theorem 1, they choose a fully diversified portfolio. Furthermore, this equilibrium is efficient.*

Given that the banks cannot commit to specific investment decisions, they choose a fully diversified portfolio regardless of the presence (or lack thereof) of any overlap between the sets of assets available to them. Under either scenario, more diversification implies a lower default probability and hence, higher profits. Yet, with non-overlapping assets, full diversification no longer leads to correlated returns and hence, a higher likelihood of systemic crises. Thus, investment decisions that minimize the probability of individual defaults are also desirable from a social welfare point of view. This theorem also clarifies that the inefficiency identified in Theorem 2 is a direct consequence of the presence of overlapping assets and the induced correlation.

4.4 Exponential Returns

So far, our focus was on providing conditions under which the equilibrium portfolios chosen by the banks are overly diversified, while assuming that the short-term returns of the assets have a general distribution $f(z)$. In the remainder of this section, we assume that the short-term returns are exponentially distributed with mean $\mu = 1/\lambda$; that is, $f(z) = \lambda e^{-\lambda z}$. This particular parametric assumption enables us to determine and compare the equilibrium and efficient levels of diversification as a function of the expected returns of the assets. The following proposition summarizes our results.

Proposition 1. *There exist constants $\underline{\lambda}$ and $\bar{\lambda}$ and functions $\underline{\kappa}(\cdot)$ and $\overline{\kappa}(\cdot)$ such that,*

(a) For all $\lambda < \underline{\lambda}$ and $\underline{\kappa}(\lambda) < k < \overline{\kappa}(\lambda)$, the banks choose a fully diversified portfolio. If the pairs of assets available to the banks are identical, then this equilibrium is inefficient. On the other hand, with non-overlapping assets, the equilibrium is efficient.

(b) For all $\lambda > \bar{\lambda}$ and $\underline{\kappa}(\lambda) < k < \overline{\kappa}(\lambda)$, the banks choose a non-diversified portfolio in equilibrium. Furthermore, the no diversification equilibrium is efficient, regardless of the composition of the assets.

Part (a) simply states that for small values of λ — which equivalently corresponds to high expected short-term returns — the banks invest equally in both assets. This is not surprising in view of the fact that with high expected short-term returns, a fully diversified portfolio minimizes the probability of individual bank runs. Furthermore, it states that if both banks have to invest in the same pair of assets, a fully diversified equilibrium is never socially optimal. In other words, even though full diversification minimizes the risk of runs on individual banks, it actually increases the probability of simultaneous runs on both banks, increasing the aggregate risk faced by the depositors. Thus, this is the counterpart of Theorem 2 for the assets with exponential returns. The last statement of part (a) establishes that the inefficiency identified is indeed a consequence of the fact that banks invest in the same set of assets. As in Theorem 3, with non-overlapping sets of assets, the incentives of all parties would be fully aligned, as more diversification would no longer lead to higher inter-bank correlations. Taken together, part (a) of Proposition 1 also shows that the assumptions in the statement of Theorem 1 are not necessary for the existence of inefficient equilibria entailing full diversification.

Part (b) of Proposition 1, on the other hand, shows that for large enough values of λ — which correspond to low expected short-term returns — the banks choose a non-diversified portfolio. This observation rests on the fact that, unlike part (a), a non-diversified portfolio is what minimizes the probability of a run on each of the banks. In particular, when expected short-term returns are sufficiently smaller than the run thresholds, the banks' optimal strategy entails maximum risk-taking. Finally, the last statement of part (b) states that the no diversification equilibrium is precisely the socially efficient portfolio choice, even if the banks invest in the same pair of assets.

5 Conclusions

The main insight suggested by this paper is that in the presence of liquidation costs, risk diversification by financial institutions may be socially inefficient. The paper presents a stylized model in which banks have an incentive to hold diversified portfolios, as diversification reduces the probability of bank runs. As a side effect, however, diversification also increases the correlation between the returns on banks' investments. Such correlations make systemic crises in which multiple banks fail simultaneously more likely. Our results show that it is indeed possible that the welfare loss due to joint failures of financial intermediaries outweigh the gains of diversification. In particular, we establish that in the presence of risk-averse depositors, large enough returns and entry costs guarantee that full diversification is inefficient.

Unlike the rest of the literature, we show that diversification may increase systemic risk even if there are no negative inter-bank externalities. Rather, the inefficiency in our model is due to the fact that banks do not internalize the adverse effects of diversification on the aggregate risk faced by their depositors: the diversification undertaken by the banks reduces the set of contingencies in which the depositors are paid above the liquidation value, effectively increasing the likelihood of a worst-case outcome for the depositors.

Our results have a number of implications. First, the potential externality that the banks' diversification decisions impose on the depositors suggests that, under certain conditions, a regulator may be able to increase social welfare by restricting the extent of diversification undertaken by individual financial institutions. This goal can be achieved by either imposing an outright limit on the set of assets that different financial institutions can invest in or introducing prohibitions against certain banking activities. Alternatively, a regulator can discourage diversification by, say, increasing the capital requirements for excessively diversified financial institutions. Such regulations would effectively decrease the correlation between the returns of different intermediaries as one of the main side effects of diversification, and as a result, reduce the likelihood of a systemic collapse of the financial system.

On a broader level, our analysis highlights that regulatory mechanisms that only focus on reducing each institution's risk in isolation may be insufficient for mitigating risks at a systemic level. This is due to the fact that reducing the risk in each institution's portfolio is not necessarily equivalent to reducing the risks to the banking system from an aggregate perspective. Therefore, an effective regulatory policy may need to take the correlations between different banks' portfolios into account.

Relatedly, our results suggest that the overall welfare implications of new financial instruments that facilitate risk-sharing and diversification may be non-trivial. In particular, even though the introduction of complex financial instruments may make individual institutions more stable, the system as a whole may become more prone to systemic crises. Thus, the well-known benefits of financial innovations (such as better risk-sharing, reducing transaction costs of investing, etc.), may be inseparable from the so called "curse of financial engineering."

We emphasize that even though we focused on an admittedly stylized micro-founded model, our conclusions do not hinge on its specific details or the simplifying assumptions we made (such as demand deposit contracts, competitive banking sectors, possibility of banks runs, etc.). Rather, as in the reduced-form model of Section 2, the key ingredient is the presence of non-convexities in the payments to the bankers and the depositors. Such non-convexities not only may create a misalignment of incentives between different parties, but can also lead to a non-smooth allocation of financial losses.

Appendix

A Equilibrium Characterization

This appendix characterizes the equilibria of the subgames that follow the investment decisions of the banks. In particular, it characterizes the set of states in which patient depositors run on the banks, the transfers to the depositors, and the banks' expected profits as a function of the face value of the contracts and the short-term returns of the assets. Throughout, for notational simplicity, we drop the bank index i and denote the returns of its portfolio by $x = \gamma z_1 + (1-\gamma)z_2$.

We restrict our attention to subgames in which banks offer demand deposit contracts $c = (c_1, c_2)$ such that $c_1 \leq c_2$. In light of the fact that offering more to depositors who withdraw at $t = 1$ is a strictly dominated strategy, this assumption is without loss of generality. Also note that as in the standard model of Diamond and Dybvig (1983), there may exist equilibria in which all patient depositors withdraw their deposits at $t = 1$, regardless of the returns of the assets. In this paper, however, we restrict our attention to "essential" runs and ignore those that arise due to coordination failures between the depositors. Finally, given the argument at the end of Section 3, we can restrict our attention to symmetric equilibria in which each consumer deposits her funds equally in the two banks.

Depending on the face value of the contracts, we consider two different cases. In particular, we study the equilibrium depending on whether c_1 is smaller or larger than $R/(1-\pi)$.

A.1

First, suppose that the face value of the contract offered by the bank (and accepted by the depositors) satisfies $R < (1-\pi)c_1$. We show that, in this case, there is a run on the bank if and only if the short-term return on its investment is less than $c_1 - R$.

If $x > c_1 - R$, then the bank does not need to liquidate its assets to pay the impatient depositors. Moreover, the bank would be able pay at least c_1 to the depositors who withdraw at $t = 2$. Therefore, the patient depositors have no incentive to run on the bank at $t = 1$. In this case, the impatient and patient depositors get c_1 and $\min\{c_2, \frac{R+x-\pi c_1}{1-\pi}\}$, respectively. The payoff to the bank would then be equal to $\max\{R + x - \pi c_1 - (1-\pi)c_2, 0\}$.

On the other hand, if $x < \pi c_1$, the bank needs to liquidate its assets, as it cannot meet the demand at $t = 1$. Furthermore, $x < \pi c_1$ implies that $R + x < c_1$, and hence, the bank would not be able to guarantee c_1 to depositors who withdraw at $t = 2$, inducing a run. In this case, all depositors get x and the bank gets nothing.

Finally, if $\pi c_1 < x < c_1 - R$, even though the bank does not need to liquidate its portfolio to pay the impatient depositors, there will still be a bank run. This is due to the fact that assuming that no other patient depositor withdraws at $t = 1$, an impatient depositor has a strictly dominant strategy to withdraw at $t = 1$ and get c_1 instead of $\frac{R+x-\pi c_1}{1-\pi} < c_1$, inducing a run. In this case, the fact that $x < c_1$ implies that the bank gets nothing, whereas all depositors get x.

To summarize, the transfer from the bank to the depositors as a function of the face value of the contract and the realized returns of its portfolio is given by

$$T^{\theta}(c,\gamma) = \begin{cases} x & \text{if } x < c_1 - R \\ c_1 & \text{if } x > c_1 - R, \theta = s \\ \min\{c_2, \frac{R+x-\pi c_1}{1-\pi}\} & \text{if } x > c_1 - R, \theta = \ell. \end{cases}$$

The banks' expected profits, on the other hand, are

$$\Pi(c,\gamma) = \mathbb{E}\Big[\max\big\{R + x - \pi c_1 - (1-\pi)c_2, 0\big\}\Big].$$

A straightforward consequence of the above characterization is that in the subgames in which bank i offers a contract with face value $c_{i1} > R/(1-\pi)$, it chooses a non-diversified portfolio. This is in light of the fact that the expected profit profit function $\Pi_i(c_i,\gamma_i)$ is convex in x_i, and hence, is maximized by choosing $\gamma_i \in \{0,1\}$.

A.2

Now suppose that $R > (1-\pi)c_1$. We show that in this case, there is a run on the bank if and only if the short-term return on the bank's investments satisfy $x < \pi c_1$.

If $x < \pi c_1$, then the bank needs to liquidate its assets, as it is incapable of meeting its commitment to the impatient depositors. Moreover, $x < c_1$ implies that following liquidation, it cannot meet its commitment to the patient depositors either. This induces a run on the bank, and as a consequence, the bank gets nothing whereas all depositors get x.

If on the other hand $x > \pi c_1$, then the bank does not need to liquidate its assets to pay the impatient depositors. Moreover, the fact that $R + x - \pi c_1 > R > (1-\pi)c_1$ implies that the patient depositors have no incentive to withdraw at $t = 1$, ruling out a bank run. Thus, the impatient depositors get the face value of their contracts, whereas the patient depositors get $\min\{c_2, \frac{R+x-\pi c_1}{1-\pi}\}$. In this case, the bank gets $\max\{R + x - \pi c_1 - (1-\pi)c_2, 0\}$.

To summarize, transfers to the depositors can be expressed as

$$T^{\theta}(c,\gamma) = \begin{cases} x & \text{if } x < \pi c_1 \\ c_1 & \text{if } x > \pi c_1, \theta = s \\ \min\{c_2, \frac{R+x-\pi c_1}{1-\pi}\} & \text{if } x > \pi c_1, \theta = \ell. \end{cases}$$

The payoff to the banks, on the other hand, is equal to

$$\Pi(c,\gamma) = \mathbb{E}\Big[\max\big\{R + x - \pi c_1 - (1-\pi)c_2, 0\big\}\mathbf{1}_{\{x > \pi c_1\}}\Big], \tag{2}$$

where $\mathbf{1}$ denotes the indicator function. Note that unlike the previous case, $\Pi_i(c_i,\gamma_i)$ is not necessarily convex in x_i anymore. Furthermore, if $(1-\pi)c_{i2} < R$, then it exhibits a discontinuity at $x_i = \pi c_{i1}$.

B Planner's Problem

This appendix characterizes the problem of a social planner who intends to maximize social welfare by choosing the extent of diversification in banks' investments (i.e., γ_a and γ_b), subject to the incentive compatibility constraints of the banks who offer contracts that maximize their profits.

The social welfare is the sum of banks' profits and the expected utility of the depositors; that is,

$$W(\gamma_a, \gamma_b) = \widehat{V}(c_a, c_b, \gamma_a, \gamma_b) + \Pi_a(c_a, \gamma_a) + \Pi_b(c_b, \gamma_b) - 2k, \tag{3}$$

where c_a and c_b are the face value of the contracts offered by the banks under competition. We have the following lemma:

Lemma 1. *Given γ_i, γ_j and c_j, the demand deposit contract c_i is offered by bank i only if it solves the problem*

$$\max_{\hat{c}_i} \quad \widehat{V}(\hat{c}_i, c_j, \gamma_i, \gamma_j) \tag{4}$$

$$\text{s.t.} \quad \Pi_i(\hat{c}_i, \gamma_i) \geq k. \tag{5}$$

Furthermore, the constraint binds at the optimal solution.

Proof: Suppose that $c_i = (c_{i1}, c_{i2})$ is a contract that is offered by a bank in sector i. Clearly, the bank can always guarantee itself zero profits by deciding not to enter at all. Therefore, c_i is offered in equilibrium only if $\Pi_i(c_i, \gamma_i) \geq k$, implying that the contract must satisfy constraint (5). Now, suppose that there exists another contract $\hat{c}_i = (\hat{c}_{i1}, \hat{c}_{i2})$ such that $\widehat{V}(\hat{c}_i, c_j, \gamma_i, \gamma_j) > \widehat{V}(c_i, c_j, \gamma_i, \gamma_j)$ while satisfying constraint (5). Due to the continuity of $\widehat{V}$, there exists a small enough $\delta > 0$ such that the contract $\tilde{c}_i = (\hat{c}_{i1} - \delta, \hat{c}_{i2})$ satisfies

$$\widehat{V}(\tilde{c}_i, c_j, \gamma_i, \gamma_j) > \widehat{V}(c_i, c_j, \gamma_i, \gamma_j)$$

$$\Pi_i(\tilde{c}_i, \gamma_i) > \Pi_i(\hat{c}_i, \gamma_i) \geq k,$$

where the second inequality is a consequence of the fact that $\Pi_i(c_i, \gamma_i)$ is decreasing in c_{i1} for all γ_i. Thus, another bank can make strictly positive profits by offering $\tilde{c}_i$. Such a deviation would undercut the bank that had offered c_i and hence, would attract all the depositors. This contradicts the assumption that c_i was offered in equilibrium.

We next show that constraint (5) binds at the optimal solution. Suppose, that it does not bind for some solution c_i; that is, $\Pi_i(c_i, \gamma_i) > k$. Then, there exists a small enough δ such that the contract $\tilde{c}_i = (c_{i1}, c_{i2} + \delta)$ satisfies

$$\begin{aligned} \widehat{V}(\tilde{c}_i, c_j, \gamma_i, \gamma_j) &> \widehat{V}(c_i, c_j, \gamma_i, \gamma_j) \\ \Pi_i(\tilde{c}_i, \gamma_i) &> k, \end{aligned}$$

which is a contraction. ■

The above lemma shows that once the planner chooses γ_a and γ_b, banks in sector i offer a contract c_i in order to maximize the expected utility of the depositors, subject to making zero profits. Thus, the planner's problem reduces to choosing γ_a and γ_b in order to maximize $\widehat{V}(c_a, c_b, \gamma_a, \gamma_b)$ subject to the constraint that c_i solves problem (4).

C Proofs

C.1 Proof of Theorem 1

We first state and prove a simple lemma.

Lemma 2. $\Pi_i(c_i, \gamma_i^*(c_i))$ *is continuous and strictly decreasing in* c_{i1} *and* c_{i2}.

Proof: The equilibrium characterization provided in Appendix A shows that $\Pi_i(c_i, \gamma_i)$ is continuous in c_{i2} and γ_i. This is a consequence of the assumption that the distribution function of the returns has a well-defined density with full support over positive reals. Furthermore, it is easy to verify that the expected profit function is continuous in c_{i1} in both cases A.1 and A.2. Therefore, to establish continuity in c_{i1}, it is sufficient to show that $\Pi_i(c_i, \gamma_i)$ is continuous at $c_{i1} = R/(1-\pi)$. At this point, we have $\pi c_{i1} + (1-\pi)c_{i2} - R \geq \pi c_{i1}$, which implies

$$\mathbb{E}\Big[\max\left\{R + x_i - \pi c_{i1} - (1-\pi)c_{i2}, 0\right\}\Big] = \mathbb{E}\Big[\max\left\{R + x_i - \pi c_{i1} - (1-\pi)c_{i2}, 0\right\}\mathbf{1}_{\{x_i > \pi c_{i1}\}}\Big].$$

Thus, $\Pi_i(c_i, \gamma_i)$ is continuous in all arguments. Berge's Maximum Theorem then guarantees that $\Pi_i(c_i, \gamma_i^*(c_i))$ is continuous in c_i.

In order to prove monotonicity, note that, keeping γ_i fixed, the profits of bank i are strictly decreasing in c_{i1}. Therefore, if $\tilde{c}_i = (c_{i1} - \delta, c_{i2})$ for some $\delta > 0$, then

$$\Pi_i(\tilde{c}_i, \gamma_i^*(\tilde{c}_i)) \geq \Pi_i(\tilde{c}_i, \gamma_i^*(c_i)) > \Pi_i(c_i, \gamma_i^*(c_i)),$$

where the first inequality is a consequence of the fact that, by definition, $\gamma_i^*(\tilde{c}_i)$ maximizes the profits of bank i if it offers contract $\tilde{c}_i$ to the depositors. A similar argument implies that $\Pi_i(c_i, \gamma_i^*(c_i))$ is strictly decreasing in c_{i2}. ■

Proof of Theorem 1: Given the symmetry between the two assets, we restrict our attention to the case that $\gamma_i \in [0, 1/2]$ and show that bank i's profit is maximized at $\gamma_i = 1/2$ which corresponds to a fully diversified portfolio.

Fix a constant $0 < \bar{c} < \bar{z}/(2\pi)$ and suppose that $\underline{\kappa}(R) < k < \overline{\kappa}(R)$, where

$$\begin{aligned}
\overline{\kappa}(R) &= R + \mu \\
\underline{\kappa}(R) &= R + \mu - (1-\pi)\bar{c}.
\end{aligned}$$

Assumption $k < R + \mu$ ensures that the entry cost is not too high for the banks to enter in equilibrium. Otherwise, bank i would make negative expected profits even if it offers $c_i = (0, 0)$ to the depositors.

On the other hand, assumption $k > \underline{\kappa}(R)$ ensures that as long as $R > (1-\pi)\bar{c}$, bank i would never offer a contract in which $c_{i2} > \bar{c}$. Note that given the characterization in Appendix A, if the bank offers $c_i = (0, \bar{c})$, its expected payoffs following entry would satisfy

$$\Pi_i(c_i, \gamma_i) = R + \mu - (1-\pi)\bar{c},$$

which is strictly smaller than the entry cost k. Thus, given the monotonicity of $\Pi_i(c_i, \gamma_i^*(c_i))$ in c_{i1} and c_{i2} established in Lemma 2, the bank would only offer contracts in which $c_{i1} \leq c_{i2} < \bar{c}$.

Next, note that as long as $R > (1-\pi)\bar{c}$, bank i's expected profit as a function of the face value of the contract c_i and its investment decision γ_i is given by (2). Hence, the bank chooses γ_i in order to maximize

$$\Pi_i(c_i, \gamma_i) = \int_{\gamma_i z_1 + (1-\gamma_i) z_2 \geq \pi c_{i1}} \big[R - \pi c_{i1} - (1-\pi) c_{i2} + \gamma_i z_1 + (1-\gamma_i) z_2\big] f(z_1) f(z_2) dz_1 dz_2.$$

The derivative of the above expression with respect to γ_i is equal to

$$\frac{\partial \Pi_i}{\partial \gamma_i} = -\int_0^{\frac{\pi c_{i1}}{\gamma_i}} \int_0^{\frac{\pi c_{i1} - \gamma_i z_1}{1-\gamma_i}} (z_1 - z_2) f(z_1) f(z_2) dz_2 dz_1 + \frac{R - (1-\pi) c_{i2}}{(1-\gamma_i)^2} \int_0^{\pi c_{i1}/\gamma_i} (z - \pi c_{i1}) f(z) f\left(\frac{\pi c_{i1} - \gamma_i z}{1-\gamma_i}\right) dz.$$

It is easy to verify that the first term on the right-hand side is non-positive over $[0, 1/2]$, whereas the second term is equal to zero at $\gamma = 1/2$. Furthermore, the integral in the second term is strictly positive over $[0, 1/2)$. This can be verified by considering the derivative

$$\begin{aligned} \frac{\partial}{\partial \gamma_i} \int_0^{\frac{\pi c_{i1}}{\gamma_i}} (z - \pi c_{i1}) f(z) f\left(\frac{\pi c_{i1} - \gamma_i z}{1-\gamma_i}\right) dz \quad &= \quad -\frac{\pi^2 c_{i1}^2}{\gamma_i^3} f(0) f\left(\frac{\pi c_{i1}}{\gamma_i}\right) \\ &\quad - \int_0^{\frac{\pi c_{i1}}{\gamma_i}} \frac{(z - \pi c_{i1})^2}{(1-\gamma_i)^2} f(z) f'\left(\frac{\pi c_{i1} - \gamma_i z}{1-\gamma_i}\right) dz, \end{aligned} \tag{6}$$

which is negative given the assumption that $f'(\cdot)$ is positive over $[0, \bar{z}]$. Thus, $h_i(\gamma_i) \geq h_i(1/2) = 0$, where

$$h_i(\gamma_i) = \int_0^{\frac{\pi c_{i1}}{\gamma_i}} (z - \pi c_{i1}) f(z) f\left(\frac{\pi c_{i1} - \gamma_i z}{1-\gamma_i}\right). \tag{7}$$

Therefore, as long as

$$R > \hat{R} = \max_{\substack{\gamma_i \in [0, 1/2) \\ c_{i1}, c_{i2} < \bar{c}}} \frac{(1-\gamma_i)^2}{h_i(\gamma_i)} \int_{x_i < \pi c_{i1}} (z_1 - z_2) f(z_1) f(z_2) dz_2 dz_1 + (1-\pi) c_{i2}, \tag{8}$$

then $\partial \Pi_i / \partial \gamma_i$ is non-negative for all $\gamma_i \in [0, 1/2)$, implying that the bank's profits are maximized by choosing a fully diversified portfolio. Hence, for $R > \overline{R} = \max\{\hat{R}, (1-\pi)\bar{c}\}$ and $\kappa(R) < k < R + \mu$, the equilibrium entails full diversification.

The proof is complete once we verify that $\hat{R}$ is finite. In particular, it is sufficient to show that the ratio in (8) is finite as $\gamma_i \to 1/2$, as the numerator and the denominator are, respectively, finite and non-zero for all other values of $\gamma_i \in [0, 1/2)$. To this end, we apply L'Hopital's rule. The expression in (6) immediately implies that the derivative of $h_i(\gamma_i)$ in the denominator with respect to γ_i evaluated at $1/2$ is non-zero. Hence, $\hat{R}$ is finite, completing the proof. ■

C.2 Proof of Theorem 2

In order to establish the inefficiency of equilibrium, it is sufficient to show that the social planner can increase welfare by forcing the banks to hold less diversified portfolios than what they choose in equilibrium. To this end, we show that $\partial W/\partial \gamma_a$ is strictly negative at $\gamma_a = \gamma_b = 1/2$, where $W(\gamma_a, \gamma_b)$ is the social welfare function defined in (3).

By Lemma 1, the expected profit of each bank after entry is exactly equal to the entry cost k, regardless of the values of γ_a and γ_b. Hence, it is sufficient to study the change in the depositors' expected utility,

$$\frac{\partial W}{\partial \gamma_a} = \frac{\partial \widehat{V}}{\partial \gamma_a} + \sum_{i \in \{a,b\}} \left(\frac{\partial \widehat{V}}{\partial c_{i1}} \frac{\partial c_{i1}}{\partial \gamma_a} + \frac{\partial \widehat{V}}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial \gamma_a} \right).$$

Furthermore, Lemma 1 also shows that c_i solves problem (4). Thus, the first-order conditions imply for $i \in \{a, b\}$

$$\begin{aligned} \frac{\partial \widehat{V}}{\partial c_{i1}} + \zeta_i \left(\frac{\partial \Pi_i}{\partial c_{i1}} \right) &= 0 \\ \frac{\partial \widehat{V}}{\partial c_{i2}} + \zeta_i \left(\frac{\partial \Pi_i}{\partial c_{i2}} \right) &= 0. \end{aligned}$$

where ζ_i is a Lagrange multiplier. Therefore,

$$\frac{\partial W}{\partial \gamma_a} = \frac{\partial \widehat{V}}{\partial \gamma_a} - \sum_{i \in \{a,b\}} \zeta_i \left(\frac{\partial \Pi_i}{\partial c_{i1}} \frac{\partial c_{i1}}{\partial \gamma_a} + \frac{\partial \Pi_i}{\partial c_{i2}} \frac{\partial c_{i2}}{\partial \gamma_a} \right). \tag{9}$$

On the other hand, the fact that $\Pi_i(c_i, \gamma_i) = k$ for all γ_i implies

$$\begin{aligned} \frac{\partial \Pi_a}{\partial c_{a1}} \left(\frac{\partial c_{a1}}{\partial \gamma_a} \right) + \frac{\partial \Pi_a}{\partial c_{a2}} \left(\frac{\partial c_{a2}}{\partial \gamma_a} \right) &= -\partial \Pi_a / \partial \gamma_a \\ \frac{\partial \Pi_b}{\partial c_{b1}} \left(\frac{\partial c_{b1}}{\partial \gamma_a} \right) + \frac{\partial \Pi_b}{\partial c_{b2}} \left(\frac{\partial c_{b2}}{\partial \gamma_a} \right) &= 0. \end{aligned}$$

Replacing the above equalities in (9) leads to

$$\frac{\partial W}{\partial \gamma_a} = \frac{\partial \widehat{V}}{\partial \gamma_a} + \zeta_a \left(\frac{\partial \Pi_a}{\partial \gamma_a} \right). \tag{10}$$

Since $\Pi_a(c_a, \gamma_a)$ is a symmetric and differentiable function of γ_a around $1/2$, its derivative at that point has to be equal to zero. Hence, the proof is complete once we show that $\lim_{\gamma_a \to \frac{1}{2}} \partial \widehat{V}/\partial \gamma_a$ is strictly negative. In light of the fact that in a symmetric equilibrium banks offer identical contracts, without loss of generality, we can restrict our attention to the case that $c_a = c_b = (c_1, c_2)$. Furthermore, we assume that $\gamma_a \leq 1/2 \leq \gamma_b$.

Recall that under the assumptions of Theorem 1, the face value of the contracts offered in equilibrium satisfy $c_{i1}, c_{i2} \leq R/(1-\pi)$. Therefore, by the characterization provided in Appendix A, the expected utility of the impatient depositors when both banks offer contract $c = (c_1, c_2)$ and choose diversification levels γ_a and γ_b is equal to

$$\begin{aligned}\widehat{V}^s(c,c,\gamma_a,\gamma_b) &= u(2c_1)\mathbb{P}(x_a, x_b > \pi c_1) + \mathbb{E}[u(x_a+x_b)\mathbf{1}_{\{x_a,x_b<\pi c_1\}}] \\ &+ \mathbb{E}[u(c_1+x_b)\mathbf{1}_{\{x_a>\pi c_1>x_b\}}] + \mathbb{E}[u(x_a+c_1)\mathbf{1}_{\{x_b>\pi c_1>x_a\}}],\end{aligned}$$

where $\widehat{V}^s(c,c,\gamma_a,\gamma_b) = \mathbb{E}u\left(T_a^s(c,\gamma_a)+T_b^s(c,\gamma_b)\right)$ is the expected utility of the impatient depositors. Expanding the terms, we obtain

$$\begin{aligned}\widehat{V}^s(c,c,\gamma_a,\gamma_b) &= u(2c_1) + \int_0^{\pi c_1}\int_{z_2}^{\frac{\pi c_1-(1-\gamma_b)z_2}{\gamma_b}} \Big[u\big((\gamma_a+\gamma_b)(z_1-z_2)+2z_2\big)-u(2c_1)\Big]f(z_1)f(z_2)dz_1dz_2 \\ &+ \int_0^{\pi c_1}\int_{z_1}^{\frac{\pi c_1-\gamma_a z_1}{1-\gamma_a}} \Big[u\big((\gamma_a+\gamma_b)(z_1-z_2)+2z_2\big)-u(2c_1)\Big]f(z_1)f(z_2)dz_2dz_1 \\ &+ \int_0^{\pi c_1}\int_{\frac{\pi c_1-(1-\gamma_b)z_2}{\gamma_b}}^{\frac{\pi c_1-(1-\gamma_a)z_2}{\gamma_a}} \Big[u\big(c_1+\gamma_a z_1+(1-\gamma_a)z_2\big)-u(2c_1)\Big]f(z_1)f(z_2)dz_1dz_2 \\ &+ \int_0^{\pi c_1}\int_{\frac{\pi c_1-\gamma_a z_1}{1-\gamma_a}}^{\frac{\pi c_1-\gamma_b z_1}{1-\gamma_b}} \Big[u\big(c_1+\gamma_b z_1+(1-\gamma_b)z_2\big)-u(2c_1)\Big]f(z_1)f(z_2)dz_2dz_1.\end{aligned}$$

Differentiating the above with respect to γ_a implies

$$\begin{aligned}\lim_{\gamma_a,\gamma_b\to\frac{1}{2}}\frac{\partial\widehat{V}^s}{\partial\gamma_a} &= \int_{z_1+z_2<2\pi c_1} u'(z_1+z_2)(z_1-z_2)f(z_1)f(z_2)dz_1dz_2 \\ &+ 4\int_0^{\pi c_1}(\pi c_1-z)\big[u(2c_1)+u(2\pi c_1)-2u(c_1+\pi c_1)\big]f(z)f(2\pi c_1-z)dz.\end{aligned}$$

Due to symmetry, the first term on the right-hand side is equal to zero. The second term, on the other hand, is strictly negative as long as $u(\cdot)$ is strictly concave. Thus, by forcing bank a away from full diversification, the social planner can strictly increase the expected utility of the impatient depositors. A similar argument implies $\lim_{\gamma_a,\gamma_b\to\frac{1}{2}}\frac{\partial\widehat{V}^\ell}{\partial\gamma_a} < 0$, where $\widehat{V}^\ell(c,c,\gamma_a,\gamma_b) = \mathbb{E}u\left(T_a^\ell(c,\gamma_a)+T_b^\ell(c,\gamma_b)\right)$ is the expected utility of the patient depositors. Thus, the social planner can increase the expected welfare of the depositors by forcing banks away from full diversification, establishing that the full diversification equilibrium is inefficient. ■

C.3 Proof of Theorem 3

Once the contracts are accepted by the depositors, the optimal level of diversification for each bank is independent of the overlap between the sets of assets available to each one of them. Therefore, by Theorem 1, the banks would choose a fully diversified portfolio. In the rest of the proof, we show that full diversification is indeed efficient.

In order to prove the efficiency of equilibrium in the presence of non-overlapping assets, it is sufficient to show that $\partial W_n/\partial\gamma_a$ is non-negative for $\gamma_a \in (0, 1/2)$, where $W_n(\gamma_a,\gamma_b)$ is the social welfare function when the assets are non-overlapping and is equal to the sum of the banks' profits and the expected utility of the depositors.[11] In particular,

$$W_n(\gamma_a, \gamma_b) = \widehat{V}_n(c_a, c_b, \gamma_a, \gamma_b) + \Pi_a(c_a, \gamma_a) + \Pi_b(c_b, \gamma_b) - 2k,$$

in which $\widehat{V}_n(c_a, c_b, \gamma_a, \gamma_b)$ is the ex ante expected utility of the depositors when the banks offer contracts with face values c_a and c_b and choose diversification levels γ_a and γ_b. As in the proof of Theorem 2, we have

$$\frac{\partial W_n}{\partial \gamma_a} = \frac{\partial \widehat{V}_n}{\partial \gamma_a} + \zeta_a \left(\frac{\partial \Pi_a}{\partial \gamma_a}\right).$$

By Theorem 1, we know that $\partial \Pi_a / \partial \gamma_a$ is non-negative for $\gamma_a \in (0, 1/2)$. Given that the Lagrange multiplier ζ_a is also non-negative, it is sufficient to show that $\widehat{V}_n$ is non-decreasing in γ_a over $(0, 1/2)$. By the characterization in Appendix A,

$$\begin{aligned}
\widehat{V}_n(c_a, c_b, \gamma_a, \gamma_b) \;&=\; \big[\pi u(c_{a1} + c_{b1}) + (1-\pi) u(c_{a2} + c_{b2})\big] \cdot \mathbb{P}(x_a > \pi c_{a1}) \cdot \mathbb{P}(x_b > \pi c_{b1}) \\
&+\; \mathbb{E}\Big[\big(\pi u(c_{a1} + x_b) + (1-\pi) u(c_{a2} + x_b)\big) \mathbf{1}_{\{x_b < \pi c_{b1}\}}\Big] \cdot \mathbb{P}(x_a > \pi c_{a1}) \\
&+\; \mathbb{E}\Big[\big(\pi u(x_a + c_{b1}) + (1-\pi) u(x_a + c_{b2})\big) \mathbf{1}_{\{x_a < \pi c_{a1}\}}\Big] \cdot \mathbb{P}(x_b > \pi c_{b1}) \\
&+\; \mathbb{E}\left[u(x_a + x_b) \mathbf{1}_{\{x_a < \pi c_{a1}\}} \mathbf{1}_{\{x_b < \pi c_{b1}\}}\right],
\end{aligned} \tag{11}$$

where $x_i = \gamma_i z_1^i + (1 - \gamma_i) z_2^i$ denotes the short-term return of bank i's portfolio and (z_1^i, z_2^i) are the short-term returns on the assets available to i. Note that,

$$\frac{\partial}{\partial \gamma_a} \mathbb{P}(x_a > \pi c_{a1}) = \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}, \tag{12}$$

where $h_a(\cdot)$ is defined in (7). Furthermore,

$$\begin{aligned}
\frac{\partial}{\partial \gamma_a} \mathbb{E}\left[u(x_a + x_b) \mathbf{1}_{\{x_a < \pi c_{a1}\}} \mathbf{1}_{\{x_b < \pi c_{b1}\}}\right] \;&=\; \mathbb{E}\left[u'(x_a + x_b)(z_1^a - z_2^a) \mathbf{1}_{\{x_a < \pi c_{a1}\}} \mathbf{1}_{\{x_b < \pi c_{b1}\}}\right] \\
&\quad - \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\, \mathbb{E}\Big[u(\pi c_{a1} + x_b) \mathbf{1}_{\{x_b < \pi c_{b1}\}}\Big].
\end{aligned} \tag{13}$$

Given that the first term on the right-hand side of the above equality is non-negative for $\gamma_a \in (0, 1/2)$, we obtain

$$\frac{\partial}{\partial \gamma_a} \mathbb{E}\left[u(x_a + x_b) \mathbf{1}_{\{x_a < \pi c_{a1}\}} \mathbf{1}_{\{x_b < \pi c_{b1}\}}\right] \;\geq\; \frac{-h_a(\gamma_a)}{(1-\gamma_a)^2}\, \mathbb{E}\Big[u(\pi c_{a1} + x_b) \mathbf{1}_{\{x_b < \pi c_{b1}\}}\Big]. \tag{14}$$

Similarly,

$$\frac{\partial}{\partial \gamma_a} \mathbb{E}\Big[\big(\pi u(x_a + c_{b1}) + (1-\pi) u(x_a + c_{b2})\big) \mathbf{1}_{\{x_a < \pi c_{a1}\}}\Big] \geq \frac{-h_a(\gamma_a)}{(1-\gamma_a)^2} \Big[\pi u(\pi c_{a1} + c_{b1}) + (1-\pi) u(\pi c_{a1} + c_{b2})\Big]. \tag{15}$$

[11] The subscript in W_n and $\hat{V}_n$ reflects the assumption that, as opposed to the previous results, banks have access to two distinct pairs of non-overlapping assets.

Differentiating (11) with respect to γ_a and using (12)–(15) lead to

$$\begin{aligned}\frac{\partial \widehat{V}_n}{\partial \gamma_a} \geq\ & \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\big[\pi u(c_{a1}+c_{b1})+(1-\pi)u(c_{a2}+c_{b2})\big]\cdot \mathbb{P}(x_b>\pi c_{b1}) \\ +\ & \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\mathbb{E}\Big[\big(\pi u(c_{a1}+x_b)+(1-\pi)u(c_{a2}+x_b)\big)\mathbf{1}_{\{x_b<\pi c_{b1}\}}\Big] \\ -\ & \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\Big[\pi u(\pi c_{a1}+c_{b1})+(1-\pi)u(\pi c_{a1}+c_{b2})\Big]\cdot \mathbb{P}(x_b>\pi c_{b1}) \\ -\ & \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\,\mathbb{E}\Big[u(\pi c_{a1}+x_b)\mathbf{1}_{\{x_b<\pi c_{b1}\}}\Big].\end{aligned}$$

Given the fact that $c_{a2}\geq c_{a1}$, we obtain

$$\frac{\partial \widehat{V}_n}{\partial \gamma_a} \geq A\cdot h_a(\gamma_a) \tag{16}$$

for some positive constant A. Finally, as established in the proof of Theorem 1, $h_a(\gamma_a)\geq 0$ for all $\gamma_a\in(0,1/2)$, hence, implying that $\widehat{V}_n$ is maximized at $\gamma_a=1/2$. This completes the proof. ■

C.4 Proof of Proposition 1

We first state two simple lemmas, which we will later use in the proof of part (b).

Lemma 3. *Suppose that* $g:[0,a]\to\mathbb{R}$ *is differentiable, where* a *is a positive constant. Then,*

$$\lim_{\lambda\to\infty}\int_0^a g(z)\lambda e^{-\lambda z}dz = g(0).$$

Lemma 4. *For any continuous function* $g(z_1,z_2)$*,*

$$\lim_{\lambda\to\infty}\lambda e^{\frac{\lambda\pi c_1}{1-\gamma}}\int_0^{\frac{\pi c_1}{\gamma}}\int_0^{\frac{\pi c_1-\gamma z_1}{1-\gamma}} g(z_1,z_2)e^{-\lambda(z_1+z_2)}dz_2dz_1 = 0.$$

Proof of Proposition 1: To prove part (a), consider a sequence of economies characterized by $\{\lambda_r,k_r\}_{r\in\mathbb{N}}$ such that

$$\begin{gathered}\lim_{r\to\infty}\lambda_r = 0 \\ \underline{\kappa}(\lambda_r)<k_r<\overline{\kappa}(\lambda_r),\end{gathered}\tag{17}$$

where $\overline{\kappa}(\lambda)=R+\lambda^{-1}$ and $\underline{\kappa}(\lambda)=\overline{\kappa}(\lambda)-\frac{(1-\pi)R}{1-2\pi/3}$. As in the proof of Theorem 1, condition (17) guarantees that banks enter in equilibrium and that they never offer a contract in which $c_{i2}>\frac{R}{1-2\pi/3}$. Therefore, bank i's expected profit as a function of the face value of the contract c_i and its investment decision γ_i is given by (2).

The derivative of bank i's objective function in a subgame in which it offers contract $c_i=(c_{i1},c_{i2})$ to depositors satisfies

$$\left(\frac{1}{\lambda_r^2}\right)\frac{\partial \Pi_i}{\partial\gamma_i} = \int_0^{\frac{\pi c_{i1}}{\gamma_i}}\int_0^{\frac{\pi c_{i1}-\gamma_i z_1}{1-\gamma_i}}(z_2-z_1)e^{-\lambda_r(z_1+z_2)}dz_1dz_2+\frac{[R-(1-\pi)c_{i2}]e^{-\frac{\lambda_r\pi c_{i1}}{1-\gamma_i}}}{(1-\gamma_i)^2}\int_0^{\frac{\pi c_{i1}}{\gamma_i}}(z-\pi c_{i1})e^{\frac{\lambda_r z(2\gamma_i-1)}{1-\gamma_i}}dz.$$

Straightforward algebraic manipulations imply

$$\lim_{r\to\infty}\left(\frac{1}{\lambda_r^2}\right)\frac{\partial \Pi_i}{\partial \gamma_i}=\frac{(\pi c_{i1})^2(1-2\gamma_i)}{2\gamma_i^2(1-\gamma_i)^2}\left(R-(1-\pi)c_{i2}-\frac{\pi c_{i1}}{3}\right),$$

which is positive for $\gamma_i\in(0,1/2)$ and negative for $\gamma_i\in(1/2,1)$. Therefore, for large enough values of r, and hence, small enough values of λ, the expected profit of the bank is maximized at $\gamma_i^*(c_i)=1/2$ in all subgames. Thus, under the conditions of part (a), banks invest equally in both assets.

The proof of the second statement of part (a) is parallel to the proof of Theorem 2, which shows that the social planner never chooses a fully diversified portfolio. Thus, under the conditions of the proposition, a regulator can improve the welfare of the depositors by forcing banks to choose a less diversified portfolio.

Finally, proving that full diversification is efficient with non-overlapping pairs of assets follows a logic parallel to that of the proof of Theorem 3. In particular, recall that as established in the proof of Theorem 3, the full diversification equilibrium is efficient if $\partial\widehat{V}/\partial\gamma_a$ is positive for all $\gamma_a\in(0,1/2)$. Furthermore, by (16), it is sufficient to show that

$$h_a(\gamma_a)=\int_0^{\frac{\pi c_{a1}}{\gamma_a}}(z-\pi c_{a1})f(z)f\left(\frac{\pi c_{a1}-\gamma_a z}{1-\gamma_a}\right)dz>0.$$

Given that the returns are exponentially distributed, we have

$$\lim_{r\to\infty}\left(\frac{1}{\lambda_r^2}\right)h_a(\gamma_a)=\frac{(\pi c_{a1})^2(1-2\gamma_a)}{2\gamma_a^2(1-\gamma_a)^2},$$

which is positive for $\gamma_a<1/2$. Therefore, for large enough values of r, i.e., small enough λ, the social welfare is maximized with fully diversified portfolios, completing the proof.

To prove part (b), consider a sequence of economies characterized by $\{\lambda_r,k_r\}_{n\in\mathbb{N}}$ such that

$$\lim_{r\to\infty}\lambda_r=\infty$$

$$\underline{\kappa}(\lambda_r)<k_r<\overline{\kappa}(\lambda_r),$$

where $\overline{\kappa}(\lambda)$ and $\underline{\kappa}(\lambda)$ are defined as before. Once again, the banks enter in equilibrium and their profit functions are given by (2). Thus, as above, the derivative of bank i's objective function satisfies

$$\left(\frac{1}{\lambda_r^2}\right)\frac{\partial \Pi_i}{\partial \gamma_i}=\int_0^{\frac{\pi c_{i1}}{\gamma_i}}\int_0^{\frac{\pi c_{i1}-\gamma_i z_1}{1-\gamma_i}}(z_2-z_1)e^{-\lambda_r(z_1+z_2)}dz_1dz_2+\frac{[R-(1-\pi)c_{i2}]e^{-\frac{\lambda_r\pi c_{i1}}{1-\gamma_i}}}{(1-\gamma_i)^2}\int_0^{\frac{\pi c_{i1}}{\gamma_i}}(z-\pi c_{i1})e^{\frac{\lambda_r z(2\gamma_i-1)}{1-\gamma_i}}dz.$$

Furthermore,

$$\lim_{r\to\infty}\left(\frac{1}{\lambda_r}e^{\frac{\lambda\pi c_{i1}}{1-\gamma_i}}\right)\frac{\partial \Pi_i}{\partial \gamma_i}=\frac{\pi c_{i1}\left(R-(1-\pi)c_{i2}\right)}{(1-\gamma_i)(2\gamma_i-1)},$$

which is negative for $\gamma_i\in(0,1/2)$ and positive for $\gamma_i\in(0,1/2)$. Therefore, for large enough values of r, and hence, large enough values of λ, the expected profit of the bank is maximized at $\gamma_i^*(c_i)=0$

in all subgames — corresponding to a non-diversified portfolio. Thus, in equilibrium, banks do not diversify at all.

In order to prove that the no diversification equilibrium is indeed efficient, note that in view of (10) and the fact that $\frac{\partial \Pi_a}{\partial \gamma_a} < 0$ for large enough values of λ and $\gamma_a \in (0, 1/2)$, it is sufficient to show that $\lim_{r\to\infty} \frac{\partial \widehat{V}}{\partial \gamma_a} < 0$. Considering the impatient depositors first, we have,

$$
\begin{aligned}
\frac{\partial \widehat{V}^s}{\partial \gamma_a} &= \int_{\substack{\gamma_a z_1 + (1-\gamma_a) z_2 < \pi c_1 \\ \gamma_b z_1 + (1-\gamma_b) z_2 < \pi c_1}} u'((\gamma_a + \gamma_b)(z_1 - z_2) + 2 z_2)(z_1 - z_2) f(z_1) f(z_2) dz_1 dz_2 \\
&+ \int_{\substack{\gamma_a z_1 + (1-\gamma_a) z_2 < \pi c_1 \\ \gamma_b z_1 + (1-\gamma_b) z_2 > \pi c_1}} u'(c_1 + \gamma_a z_1 + (1-\gamma_a) z_2)) f(z_1) f(z_2) dz_1 dz_2 \\
&+ \frac{1}{\gamma_a^2} \int_0^{\pi c_1} (z - \pi c_1) \left[u(c_1 + \pi c_1) - u(2c_1) \right] f(z) f\left(\frac{\pi c_1 - (1-\gamma_a) z}{\gamma_a} \right) dz \\
&+ \frac{1}{(1-\gamma_a)^2} \int_0^{\pi c_1} (\pi c_1 - z) \cdot u\left(\pi c_1 + \frac{1-\gamma_b}{1-\gamma_a} \pi c_1 + \frac{\gamma_b - \gamma_a}{1-\gamma_a} z \right) f(z) f\left(\frac{\pi c_1 - \gamma_a z}{1-\gamma_a} \right) dz \\
&+ \frac{1}{(1-\gamma_a)^2} \int_0^{\pi c_1} (z - \pi c_1) \cdot u\left(c_1 + \gamma_b z + \frac{1-\gamma_b}{1-\gamma_a} (\pi c_1 - \gamma_a z) \right) f(z) f\left(\frac{\pi c_1 - \gamma_a z}{1-\gamma_a} \right) dz.
\end{aligned} \tag{18}
$$

Simple applications of Lemmas 3 and 4 imply

$$
\lim_{r\to\infty} \left(\frac{1}{\lambda_r} e^{\frac{\lambda_r \pi c_1}{1-\gamma_a}} \right) \frac{\partial \widehat{V}^s}{\partial \gamma_a} = \frac{\pi c_1 (1 - 2\gamma_a)}{1-\gamma_a} \left[u\left(\pi c_1 + \frac{1-\gamma_b}{1-\gamma_a} \pi c_1 \right) - u\left(c_1 + \frac{1-\gamma_b}{1-\gamma_a} \pi c_1 \right) \right]. \tag{19}
$$

In particular, after multiplying both sides of (18) by $\lambda_r^{-1} \exp\left(\frac{\lambda_r \pi c_1}{1-\gamma_a} \right)$, Lemma 4 implies that the limits of the first two terms on the right-hand side are equal to zero. On the other hand, Lemma 3 implies that the limit of the third term is equal to zero, whereas the last two terms have finite limits. Now, it is easy to verify that the right-hand side of (19) is negative for $\gamma_a \in (0, 1/2)$. Similarly, we can show that $\lim_{r\to\infty} \frac{\partial \widehat{V}^\ell}{\partial \gamma_a} < 0$ for $\gamma_a \in (0, 1/2)$. Therefore, we conclude that for large values of r, and consequently large values of λ, a non-diversified portfolio is efficient.

Finally, to complete the proof of part (b), we need to show that for large values of λ, non-diversified portfolios maximize social welfare even when banks invest in non-overlapping pairs of assets. Recall from the proof of Theorem 3 that

$$
\frac{\partial W_n}{\partial \gamma_a} = \frac{\partial \widehat{V}_n}{\partial \gamma_a} + \zeta_a \left(\frac{\partial \Pi_a}{\partial \gamma_a} \right).
$$

Given that $\partial \Pi_a / \partial \gamma_a$ is negative for large enough values of λ, it is sufficient to show

$$
\lim_{r\to\infty} \left(\frac{1}{\lambda_r} e^{\frac{\lambda_r \pi c_1}{1-\gamma_a}} \right) \frac{\partial \widehat{V}_n}{\partial \gamma_a} < 0.
$$

Differentiating (11) with respect to γ_a, we obtain

$$\begin{aligned}\frac{\partial \widehat{V}_n}{\partial \gamma_a} &= \left[\pi u(c_{a1}+c_{b1}) + (1-\pi)u(c_{a2}+c_{b2})\right] \cdot \mathbb{P}(x_b > \pi c_{b1}) \cdot \frac{\partial}{\partial \gamma_a}\mathbb{P}(x_a > \pi c_{a1}) \\ &+ \mathbb{E}\Big[\big(\pi u(c_{a1}+x_b) + (1-\pi)u(c_{a2}+x_b)\big)\mathbf{1}_{\{x_b<\pi c_{b1}\}}\Big] \cdot \frac{\partial}{\partial \gamma_a}\mathbb{P}(x_a > \pi c_{a1}) \\ &+ \mathbb{P}(x_b > \pi c_{b1}) \cdot \frac{\partial}{\partial \gamma_a}\mathbb{E}\left[u(x_a+x_b)\mathbf{1}_{\{x_a<\pi c_{a1}\}}\mathbf{1}_{\{x_b<\pi c_{b1}\}}\right] \\ &+ \frac{\partial}{\partial \gamma_a}\mathbb{E}\left[u(x_a+x_b)\mathbf{1}_{\{x_a<\pi c_{a1}\}}\mathbf{1}_{\{x_b<\pi c_{b1}\}}\right]. \end{aligned} \tag{20}$$

Recall from (12) that

$$\frac{\partial}{\partial \gamma_a}\mathbb{P}(x_a > \pi c_{a1}) = \frac{h_a(\gamma_a)}{(1-\gamma_a)^2},$$

and from (13) that

$$\begin{aligned}\frac{\partial}{\partial \gamma_a}\mathbb{E}\left[u(x_a+x_b)\mathbf{1}_{\{x_a<\pi c_{a1}\}}\mathbf{1}_{\{x_b<\pi c_{b1}\}}\right] &= \mathbb{E}\left[u'(x_a+x_b)(z_1^a - z_2^a)\mathbf{1}_{\{x_a<\pi c_{a1}\}}\mathbf{1}_{\{x_b<\pi c_{b1}\}}\right] \\ &\quad - \frac{h_a(\gamma_a)}{(1-\gamma_a)^2}\,\mathbb{E}\Big[u(\pi c_{a1}+x_b)\mathbf{1}_{\{x_b<\pi c_{b1}\}}\Big].\end{aligned}$$

Replacing the above expressions in (20), multiplying both sides by $\lambda_r^{-1}\exp\left(\frac{\lambda_r \pi c_1}{1-\gamma_a}\right)$, and noticing that by Lemma 3 the limit

$$\lim_{r\to\infty}\frac{1}{\lambda_r}h_a(\gamma_a)\exp\left(\frac{\lambda_r \pi c_1}{1-\gamma_a}\right) = -\frac{\pi c_{a1}(1-\gamma_a)}{1-2\gamma_a}$$

is finite, imply

$$\lim_{r\to\infty}\left(\frac{1}{\lambda_r}e^{\frac{\lambda_r \pi c_1}{1-\gamma_a}}\right)\frac{\partial \widehat{V}_n}{\partial \gamma_a} = -\frac{\pi c_{a1}}{(1-2\gamma_a)(1-\gamma_a)}\Big[\pi u(c_{a1}) + (1-\pi)u(c_{a2}) - u(\pi c_{a1})\Big]. \tag{21}$$

The right-hand side of the above expression is negative for $\gamma_a \in (0, 1/2)$. Hence, for large enough values of λ, the welfare is maximized if the banks' portfolios are not diversified, completing the proof.

■

References

Acharya, Viral V. (2009), "A theory of systemic risk and design of prudential bank regulation." *Journal of Financial Stability*, 5, 224–255.

Acharya, Viral V., Lasse H. Pedersen, Thomas Philippon, and Matthew Richardson (2010), "Measuring systemic risk." Working paper, New York University, Stern School of Business.

Acharya, Viral V. and Tanju Yorulmazer (2007), "Too many to fail — an analysis of time-inconsistency in bank closure policies." *Journal of Financial Intermediation*, 16, 1–31.

Acharya, Viral V. and Tanju Yorulmazer (2008), "Cash-in-the-market pricing and optimal resolution of bank failures." *Review of Financial Studies*, 21, 2705–2742.

Adrian, Tobias and Markus K. Brunnermeier (2011), "CoVaR." NBER Working Paper No. 17454.

Allen, Franklin, Ana Babus, and Elena Carletti (2012), "Asset commonality, debt maturity and systemic risk." *Journal of Financial Economics*, 104, 519–534.

Allen, Franklin and Douglas Gale (2000), "Financial contagion." *Journal of Political Economy*, 108, 1–33.

Allen, Franklin and Douglas Gale (2007), *Understanding Financial Crises*. Oxford University Press.

Bennardo, Alberto and Pierre-André Chiappori (2003), "Bertrand and Walras equilibria under moral hazard." *Journal of Political Economy*, 111, 785–817.

Brunnermeier, Markus K., Gary Gorton, and Arvind Krishnamurthy (2011), "Risk topography." *NBER Macroeconomics Annual*, 26, pp. 149–176.

Buffa, Andrea M. (2013), "Strategic risk taking with systemic externalities." Working paper.

Dasgupta, Amil (2004), "Financial contagion through capital connections: A model of the origin and spread of bank panics." *Journal of the European Economic Association*, 2, 1049–1084.

Diamond, Douglas W. and Philip H. Dybvig (1983), "Bank runs, deposit insurance, and liquidity." *Journal of Political Economy*, 91, 401–419.

Farhi, Emmanuel and Jean Tirole (2012), "Collective moral hazard, maturity mismatch, and systemic bailouts." *American Economic Review*, 102, 60–93.

Financial Crisis Inquiry Commission (2011), "The Financial Crisis Inquiry Report: Final Report of the National Commission on the Causes of the Financial and Economic Crisis in the United States." Technical report, U.S. Government Printing Office, Washington, DC.

Goldstein, Itay and Ady Pauzner (2004), "Contagion of self-fulfilling financial crises due to diversification of investment portfolios." *Journal of Economic Theory*, 119, 151–183.

Greenspan, Alan (1997), "H.R. 10, the Financial Services Competitiveness Act of 1997." Testimony before the House Committee on Banking and Financial Services, U.S. House of Representatives.

Haldane, Andrew G. (2009), "Rethinking the financial network." Speech delivered at the Financial Student Association in Amsterdam, The Netherlands, April 2009. www.bankofengland.co.uk/archive/Documents/historicpubs/speeches/2009/speech386.pdf.

Ibragimov, Rustam, Dwight Jaffee, and Johan Walden (2011), "Diversification disasters." *Journal of Financial Economics*, 99, 333–348.

Kubler, Felix and Karl Schmedders (2012), "Financial innovation and asset price volatility." *American Economic Review*, 102, 147–51.

Rothschild, Michael and Joseph E. Stiglitz (1976), "Equilibrium in competitive insurance markets: An essay on the economics of imperfect information." *Quarterly Journal of Economics*, 90, 629–649.

Shaffer, Sherrill (1994), "Pooling intensifies joint failure risk." *Research in Financial Services*, 6, 249–280.

Simsek, Alp (2013), "Speculation and risk sharing with new financial assets." *The Quarterly Journal of Economics*, 128, 1365–1396.

Stiglitz, Joseph E. (2010a), "Contagion, liberalization, and the optimal structure of globalization." *Journal of Globalization and Development*, 1.

Stiglitz, Joseph E. (2010b), "Risk and global economic architecture: Why full financial integration may be undesirable." *The American Economic Review, Papers and Proceedings*, 100, 388–392.

Wagner, Wolf (2010), "Diversification at financial institutions and systemic crises." *Journal of Financial Intermediation*, 19, 373–386.

Wagner, Wolf (2011), "Systemic liquidation risk and the diversity-diversification trade-off." *Journal of Finance*, 66, 1141–1175.