

Social Media Competition: Differentiation with User-Generated Content

Kaifu Zhang

Miklos Sarvary¹

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¹Kaifu Zhang is a Ph.D. student in Marketing and Miklos Sarvary is Professor of Marketing at INSEAD, Bd. de Constance, Fontainebleau, France. kaifu.zhang@insead.edu, miklos.sarvary@insead.edu.

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Abstract

This paper studies the competition between social media sites in a game theoretic framework. We model three important features of these institutions: (i) firms' content is usually user-generated; (ii) consumers' content preferences are governed by local network effects, and (iii) consumers have strong tendencies to multi-home. Our analyses reveal that ex-ante identical sites can acquire differentiated market positions that spontaneously emerge from user-generated content. Moreover, sites may obtain unanticipated and sometimes ambiguous market positions, wherein one site simultaneously attracts distinct and isolated consumer segments that seldom interact. Spontaneous differentiation reduces firm competition but may imply too much consumer segregation and lower social welfare. In most equilibria, a subset of consumers multi-home. We show that the degree of spontaneous differentiation increases in the localness of network effects. Interestingly, more multi-homing consumers imply reduced differentiation and intensify site competition. In one extension, we allow each firm to explicitly target a consumer segment by introducing certain design features. The results show that user-generated content can either enhance or override firms' product designs, leading to interesting situations where competing firms attract the users they do not intend to target. In equilibrium, the firms may either pursue maximal differentiation or choose identical designs, depending on the relative importance of UGC and site features.

1 Introduction

Social media applications, such as social networks (e.g., Facebook, Orkut), video sharing sites (e.g., YouTube), virtual world platforms (e.g., Second Life), on-line dating communities (e.g., eHarmony, Match.com) represent a diverse and rapidly growing industry. In this industry, typically, multiple sites compete in a relatively well-defined category (e.g., on-line dating). While these categories are quite different, social media applications share a number of important features: First, most of these sites rely extensively on user-generated content where *consumers* largely define the firms' product offerings. Typically, users have heterogeneous content preferences and favor content generated by similar users, leading to large but local network externalities. In addition, it is easy for consumers to join multiple communities (multi-homing), and sites compete for share of consumer time. While the overall business impact of social media has been well documented, the competitive implications of these novel economic properties have not been formally addressed. The goal of this study is to close this gap. We study the competition between social media sites defined by the above features in a game theoretic framework.

Although the industry is still young, a few stylized facts seem to emerge. First, as a consequence of user-generated content, the content positioning of competing firms can sometimes be determined by their *users*. Firms may acquire largely unintended and sometimes ambiguous product positioning. Consider the early players in the social networking industry. Myspace, Friendster and Google's Orkut are notable competitors before the ascent of Facebook. All three websites started in California and targeted the US market initially. Over time, however, Myspace became the largest player in the US market, Friendster remained popular only in South East Asian countries, and Orkut has become one of the most visited websites in three culturally distinct countries: Brazil, India and Estonia. While the sites were still competing for consumer time (as evidenced by the large number of consumers who join multiple networks), they acquired differentiated positions defined by distinct languages and cultures. There were clear evidences that this differentiation was not a

consequence of the firms' deliberate strategic choices. Differentiation also spontaneously emerged between Myspace and Facebook, the major contestants for US market leadership between 2006-2009. In an ethnographic study, Boyd (2010) documents a so-called 'white flight' from MySpace to Facebook, and suggests that the two leading players in the US social networking market acquired differentiated market positions with socio-economic connotations. Anecdotes in the on-line dating domain also suggest that consumers play important roles in shaping firms' market positioning. Match.com, one of the earliest entrant in this industry, had an ambitious positioning which catered to daters with a range of different objectives². Its major competitor eHarmony, however, specifically targeted the serious, marriage-minded daters by marketing its sophisticated personality matching algorithm. eHarmony became so successful among serious daters that Match.com soon (unwillingly) acquired a reputation of being more popular among daters who only seek short-term relationships. To compete with eHarmony, Match.com launched a premium service called Chemistry.com based on a different personality matching algorithm. Some consumers considered Chemistry.com's algorithm to be superior. But even to these consumers, eHarmony often remained more attractive due to its 'high quality pool' of serious daters³. The above examples illustrate some interesting cases where consumers play key roles in determining the firms' market positions. In the case of social networking sites, the firms acquired largely unintended market positions which the consumers 'chose' for them, a phenomenon we describe as 'spontaneous product differentiation'.

Second, while network externalities are clearly significant in all social media markets, different social media categories exhibit widely varying levels of concentrations. In some markets, we observe the emergence of a dominant site (e.g., YouTube in the video-sharing industry and most recently Facebook in the social-networking industry) and a 'winner-take-all' market structure, which is the typical market outcome in traditional network industries. In other markets, as discussed above, competing firms are able to coexist with differentiated positions despite strong

²Match.com states that 'Whether you are looking for marriage, a long term relationship, or just a friend, you will find what you're looking for at Match.com.'

³See <http://www.edatereview.com/> for consumers' comparison of these services

	Approx. Herfindahl Index
Mobile Community	0.1427
Casual Gaming	0.1583
Social Networks	0.3919
Auto Classified	0.4861
Video Sharing	0.5840

Table 1: Herfindahl Indices in different social media categories

network externalities. Table 1 lists the approximate Herfindahl indices⁴ for five social media markets and shows that the index exhibits large variations across these domains.

Third, some consumers have strong tendencies to multi-home in competing communities while others are loyal to one site. A survey by *Pew Research* on North American adult social network users reveals that over 40% of the respondents actively maintain multiple profiles on different websites while 43% of respondents state that they only maintain one profile in a single community⁵.

Our paper seeks to shed some light on these stylized patterns and trends in the social media industry. Beyond replicating market outcomes, we are interested in identifying the determinants of firm profits and study competing firms' strategic choices. To do so, we develop a model of competing social media sites with the following main features:

- **User-Generated Content:** We assume that firms host user-generated content. In the baseline model, we further assume that the sites don't produce content on their own. Each consumer

⁴The index has been derived from the top ten sites in each category, with the following formula $H = \sum_{i=1 \dots N} s_i^2$, where s_i is the market share of firm i (Hirschman 1964). Data source: Hitwise 2009.

⁵See the report 'Social Networks Grow: Friending Mom and Dad' (Lenhart 2009).

(user) generates content consistent with her own preferences. Consumers derive utility from consuming the content generated by all the other consumers in the same community.

- **Local Network Effects:** The marginal utility from consuming a piece of content depends on the similarity between the consumer who contributes the content and the consumer who consumes the content. Consumers have stronger preferences for content generated by similar others⁶.
- **Saturation from Content Consumption:** Repeated consumption of similar content yields decreasing returns to consumers.

Besides these main features, our model assumes that consumers develop expectations about firms' customer bases and maximize utility by freely allocating a limited amount of time between competing websites. On the supply side, we consider a duopoly of social media firms who profit from advertising. We start with a base-line model where the competing firms pursue identical product designs.

Our first set of results speak to the equilibrium market structures. The analysis reveals the existence of three qualitatively different types of equilibria. When network effects are relatively global, there exists a winner-take-all equilibrium where all consumers join a single dominant firm's network. When network effects are relatively local, ex-ante identical sites can obtain differentiated market positions that emerge spontaneously from user-generated content. The sites attract different but overlapping consumer segments who then generate content consistent with their respective tastes. When network effects are sufficiently local, there exists an interesting equilibrium where one sites attracts two distinct consumer segments who do not value the content generated by each other. Despite its ambiguous positioning, this site coexists with its competitor who has a clear market position. Importantly, we show that the type of market outcome depends on the localness

⁶As opposed to local network effects, we say that the network effects are global when a consumer's content preferences don't depend on who generates the content.

of network effects, not the magnitude of network effects. Firms are able to coexist under large network effects and winner-take-all outcome can emerge even when network effects are relatively small. In most equilibria, we also observe a segment of consumers who multi-home. Stronger saturation from content consumption enlarges this segment.

Our second set of results shed light on the properties of spontaneous differentiation and the determinants of firm profits. On the firm side, we show that spontaneous differentiation reduces firm competition similar to the case of classic horizontal differentiation. As expected, the degree of spontaneous differentiation is increasing in the localness of network effects. Thus, firm profits rise when members strongly favor the content generated by similar members. Interestingly, more multi-homing consumers result in fiercer competition between the communities and lead to lower profits. We show that this is a unique implication of user-generated content. It arises from the fact that as more users multi-home, the competing communities end up hosting overlapping content and face reduced differentiation. On the consumer side, we show that spontaneous differentiation may emerge even when consumers collectively prefer to join the same community. Thus, spontaneous differentiation may imply ‘too much’ consumer segregation from a social welfare perspective. These results resonate to many of the stylized facts mentioned above.

In one extension, we allow each firm to explicitly target a consumer segment by choosing website features. For example, a dating site can introduce personality test and compatibility matching algorithms to attract the users who value long term relationships. A social networking service can introduce on-line CV features to appeal to professional users. In these cases, the consumers value both website features and user-generated content. We find that in equilibrium, user-content can either enhance or override firm design. In the former case, both user-generated content and site designs contribute to product differentiation. When user-generated content overrides site design, each firm can attract the consumer segment that it did not intend to target. These results have implications for the firms site design strategies. Firms may either pursue maximal or min-

imal differentiation depending on the relative importances of design features and user-generated content.

The rest of the paper is organized as follows. In Section 2, we review the relevant literature in marketing and economics. Section 3 presents the model. Section 4 presents the analyses and discuss the equilibrium results. We present the extensions in Section 5. Section 6 discusses other aspects of the social media industry and concludes. To facilitate reading all proofs have been relegated to an appendix.

2 Literature Review

Our paper is related to several literature streams. First, it is related to the emerging literature on user-generated content and social media. Previous works have examined, for example, the users' incentives to share content (Berger 2011, Berger and Milkman 2011, Huang, Singh, and Ghose 2011), the interplay between content generation and content consumption (Ghose and Han 2011, Yang, Hu, Assael, and Winer 2011) and the impact of user-generated content on sales (Chevalier and Mayzlin 2006, Trusov, Bucklin, and Pauwels 2009). The emphasize of this paper is on the competition, in particular product differentiation, between social media sites.

First, it is related to the economics literature on product differentiation. Classic product differentiation models often assume a two-stage process where competing firms choose their product positioning in the first stage and then compete in prices (d'Aspremont, Gabszewicz, and Thisse 1979, Shaked and Sutton 1982). In a user-generated content context, we study product differentiation in a model where "content positioning" depends on which users a site attracts. This setup is similar to Dmitri and Shachar (2010) where a brand's identity depends on the consumers who own it. We study competitive outcomes in this 'spontaneous' differentiation context and compare it with classic horizontal differentiation.

Second, our study is closely related to the vast literature on network externalities, in both

economics (Katz and Shapiro 1985, 1986, Farrell and Klemperer 2005) and marketing (Xie and Sirbu 1995, Ofek and Sarvary 2001, Sun, Xie, and Cao 2004, Chen and Xie 2007, Goldenberg, Libai, and Muller 2010, Tucker and Zhang 2010). Most of these models assume a consumer utility function that increases linearly in network size. This simple assumption is sufficient to explain general industry outcomes such as the winner-take-all market structure. However, the social media industry is typically characterized by local, as opposed to global network effects. Local network effects have been studied by a few recent papers in economics (Fjeldstad, Moen, and Riis 2009, Banerji and Dutta 2009). Our model is similar to these papers but, in line with the social media context, has other features such as saturation from repeated content consumption. More importantly, we apply a more general solution concept to the game. To our knowledge, ours is the first model with local network effects that yields the classic global network effect model and winner-take-all outcome as a special case.

Third, to model advertising competition between communities, we adopt the standard ‘advertising disutility’ paradigm (Dukes and Gal-Or 2003, Dukes 2004, Gabszewicz, Laussel, and Sonnac 2004, Anderson and Coate 2005, Anderson and Gans 2010). This framework assumes that consumers consider advertising as nuisance. The tendency of ad avoidance has found much empirical support (see Wilbur (2008) for a recent example).

Finally, we assume consumer multi-homing and as a result, the paper is also related to papers on multiple buying and variety seeking (Kahn 1995, Seetharaman and Che 2009, Sajeesh and Raju 2010, Caillaud and Jullien 2003, Doganoglu and Wright 2006, Guo 2006, Xiang and Sarvary 2007). In particular, Caillaud and Jullien (2003), Doganoglu and Wright (2006) both study the impact of multi-homing behavior on platform competition under network effects. Both papers consider network products sold via fixed prices. Multi-homing implies paying for both products and product utilities are also additive. Our model introduces features specific to the social media context. We assume that consumers allocate a fixed amount of time between the communities.

Both advertising disutility and consumption utility are proportional to the amount of time spent in a site, and repeated content consumption yields decreasing return. Importantly to our context, this aspect of the demand also affects our supply function: the amount of content a user generates for the community depends on how much time she allocates to the site.

3 The Model

We consider a simple social media market with two ex-ante identical sites indexed $i = 1, 2$ competing for a heterogeneous set of consumers. Sites earn profits from advertising⁷. A site's subscribers derive utility from consuming the content generated by other members in the same community and choose to allocate their limited amount of time between the competing sites (multi-homing). Site's content depends on the type of consumers they attract (user-generated content) and the amount of time these consumers spend at the sites. Consumers prefer content generated by similar users (local network effect) and derive disutility from advertising.

The game consists of the following stages. First, all parties (both consumers and firms) form expectations about which users will join which website and how much time they will spend on the sites. Firms set advertising levels according to their expectations about the type and amount of content they will host. Then consumers make time allocation decisions based on the advertising levels and the expected type and amount of content in each community. We seek the Fulfilled Expectation Equilibrium where the expected consumer time allocation pattern coincides with the realized time allocation pattern (Katz and Shapiro 1985, Farrell and Klemperer 2005). Below, we elaborate on these features in greater details.

⁷There are three major revenue models for social media websites: advertising (as in YouTube), membership fees (as in the case of dating websites) and taxing the virtual economy (as in the case of Second Life). In an appendix available from the authors, we show that all three revenue models can be modeled in a mathematically equivalent way and we use the term advertising throughout the paper to facilitate reading.

3.1 Consumers

Consumers are heterogeneous and we assume that their types are uniformly distributed on a linear city $C = [0, 1]$. Each user is simultaneously ‘content consumer’ and ‘content contributor’ and each consumer’s preference is correlated with the content generated by her. Specifically, a consumer located at $x \in [0, 1]$ generates a piece of content at the same location in each unit of her time. We assume that consumers have access to the content generated by the other consumers in the same community⁸. Thus, the total content consumption benefit consumer x derives from joining community i , v_x^i is:

$$v_x^i = \int_{y \in C} \delta(x, y) T_i^e(y) dy, \quad (1)$$

where $\delta(x, y)$ denotes the marginal utility consumer x derives from consuming the content generated by consumer y and $T_i^e(y)$ is the market expectation about the amount of time consumer y will spend in community i . It also measures the amount of content consumer y is expected to contribute to community i . Under single-homing, $T_i^e(y)$ can be modeled as an indicator function, taking the value of 1 if consumer y is in community i and 0 otherwise.

A consumer’s location (x) may carry different interpretations in different social media contexts. For example, in the case of global social networks, a consumer’s type may be determined by her language or culture, whereas in the case of video sharing websites, a consumer’s type corresponds to her preference for different categories of videos. While we do not explicitly model a user’s incentive to generate content, it is a reasonable assumption that users generate content according to their own preferences. For instance, Facebook consumers generate content by writing blogs, uploading pictures etc. Presumably, this content is related to the consumers’ personal experiences and reflect their preferences.

⁸We explain the model in terms of content consumption. The analysis also applies to cases where consumers derive utility from direct social interaction.

As a result, $\delta(x, y)$ depends on the similarity between the content contributor y and the content consumer x . In other words, there are local network effects, where consumers benefit more from the presence of ‘similar’ others in the same community. Specifically, we assume $\delta(x, y)$ is decreasing as x and y become more distant⁹:

$$\delta(x, y) = \alpha - \beta|x - y|. \quad (2)$$

The above formulation allows for the possibilities of negative marginal content utility. For example, it is a well documented phenomenon that some Second Life participants consider each other annoying. We complete the consumers’ utility function by incorporating advertising disutility a_i that is proportional to advertising intensity (Dukes and Gal-Or 2003) and a constant term c ¹⁰. The total utility a consumer derives from site i is therefore:

$$u_x^i = c + v_x^i - a_i. \quad (3)$$

When consumers single-home, consumer x will join network i if $u_x^i > u_x^{-i}$.

Next, we allow multi-homing where consumers allocate their time between the competing communities. For simplicity we assume that each consumer disposes only two units of time. Each consumer x chooses $T_i(x)$ based on her expectation of all the other consumers’ time allocation decisions $T_i^e(y)$, $y \in [0, 1]$. $T_i(x) = k$ if x allocates k units of her time to community i ($k = 0, 1, 2$). Clearly, $T_{-i}(x) = 2 - T_i(x)$. Multi-homing takes place when a consumer allocates 1 unit of her time

⁹It is useful to examine the network benefit function v_x^i under the special case of global network effects. When $\delta(x, y) = \alpha$, the formulation reduces to the classic network externality function proposed by Katz and Shapiro (1985, 1986): $v_x^i = \int_{y \in C} \delta(x, y) T_i^e(y) dy = \int_{y \in C} \alpha T_i^e(y) dy = \alpha x_i^e$, where x_i^e is the expected number of consumers joining network i . As such, a consumer’s utility only depends on the *size* of the community.

¹⁰We do not model the fact that under certain cases, consumers may actually derive positive utility from seeing a well-designed ad. The non-content benefit c that a consumer gets from joining a community may capture, for example, the intrinsic motivation from content contribution (e.g., making a YouTube video or writing a blog article may be fun on its own right); See Bénabou and Tirole (2006) for a discussion. This is also a standard technical assumption in the product differentiation literature that guarantees market coverage.

in each community.

We assume that y generates k units of content in community i if y allocates k units of her time in community i . As such, consumer x may repeatedly consume the content generated by y . Given $T_1^e(y)$, the number of times that consumer x consumes y 's content is $t_{xy}(T_1(x), T_1^e(y)) = \sum_{i=1,2} T_i(x)T_i^e(y)$, where $t_{xy} \in \{0, 2, 4\}$. For example, when $T_1(x) = T_2(x) = 1$ and $T_1^e(y) = T_2^e(y) = 1$, both x and y multi-home, and x consumes 2 of y 's content in two different communities. When $T_1(x) = T_2(x) = 1, T_1^e(y) = 2, T_2^e(y) = 0$, x allocates one unit of her time to community 1 while y generates two units of content in community 1. Thus, x consumes 2 of y 's content in the same community. When $T_1(x) = 2, T_1^e(y) = 2$, both x and y singe-home in community 1, and x consumes 4 of y 's content.

We consider a network utility function $\delta'(x, y, t_{xy})$ concave in t_{xy} , which implies 'saturation' from repeated consumption of the same type of content. Formally, $\delta'(x, y, 4) - \delta'(x, y, 2) < \delta'(x, y, 2) - \delta'(x, y, 0)$. Consumer x chooses $T_1(x)$ to maximize:

$$u_x = c + \int_{y \in C} \delta'(x, y, t_{xy}) dy - T_1(x)a_1 - T_2(x)a_2, \quad (4)$$

where

$$\delta'(x, y, t_{xy}) = \begin{cases} 0 & \text{if } t_{xy} = 0 \\ \delta(x, y) & \text{if } t_{xy} = 2 \\ (1 + \gamma)\delta(x, y) & \text{if } t_{xy} = 4, \end{cases} \quad (5)$$

where $0 < \gamma < 1$. Note that the time allocation decision of consumer x is a function of $T_1^e(y)$ and a_1, a_2 , which we denote as $T_1^r(x, T_1^e, a_1, a_2)$. Put differently, a consumer's time allocation decision depends on her expectation about all other consumers' time allocation decisions and the firms' advertising levels. As will be defined in Section 3.3, the equilibrium time allocation involves self-fulfilling expectation and is denoted as $T_1^*(x)$.

3.2 Firms

We consider two competing sites setting their advertising intensities $a_i > 0$. Ad intensity can be thought of as the number of ads displayed on each page. The site's profit is proportional to the number of ads multiplied by the price for each ad:

$$\Pi_i = a_i p\left(\int_{x \in C} T_i^r(x, T_1^e, a_1, a_2) dx\right). \quad (6)$$

$p(\cdot)$ is the mapping from the consumer impressions a website receives to an advertiser's willingness to pay for an ad slot on this website. We assume that advertisers have higher willingness to pay for an ad slot with more consumer impressions. Specifically,

$$p\left(\int_{x \in C} T_i^r(x, T_1^e, a_1, a_2) dx\right) = s \int_{x \in C} T_i^r(x, T_1^e, a_1, a_2) dx, \quad (7)$$

where $\int_{x \in C} T_i^r(x, T_1^e, a_1, a_2) dx$ is the total amount of consumer time spent in community i and s is the prevailing cost per impression (normalized to $\frac{1}{2}$).

Recall that we assume that displaying more ads in general leads to less enjoyable consumer experience since consumers find ads a nuisance. When consumers spend less time on a community, the advertising price on this website will also drop. The profit function captures this tradeoff between ad intensity and ad price and is a standard formulation from the literature (Dukes and Gal-Or 2003, Gabszewicz et al. 2004, Anderson and Gans 2010).

3.3 Equilibrium Concept

We generalize the solution concept of Fulfilled Expectation Equilibrium (FEE) from the network effect literature (see e.g., Katz and Shapiro). In its classic form, a Fulfilled Expectation Equilibrium consists of a network size that is a fixed point of the mapping from expected network size to realized network size $x^r = \Gamma(x^e)$ ¹¹. The FEE solution concept has a straightforward extension in our setup.

¹¹Let x^e denote the expected network size of firm 1. Firm 2's network size is therefore $1 - x^e$. The mapping Γ is derived as follows. Consumers make purchase decisions based on x^e and prices, and the demand function is

We consider the functional Γ that maps the expected time allocation function T_1^e to the realized time allocation pattern T_1^r when firms set advertising levels taking T_1^e as given. The consumer time allocation pattern in a Fulfilled Expectation Equilibrium satisfies $T_1^* = \Gamma(T_1^*)$. Equivalently, the equilibrium consists of a time allocation function T_1^* and advertising levels a_1^* and a_2^* such that:

$$\begin{cases} a_1^* = \arg \max_{a_1} a_1 p(\int_{x \in C} T_1^r(x, T_1^*, a_1, a_2) dx) \\ a_2^* = \arg \max_{a_2} a_2 p(\int_{x \in C} 1 - T_1^r(x, T_1^*, a_1, a_2) dx) \\ \forall x, T_1^*(x) = T_1^r(x, T_1^*, a_1^*, a_2^*). \end{cases} \quad (8)$$

The mapping Γ is defined as $\Gamma(T_1^*)(x) = T_1^r(x, T_1^*, a_1^*, a_2^*)$. We further restrict our interests to stable FEEs. The precise definition of stability is given in the appendix. While conceptually straightforward, extending expectation from a real number to a function leads to considerable complexity in solving the fixed-point problem of Γ , which we address in the Appendix.

4 Analysis

We first present equilibrium results from the basic model. As in the network externality literature, there are many possible equilibria and uniqueness can rarely be obtained. Our analysis focuses on existence results to highlight interesting outcomes that may relate to the stylized facts discussed in the introduction. We focus on three aspects of market outcomes: market shares, consumer multi-homing behavior and site profits. To set a benchmark, we start by showing that when the network effects are relatively global, the classic winner-take-all outcome emerges where only one

$x^r(x^e, p_1, p_2)$. Firms set prices to maximize profits, leading to $p_1^*(x^e), p_2^*(x^e)$. The mapping Γ is defined as $\Gamma(x^e) = x^r(x^e, p_1^*(x^e), p_2^*(x^e))$.

firm makes positive profit.

Proposition 1. *When $\alpha > \frac{3\beta}{2}$ and $\gamma > \frac{1}{2}$, there exist two stable winner-take-all equilibria where one firm dominates the market and consumers single-home in the dominant community. Formally, $\forall x, T_i^*(x) = 2$ ($i = 1, 2$). The dominant firm's profit is $\gamma(\alpha - \frac{\beta}{2})$ and its competitor's profit is 0.*

The winner-take-all outcome is a typical market structure in many traditional industries characterized by global network effects (see Farrell and Klemperer for empirical evidences). Our analysis further suggests that this winner-take-all outcome persists even when the network effects are 'slightly local'. Furthermore, Proposition 1 also shows that decreasing returns from content consumption (small γ) reduce the likelihood of winner-take-all outcome.

Next, we explore a more interesting outcome, namely the 'spontaneous differentiation' equilibrium where ex-ante identical sites acquire differentiated market positions that the firms cannot control.

Proposition 2. *When $\frac{\beta}{4} < \alpha < \frac{5\beta+7\beta\gamma}{8\gamma-5\gamma^2+5}$, there exist two stable 'spontaneous differentiation' equilibria where*

$$T_i^*(x) = \begin{cases} 2 & \text{if } x < \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}, \\ 1 & \text{if } \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)} \leq x \leq 1 - \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}, \\ 0 & \text{if } x > 1 - \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}. \end{cases} \quad (i = 1, 2) \quad (9)$$

Firm profits are $\frac{\beta\gamma+\beta+\alpha\gamma^2-\alpha}{2(\gamma+3)}$.

Proposition 2 describes a type of content differentiation where website i hosts more content generated by users at $x < \frac{1}{2}$ and website $-i$ hosts more content generated by users at $x > \frac{1}{2}$. Figure 1 illustrates the equilibrium multi-homing pattern. The consumers on the two extremes single-home in their preferred communities while the consumers in the middle multi-home in order to consume both types of content. We name this equilibrium outcome 'spontaneous differentiation' to reflect the fact that the firms are ex-ante identical and the differentiation is created completely with user-generated content. The spontaneous differentiation equilibrium has the following features:

- Similar to the classic horizontal differentiation, spontaneous differentiation reduces competition and leads to higher profits. Both firms earn non-zero profits even if they are ex-ante identical.
- It is sometimes impossible to differentiate with user-generated content at all. The spontaneous differentiation equilibrium only exists when network effects are sufficiently local. Comparing the conditions in Propositions 1 and 2, it can be seen that the winner-take-all outcome and the spontaneous differentiation equilibrium represent mutually exclusive market outcomes.
- Spontaneous differentiation equilibria always exist in pairs. Firms don't choose their market positions (e.g., left vs right) and market positions emerge as a result of consumer coordination. Put differently, firms may obtain 'unanticipated' market positions.

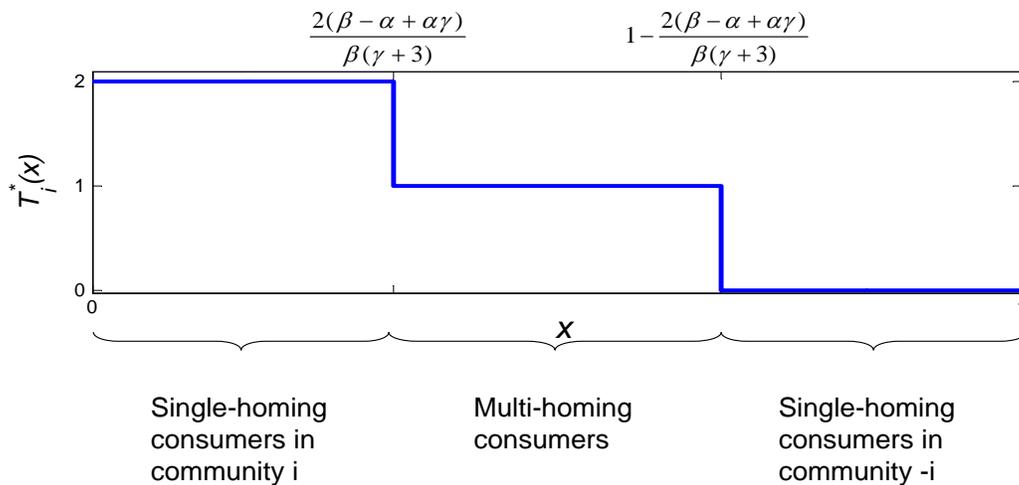


Figure 1: Spontaneous Differentiation

Clearly, spontaneous differentiation is a consequence of user-generated content as well as local network effects. But does competition also play a role in the creation of spontaneous differentiation? We find that site competition is often a necessary condition for spontaneous differentiation

to be sustained. To illustrate this point, consider a model where the firms do not interact in a competitive way (e.g., advertising levels are fixed at zero). Proposition 3 states the existence condition for spontaneous differentiation when firms don't compete.

Proposition 3. *When $a_1 = a_2 \equiv 0$, the stable spontaneous differentiation equilibrium exists if and only if $\frac{\beta}{4} < \alpha < \frac{1+3\gamma}{8\gamma-\gamma^2+1}\beta$. This condition implies negative network externalities between the consumers who single-home in different communities. Formally,*

$$\frac{\beta}{4} < \alpha < \frac{1+3\gamma}{8\gamma-\gamma^2+1}\beta \rightarrow \int \int_{(x,y) \in \{(x,y)|t_{xy}=0\}} \delta(x,y) dy dx < 0,$$

where t_{xy} is defined by the equilibrium time allocation pattern:

$$T_i^*(x) = \begin{cases} 2 & \text{if } x < \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}, \\ 1 & \text{if } \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)} \leq x \leq 1 - \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}, \quad (i = 1, 2) \\ 0 & \text{if } x > 1 - \frac{2(\beta-\alpha+\alpha\gamma)}{\beta(\gamma+3)}. \end{cases} \quad (10)$$

Proposition 3 states that when firms do not compete, spontaneous differentiation equilibrium exists only when two groups of consumers 'dislike' each other. Specifically, the time allocation function T_i^* defines two groups of single-homing consumers who do not consume the content generated by each other ($(x,y)|t_{xy}=0$). For spontaneous differentiation equilibrium to be stable, it is a necessary condition that these consumers collectively prefer to stay in different communities rather than join the same community. Conversely, if the single-homing consumers derive higher welfare from joining the same community, the spontaneous differentiation outcome is non-stable.

Comparing the conditions in Proposition 2 and Proposition 3, it is clear that the spontaneous differentiation equilibrium is more likely to exist when sites compete with each other in advertising. When firms compete, spontaneous differentiation equilibrium may exist even when consumers collectively prefer joining the same community. This is evident from the fact that differentiation can be sustained even when the marginal network externality is always strictly positive ($\alpha - \beta > 0$), such that no two consumers 'dislike' each other. In summary, spontaneous differentiation is a result

of competitive site interaction and may lead to *too much* ‘consumer segregation’ that implies lower social welfare.

The equilibrium described in Proposition 2 resonates with anecdotal evidence. The stories of Orkut and Friendster serve as lively examples where consumers with similar culture and language background joined the same website, thereby granting the websites ‘market positions’ they did not intend to obtain. For example, during its first year of launch, Orkut had the largest user base in the United States. Soon after, however, Orkut started taking off in Brazil and soon became ‘Portuguese speaking’¹². Some English speaking users started switching to competing services¹³. In fact, amazed by its unexpected popularity, Orkut’s creator visited Brazil in 2007 to ‘understand Orkut’s success in that country’¹⁴. Until very recently, Orkut remained the biggest social networking website in Brazil despite of strong competition from Facebook. Similarly in a qualitative study, Boyd (2010) suggests that a type of differentiation shaped by ‘race and class’ emerged between MySpace and Facebook during the years 2006-2009. Drawing from interview and observation data from multiple US communities, Boyd (2010) suggests that “subculturally identified teens appeared more frequently drawn to MySpace while more mainstream teens tended towards Facebook.”

Such spontaneous differentiation seemed to be a clear feature of the early social networking industry. More recently, however, Facebook is emerging as the dominant social networking service across the world. It surpassed Friendster as the most popular social networking site in the Phillipines and Orkut as the most popular site in India. Essentially, the social networking market has into a winner-take-all market. We believe that this shift is partly explained by a technological innovation around 2007. In May 2007, Facebook introduced its app platform which allowed third party developers to develop their own applications and keep 100% of the revenues. Within six months, 250,000 developers signed up. Myspace quickly followed suit later in the same year. Since

¹²<http://slashdot.org/article.pl?sid=04/07/17/2243232>

¹³<http://en.wikipedia.org/wiki/Orkut>

¹⁴<http://info.abril.com.br/aberto/infonews/042007/02042007-9.shl>

Facebook's users were considered the most valuable by advertisers, more apps were developed for Facebook. These apps were easily translated into other languages with the same collaborative translation tool, and this gave Facebook an edge over its competitors. In our terminology, when social networking users' primary activity was browsing the photos and posts by their friends, the network effects are local in scope. Photos posted by one consumer are only of interests to her friends. When third party apps entered the picture, the network effects become much more global, since apps developed for one group of consumers are most likely also of interests to another group of consumers. This eventually leads to a winner-take-all outcome.

Our finding about consumer multi-homing is also consistent with empirical observations. We find that multi-homing consumers are those who locate in the middle of the linear city. For example, Brazilians who live in the US are most likely to join both Myspace and Orkut to connect to friends in both countries¹⁵. In the *Pew Research* survey on social network users (Lenhart 2009), the most stated reasons for multi-homing include 'keeping up with friends on different sites', 'separating personal and professional life' and 'representing different parts of my personality'.

When network externalities are sufficiently local, more complicated differentiation structures can be sustained. Proposition 4 states the existence of a type of equilibrium where one site attracts two distinct groups of users:

Proposition 4. *When $\frac{\beta}{2} < \alpha < \frac{\beta(\gamma+3)}{2(3-\gamma)}$, there exist two stable equilibria where*

$$T_i^*(x) = \begin{cases} 2 & \text{if } x < A(\alpha, \beta, \gamma), \\ 1 & \text{if } A(\alpha, \beta, \gamma) \leq x \leq B(\alpha, \beta, \gamma), \\ 0 & \text{if } B(\alpha, \beta, \gamma) < x < 1 - B(\alpha, \beta, \gamma), \\ 1 & \text{if } 1 - B(\alpha, \beta, \gamma) \leq x \leq 1 - A(\alpha, \beta, \gamma), \\ 2 & \text{if } x > 1 - A(\alpha, \beta, \gamma). \end{cases} \quad (i = 1, 2) \quad (11)$$

where $B(\alpha, \beta, \gamma) > A(\alpha, \beta, \gamma)$. Exact expressions for $\lambda(\gamma), A(\alpha, \beta, \gamma), B(\alpha, \beta, \gamma)$ are given in the

¹⁵See this argument examined in <http://www.zephorio.org/thoughts/archives/2010/08/17/social-divisions-between-orkut-facebook-in-brazil.html>.

appendix¹⁶.

The equilibrium time allocation patterns are illustrated on Figure 2. Consumers located at the two ‘ends’ of the linear city prefer the content hosted in site i while the consumers residing in the ‘middle’ prefer the content hosted in site $-i$. In equilibrium, the firm with the divided clientele sets a lower advertising level to retain its consumers. Interestingly, although the mass centers of the two firms’ content overlap (at $x = \frac{1}{2}$), the sites are differentiated because of the highly localized content preferences. Furthermore, firm i attracts two distinct segments of consumers who do not enjoy the presence of each other. More specifically, the marginal network effects $\delta(x, y)$ between these two segments of consumers are small and can be negative.

This outcome is reminiscent of Orkut’s simultaneous success in three culturally distinct countries: Brazil, India and Estonia. Orkut became one of the most visited websites in Brazil and India until 2010. As of April 2010, 48% of Orkut’s traffic comes from Barzil while 39% of its traffic is from India. At the same time, it has also become the most used social network platform in Estonia¹⁷. Although these three user groups are simultaneously present in the Orkut community, they form subcommunities that seldomly interact with each other. Figure 2 can be considered an illustration of the Orkut case where Myspace (later Facebook) dominates the US market while Orkut is popular among Barzilians and Indians (i.e., two disjoint segments of consumers). In addition, multicultural consumers - such as Brazilians and Indians living in the US - are found to be the most likely multi-homers who join both Orkut and Facebook/Myspace.

To summarize, the model provides a variety of qualitatively different market outcomes and

¹⁶We provide a special case to better illustrate the intuition: when $\gamma = 1$, the equilibrium becomes:

$$T_i^*(x) = \begin{cases} 2 & \text{if } x < \frac{1}{2} - \frac{4\alpha + \beta + 2\sqrt{4\alpha^2 - 5\alpha\beta + 2\beta^2}}{14\beta} \\ 0 & \text{if } \frac{1}{2} - \frac{4\alpha + \beta + 2\sqrt{4\alpha^2 - 5\alpha\beta + 2\beta^2}}{14\beta} \leq x \leq \frac{1}{2} + \frac{4\alpha + \beta + 2\sqrt{4\alpha^2 - 5\alpha\beta + 2\beta^2}}{14\beta} \\ 2 & \text{if } x > \frac{1}{2} + \frac{4\alpha + \beta + 2\sqrt{4\alpha^2 - 5\alpha\beta + 2\beta^2}}{14\beta} \end{cases}$$

¹⁷<http://en.wikipedia.org/wiki/Orkut>

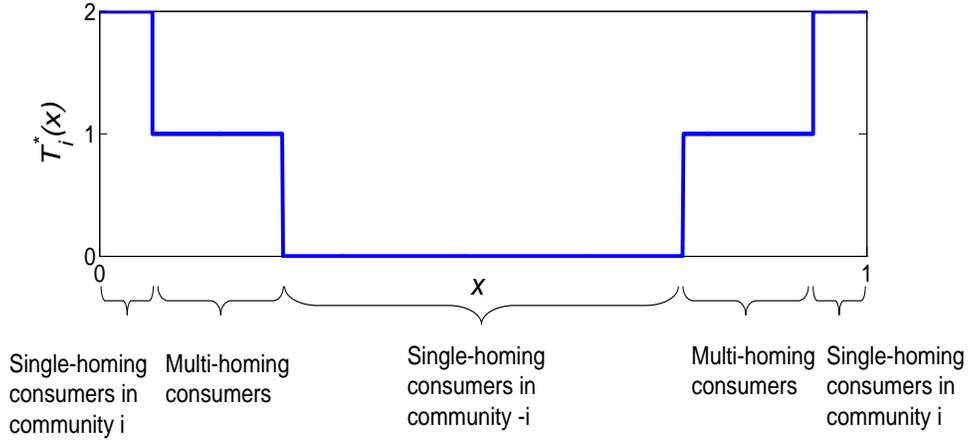


Figure 2: Spontaneous Differentiation with Divided Clientele for Firm i

may explain some of the observed patterns in the evolution of Web 2.0 communities. Importantly, the existence of different types of equilibria depends on the *localness* ($\frac{\beta}{\alpha}$), not the *magnitude* ($\delta(x,y)$), of network effects. For example, the case $\alpha = 3, \beta = 3$ represents large network externalities. The average marginal network effect is $\overline{\delta(x,y)} = 2$. On the other hand, $\alpha = 1, \beta = 0$ represent small average marginal network effects with $\overline{\delta(x,y)} = 1$. Spontaneous differentiation can be sustained in the former case but winner-take-all outcome emerges in the latter.

Being a unique feature of the social media market, what is the profit implication of spontaneous differentiation? Since users generate content, how do consumer behavior parameters impact firm profits? Next, we present a number of comparative static results to examine how the localness of network externalities (measured by β when α is fixed) and saturation from repeated content consumption (measured by γ) impact consumer behavior and firm profits. We focus on the case described in Proposition 2 where spontaneous differentiation emerges between the competing firms. Corollary 1 examines consumer behavior.

Corollary 1 (Consumer Behavior). *In the spontaneous differentiation equilibrium, more consumers multi-home when γ and β are small. Put differently, multi-homing is more likely when network effects are more global and saturation from repeated content consumption is strong.*

As expected, consumers are more likely to multi-home when saturation from repeated content consumption is strong (γ is small). Furthermore, global network effects lead to more consumers multi-homing. Global network effects imply that consumers have broader interests. Therefore, they multi-home in order to reach out to different types of content. Taken together with the findings from Proposition 1, we observe that when network effects become more global, the number of multi-homing consumers first increases — then decreases as the winner-take-all outcome emerges, in which case all consumers single-home in the same community.

Corollary 2 (Firm Profits). *In the spontaneous differentiation equilibrium, firm profits are increasing in β and γ : i.e., firm profits are higher when network effects are more local and saturation in content consumption is weaker.*

In the classic horizontal differentiation literature, the degree of product differentiation is usually measured by a ‘transportation cost’ parameter t a la Hotelling. Higher transportation cost implies higher profits. In the social media setup, we observe that the localness of network effect β is the counterpart of the transport cost parameter in the classic scenario. As Figures 1 and 2 illustrate, differentiation between social media websites stems from the different locations of their content generating users. The localness of network effects makes this differentiation more pronounced. Firm profits rise as network effects become more local.

Interestingly, we observe that consumer multi-homing coincides with lower firm profits. This stands in sharp contrast with the earlier findings in the literature. When horizontal differentiation depends on firms’ choices of market positioning, multi-buying usually diminishes the strategic incentives of price cutting since there is less need to compete for consumers who purchase both products (Guo 2006, Xiang and Sarvary 2007, Doganoglu and Wright 2006). Consequently, in the classic model, more multi-homing consumers would likely lead to higher firm profits. When content is user-generated, we observe that consumer multi-homing behavior endogenously changes the degree of spontaneous differentiation. When a user participates in competing websites, the

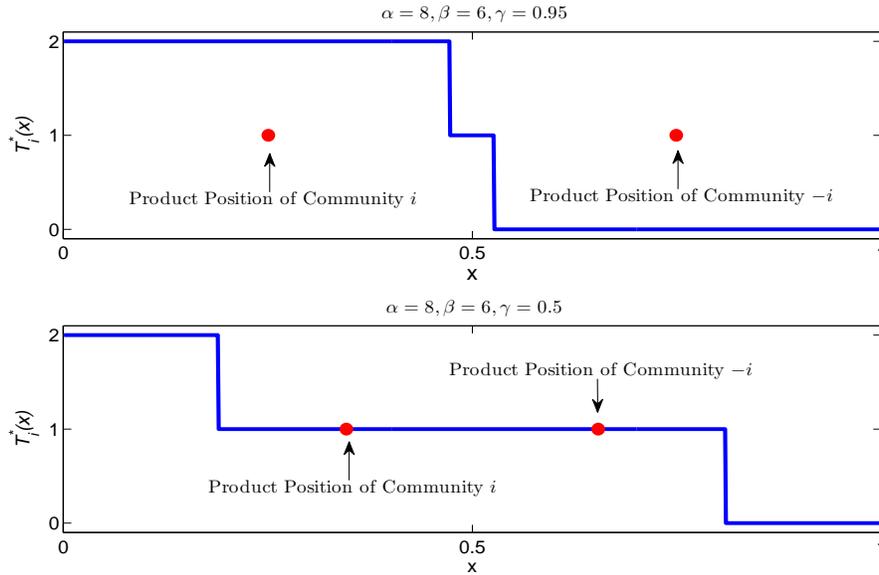


Figure 3: Content Positions under High and Low γ

content she contributes is also likely to appear on both websites. Thus, multi-homing behavior leads to greater overlap of content and therefore less product differentiation. Figure 3 illustrate the equilibrium product positions of competing firms for $\gamma = 0.95$ and $\gamma = 0.5$. Clearly, sites' product offerings become more similar under lower γ . Figure 4 illustrates the degree of product differentiation (as measured by $v_{x=0}^1 - v_{x=0}^2$) and the number of multi-homing consumers as a function of γ . The figure shows a clear negative relationship between the two variables: the degree of spontaneous differentiation is maximized when no consumer multi-homes.

It is worth pointing out that saturation from repeated content consumption is just one of the many possible causes to consumer multi-homing behavior. However, we believe that the above link between multi-homing and the degree of product differentiation is a fundamental property of user-generated content. The finding in Corollary 2 is likely to generalize to situations where consumer multi-homing is caused by other factors.

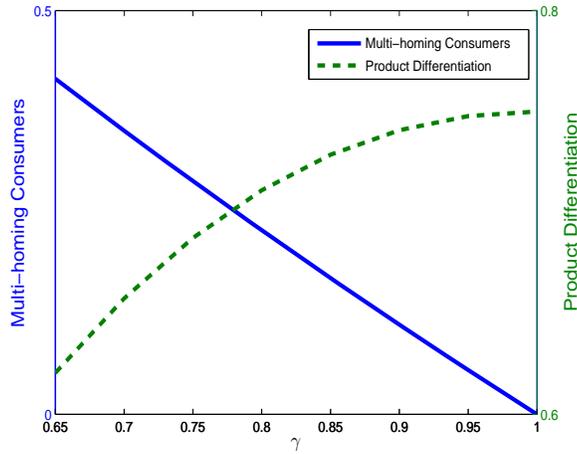


Figure 4: Multi-homing Coincides with Less Spontaneous Differentiation

5 Designing Social Media Site

In the baseline model, we consider a duopoly of social media sites that are ex-ante identical. Thus, user-generated content is the only driver of content differentiation. This setup highlights the roles played by the users in determining the sites' product positions. In most markets, however, competing firms can further differentiate their services by introducing design features that appeal to a certain consumer segments. Put differently, differentiation can be jointly determined by firm design and user-generated content.

For example, an on-line dating website can introduce personality test and compatibility matching algorithms as part of the service. These features appeal to the users who seek long-term relationships. When a user evaluates a dating site, both the design features and the quality of the user pool (i.e., match between that site's users and herself) will enter her utility function. The relative importance of user content and design features is governed by the magnitudes and localness of network effects. We consider a consumer utility function as follows:

$$u_x(T_{1x}; T_1^e(\cdot), a_1, a_2) = c + \int_{y \in [0,1]} \delta'(x, y, t_{xy}) dy - T_{1x} a_1 - (2 - T_{1x}) a_2 - T_{1x} (x - x_1)^2 t - (2 - T_{1x}) (x - x_2)^2 t,$$

The second component of the utility function is the ‘transportation cost’ term in the classic horizontal differentiation model. The relative magnitudes of α , β with respect to t determine the importance of user-generated content vs firm design. In the first stage, the site choose their location x_1 and x_2 . In the second stage, the firms set their advertising levels and the consumers make time allocation decisions. As in the previous section, we seek the Stable Fulfilled Expectation Equilibrium in the second stage.

Since the second stage subgame usually has multiple equilibria, subgame perfection in the first stage is not straightforward. We start with a discussion of the equilibrium market outcomes in the subgame where the firms maximally differentiate and the consumers do not multi-home ($\gamma = 1$). Proposition 5 outlines the equilibrium market structures.

Proposition 5. *Consider two maximally differentiated firms with $x_1 = 0$ and $x_2 = 1$:*

- *When $\alpha > \frac{3\beta}{2} + 3t$, winner-take-all is an equilibrium outcome.*
- *When $\alpha < \frac{3\beta}{2} + 3t$, there exist an equilibrium where $T_1^*(x) = 2$ if $x < \frac{1}{2}$ and $T_1^*(x) = 0$ if $x > \frac{1}{2}$.*
- *When $\alpha < \frac{3\beta}{2} - 3t$ and $\beta > 2t$, there exist an equilibrium where $T_1^*(x) = 0$ if $x < \frac{1}{2}$ and $T_1^*(x) = 2$ if $x > \frac{1}{2}$.*

Proposition 5 states that when websites pursue differentiated feature designs, the likelihood of winner-take-all outcome is reduced. However, in equilibrium, user-generated content may either

enhance or override firms' product designs. In the former case, the firm located at $x_1 = 0$ will indeed attract the consumers located at $x < \frac{1}{2}$. When user-generated content overrides the firm's product design, the firms may attract the consumer segment they do not intend to target. The firm located at $x_1 = 0$ will attract the consumers located at $x > \frac{1}{2}$. Although these consumers actually prefer the features offered by site 2, they will collectively join site 1 because of user-generated content.

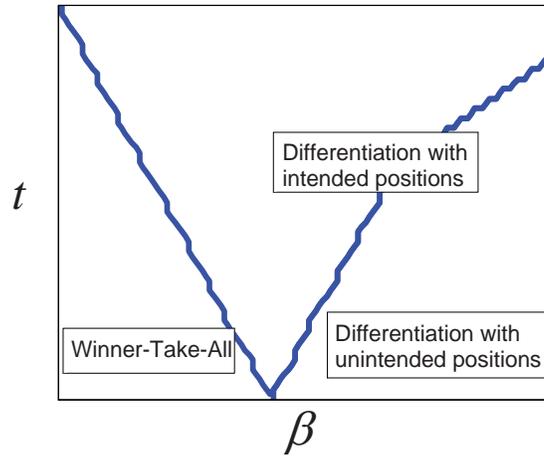


Figure 5: Equilibrium outcomes in the maximal differentiation subgame

Figure 5 illustrates the equilibrium market outcomes in the parameter space. The interaction between user content and site design has profit implications. When user-generated content enhances the firms' chosen market positions, firms profits increase in both β and t . When user-content is misaligned with site design, however, firm profits increase in β but decrease in t . The equilibrium profits can be lower compared with the cases where only one source of differentiation is present (i.e., either user content or feature design is absent). Higher β increases differentiation from user-generated content, but also increases the likelihood of such misalignment.

6 Concluding Remarks

In this paper, we study the competition between social media websites. We model three unique aspects of the social media industry, namely (i) user-generated content, (ii) local network externalities and (iii) consumer multi-homing. We find that when content is strictly user-generated, identical firms may spontaneously acquire differentiated market positions by attracting different groups of content contributors. We study the properties of this spontaneous differentiation and find that more local network effects increase the degree of differentiation while multi-homing coincides with reduced differentiation. As an extension, we consider the interaction between user-generated content and the websites' design features. While both user content and site design can lead to product differentiation, these factors may either enhance or contradict each other, leading to interesting situations where competing sites strategically pursue identical designs. Our results are consistent with a number of stylized facts observed in the social media industry.

The social media industry is a fast developing industry with many innovations in both the technology and business domains. The power of mass interaction and user generated content is being leveraged into more and more business and public policy contexts, such as distance learning and collaboration within organizations, new product ideation and open innovation contests as well as social ventures such as peer-to-peer micro-finance. Our stylized model intends to capture some fundamental features of social media competition, leaving a number of interesting issues for future research. For example, many social media sites are characterized by a mixture of user-generated content and firm-produced content. Firms can increase the site's appeal to a certain segment of consumers by interface design or by injecting relevant content into their community. Under such circumstances, the interaction between spontaneous differentiation and firm chosen differentiation becomes an interesting issue. As another example, one trend observed in the social media industry is the sharing of content between sites (see e.g., the recent content sharing agreement between LinkedIn and Twitter). Similarly, the OpenSocial standard advocated by Google greatly facilitates

the sharing of content among participating websites. An interesting research direction is to explore firms' incentives to share content in a competitive setup.

Finally, as a theoretical note, we believe that the development of user-generated content is a phenomenon that is of interest for both marketing and economics. There is broad agreement that user-generated content represents a novel situation not fully addressed by the traditional economics literature. Theoretically, our analysis suggest a close link between the notion of user-generated content and network effects. The classic model of network externalities can be considered a model of user-generated content where the quality of the product (network) depends on the number of contributing users. Ex-ante identical firms can obtain ex-post different quality levels as a result of consumer coordination. When network effects are localized, both vertical (quality) and horizontal differentiation can occur as a result of user coordination. Further developing this argument is an interesting avenue for theoretical research.

Appendix

A Stable Fulfilled Expectation Equilibrium

In this section, we provide definitions for Stable Fulfilled Expectation Equilibrium. Stability implies that in equilibrium, when there is a small perturbation in the market expectation, the consequent market outcome (consumer time allocation pattern) is not 'too different' from the equilibrium. We assume that when the market expectation changes, the market expects the marginal consumers (those who are the most likely to change their time allocation pattern) to change their time allocation decisions first.

Definition 1. (*ε -Marginal Perturbation*) In any FEE equilibrium T_1^*, a_1^*, a_2^* , a marginal consumer is defined as $x \in [0, 1]$ who is indifferent between two alternative time allocation plans. We say T_1' is an ε -marginal perturbation of T_1^* if $T_1'(x)$ and $T_1^*(x)$ are different only in ε intervals around the

marginal consumers.

Definition 2. (*Stable Fulfilled Expectation Equilibrium*) A *Stable Fulfilled Expectation Equilibrium* consists of a time allocation pattern $T_i^*(x)$ that satisfies the following condition: $\exists \delta, \forall \varepsilon$ -marginal perturbation $T_i'(x)$ of $T_i^*(x)$ where $\varepsilon < \delta$, $\|\Gamma(T_i'(x)) - \Gamma(T_i^*(x))\| < \|T_i'(x) - T_i^*(x)\|$, where $\|\cdot\|$ is the 1-norm of real-valued functions: $\|f(x)\| = \int_{x \in [0,1]} |f(x)| dx$.

Intuitively, the above condition states that given any small perturbation in market expectation, the change in realized time allocation patterns is not too large. This condition is a generalization of the stability conditions in the classic network externalities literature, which are shown to be necessary to rule out implausible outcomes¹⁸.

B Proofs for Propositions

PROOF OF PROPOSITION 1: A stable FEE meets three necessary conditions:

$$\begin{cases} a_1^* = \arg \max_{a_1} a_1 p(\int_{x \in C} T_1^r(x, T_1^*, a_1, a_2) dx) \\ a_2^* = \arg \max_{a_2} a_2 p(\int_{x \in C} 1 - T_1^r(x, T_1^*, a_1, a_2) dx) \\ \forall x, T_1^*(x) = T_1^r(x, T_1^*, a_1^*, a_2^*). \end{cases}$$

In words, the advertising levels are best responses to each other given the market expectation, and the market expectation is self-fulfilling. In addition, the market expectation is stable as defined in Definition 2. We first show that the equilibrium stated in Proposition 1 $\forall x, T_1^*(x) = 2$, $a_1^* = \gamma(\alpha - \frac{\beta}{2})$ and $a_2^* = 0$ satisfy these conditions *iff* $\alpha > \frac{3\beta}{2}$ and $\gamma > \frac{1}{2}$. Next we verify that the equilibrium is always stable.

¹⁸See Farrell and Klemperer (2005). For example, without the stability condition, the outcome that each firm has a 50% market share is always a FEE. This outcome, however, is not stable when network effects are global. Any infinitesimal perturbation in market expectation will lead to the winner-take-all outcome.

Step 1: We first provide conditions such that given market expectation $\forall x, T_1^*(x) = 2$, advertising levels $a_1^* = \gamma(\alpha - \frac{\beta}{2})$ and $a_2^* = 0$ are best responses to each other. To solve the advertising game, we first derive the demand and profit functions. When $\forall x, T_1^*(x) = 2$ and $a_2^* = 0$, from Equation (4), the consumer located at x derives utility $c + x(\alpha - \beta \frac{x}{2}) + (1-x)(\alpha - \beta \frac{1-x}{2}) - a_1 - a_2$ if she multi-homes (spends one unit of time in each community). Her utility is $c + (1+\gamma)(x(\alpha - \beta \frac{x}{2}) + (1-x)(\alpha - \beta \frac{1-x}{2})) - 2a_1$ if she spends both units of her time in community 1. Thus when $a_2 = 0$ and $a_1 < \gamma(x(\alpha - \beta \frac{x}{2}) + (1-x)(\alpha - \beta \frac{1-x}{2}))$, consumer x will prefer spending both units of her time in community 1. Otherwise, the consumer will allocate one unit of time in each community. The marginal consumer can be determined by solving the x that is indifferent between single-homing in community 1 and multi-homing. Specifically, when $a_1 \leq \gamma(\alpha - \frac{\beta}{2})$, all consumers single-home in site 1. When $\gamma(\alpha - \frac{\beta}{2}) < a_1 < \min\{\gamma(\alpha - \frac{\beta}{4}), \alpha - \frac{\beta}{2}\}$, consumers $x \in (\frac{1}{2} - \frac{\sqrt{4\alpha\beta\gamma^2 - \beta^2\gamma^2 - 4a_1\beta\gamma}}{2\beta\gamma}, \frac{1}{2} + \frac{\sqrt{4\alpha\beta\gamma^2 - \beta^2\gamma^2 - 4a_1\beta\gamma}}{2\beta\gamma})$ will single-home in community 1 while the other consumers multi-home. When $\gamma(\alpha - \frac{\beta}{4}) < a_1 < \alpha - \frac{\beta}{2}$, all consumers multi-home. When $a_1 > \alpha - \frac{\beta}{2}$, consumers on the two ends of the linear city start to single-home in community 2. Given $T_1^*(x) = 2$ and $a_2^* = 0$, site 1's profit function in the range $a_1 \in (0, \alpha - \frac{\beta}{2})$ is:

$$\Pi(a_1, a_2^*, T_1^*(x)) = \begin{cases} a_1 & \text{for } a_1 \leq \gamma(\alpha - \frac{\beta}{2}) \\ a_1(\frac{1}{2} + \frac{\sqrt{4\alpha\beta\gamma^2 - \beta^2\gamma^2 - 4a_1\beta\gamma}}{2\beta\gamma}) & \text{for } \gamma(\alpha - \frac{\beta}{2}) < a_1 < \min\{\gamma(\alpha - \frac{\beta}{4}), \alpha - \frac{\beta}{2}\} \\ \frac{1}{2}a_1 & \text{for } \gamma(\alpha - \frac{\beta}{4}) < a_1 < \alpha - \frac{\beta}{2}, \end{cases}$$

The profit function is increasing when $a_1 \leq \gamma(\alpha - \frac{\beta}{2})$ and $\gamma(\alpha - \frac{\beta}{4}) < a_1$. The profit function is concave when $\gamma(\alpha - \frac{\beta}{2}) < a_1 < \min\{\gamma(\alpha - \frac{\beta}{4}), \alpha - \frac{\beta}{2}\}$. For $a_1^* = \gamma(\alpha - \frac{\beta}{2})$ to be the optimal advertising level, it is required that:

$$\begin{cases} \frac{\partial \Pi(a_1, a_2^*, T_1^*(x))}{\partial a_1} \Big|_{a_1 = \gamma(\alpha - \frac{\beta}{2})} < 0 \\ \gamma(\alpha - \frac{\beta}{2}) > \frac{1}{2}(\alpha - \frac{\beta}{2}). \end{cases} \quad (12)$$

The first condition implies that when $a_1 = \gamma(\alpha - \frac{\beta}{2})$, site 1 has no incentives to marginally rise advertising such that the consumers located at 0 and 1 start to multi-home. The second condition implies that the site doesn't choose an advertising level such that all consumers multi-home. Since $\frac{\partial \Pi(a_1, a_2^*, T_1^*(x))}{\partial a_1} \Big|_{a_1 = \gamma(\alpha - \frac{\beta}{2})} = \frac{3\beta - 2\alpha}{2\beta}$, these conditions are satisfied *iff* $\alpha > \frac{3\beta}{2}$ and $\gamma > \frac{1}{2}$.

Finally, observe that when $a_1 > \alpha - \frac{\beta}{2}$, consumers on the two ends of the linear city start to single-home in site 2. Clearly, $\frac{\partial \Pi(a_1, a_2^*, T_1^*(x))}{\partial a_1} \Big|_{a_1 = \alpha - \frac{\beta}{2}} < \frac{\partial \Pi(a_1, a_2^*, T_1^*(x))}{\partial a_1} \Big|_{a_1 = \gamma(\alpha - \frac{\beta}{2})} < 0$. Thus, site 1 doesn't have incentives to advertise beyond $\alpha - \frac{\beta}{2}$ when $a_1^* = \gamma(\alpha - \frac{\beta}{2})$ is locally optimal.

Given $\forall x, T_1^*(x) = 2$, $a_1^* = \gamma(\alpha - \frac{\beta}{2})$, we show that site 2's best response is $a_2^* = 0$. Given site 1's advertising level, site 2 cannot gain positive demand regardless of its advertising level. Thus, any positive advertising level is weakly dominated.

Step 2: We next prove that given the advertising levels, $T_1^*(x) = 2$ is self-fulfilling. It can be easily seen that given $a_1^* = \gamma(\alpha - \frac{\beta}{2})$ and $a_2^* = 0$, all consumers prefer to single-home in community 1.

Step 3: We next prove that this equilibrium is always stable. Consider an ε -perturbation in market expectation as described in Definition 2:

$$T'(x) = \begin{cases} 1 & \text{for } x < \varepsilon \\ 2 & \text{for } \varepsilon \leq x \leq 1 - \varepsilon \\ 1 & \text{for } x > 1 - \varepsilon. \end{cases}$$

The consumers' time allocation can be characterized as follows: consumer x multi-homes if $a_1 - a_2 < \gamma((x - \varepsilon)(\alpha - \beta \frac{x - \varepsilon}{2}) + (1 - x - \varepsilon)(\alpha - \beta \frac{1 - x - \varepsilon}{2}))$ and single-homes in site 1 otherwise. The demand and profits functions can be derived accordingly. Importantly, at any advertising level, we have $\Pi(a_1, a_2, T_1'(x)) = \Pi(a_1, a_2, T_1^*(x)) + o(\varepsilon)$, where $o(\varepsilon)$ is on the same order of magnitudes as ε . It is easy to verify that $a_1 = \gamma(\alpha - \frac{\beta}{2}) - \gamma(2\alpha - \beta)\varepsilon - \beta\gamma\varepsilon^2$ and $a_2 = 0$ are best responses to each other when:

$$\begin{cases} \frac{\partial \Pi(a_1, 0, T_1'(x))}{\partial a_1} \Big|_{a_1 = \gamma(\alpha - \frac{\beta}{2}) - \gamma(2\alpha - \beta)\varepsilon - \beta\gamma\varepsilon^2} < 0 \\ \gamma(\alpha - \frac{\beta}{2}) - \gamma(2\alpha - \beta)\varepsilon - \beta\gamma\varepsilon^2 > \frac{1}{2}(\alpha - \frac{\beta}{2}) + (\beta - \alpha)\varepsilon - (\beta\varepsilon^2)/2. \end{cases} \quad (13)$$

The left-hand-sides of inequalities in condition (13) differ from the left-hand-sides of the inequalities in condition (12) by amounts that are on the order of magnitude of ε . Since the inequalities in condition (12) are not binding when $\alpha > \frac{3\beta}{2}$ and $\gamma > \frac{1}{2}$, condition (13) are also satisfied when ε is sufficiently small. Intuitively, when ε is sufficiently small, site 1 can win the entire market by a small decrease in advertising level a_1^* and will indeed do so. In other words, $\Gamma(T'(x)) = \Gamma(T^*(x))$ for sufficiently small ε . This is true for any parameter values $\alpha > \frac{3\beta}{2}$ and $\gamma > \frac{1}{2}$. The equilibrium is thus stable. This concludes the proof. □

PROOF OF PROPOSITION 2: Consider $a_1^* = a_2^* = \frac{\gamma\beta + \beta + \alpha\gamma^2 - \alpha}{2(\gamma+3)}$ and $T_1^*(x)$ as described in Proposition 2. We first verify that for any parameters, the advertising levels are best responses to each other and that the time allocation pattern is self-fulfilling. Then we show that the equilibrium is also stable *iff* $\alpha < \frac{5\beta + 7\beta\gamma}{8\gamma - 5\gamma^2 + 5}$.

Step 1: The market expectation $T_1^*(x)$ defines the marginal consumers $x_1^* = \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}$ such that consumers $y < x_1^*$ and $y > 1 - x_1^*$ are expected to be the single-homers. We first derive firm profit functions. From Equation (4) and (5), we know that when consumer y 's content is expected to appear in both communities, consumer x derives marginal utility $\delta(x, y)$ from y 's content regardless of her time allocation decision ($t_{xy} = 2$). Thus, only the unique content (i.e., content that appears in only one community) matters for consumer x 's decision of multi-homing vs single-homing. From Equation 4, we know consumer x 's decision can be characterized by the following rule:

$$T_1^r(x, T_1^*, a_1, a_2) = \begin{cases} 2 & \text{if } \gamma \int_{y < x_1^*} \delta(x, y) dy - a_1 > \int_{y > 1-x_1^*} \delta(x, y) dy - a_2 \\ 1 & \text{if } \begin{cases} \gamma \int_{y < x_1^*} \delta(x, y) dy - a_1 \leq \int_{y > 1-x_1^*} \delta(x, y) dy - a_2 \\ \gamma \int_{y > 1-x_1^*} \delta(x, y) dy - a_2 \leq \int_{y < x_1^*} \delta(x, y) dy - a_1 \end{cases} \\ 0 & \text{if } \gamma \int_{y > 1-x_1^*} \delta(x, y) dy - a_2 > \int_{y < x_1^*} \delta(x, y) dy - a_1. \end{cases} \quad (14)$$

For $a_1^* = a_2^*$ to be best responses to each other, it is necessary for them to be local best response to each other. When a_1 and a_2 are not too different, consumers $x < \frac{1}{2}$ prefer single-homing in community 1 to single-homing in community 2. Thus, $x < \frac{1}{2}$'s time allocation decision reduces to choosing between single-homing in community 1 and multi-homing. Similarly, consumers $x > \frac{1}{2}$ choose between single-homing in community 2 and multi-homing. Thus, the demand schedule can be characterized by:

$$T_1^r(x, T_1^*, a_1, a_2) = \begin{cases} 2 & \text{if } x < x_1(x_1^*, a_1, a_2) \\ 1 & \text{if } x_1(x_1^*, a_1, a_2) < x < x_2(x_1^*, a_1, a_2) \\ 0 & \text{if } x > x_2(x_1^*, a_1, a_2), \end{cases} \quad (15)$$

where $x_1(x_1^*, a_1, a_2)$ is defined by $\gamma \int_{y < x_1^*} \delta(x_1, y) dy - a_1 = \int_{y > 1-x_1^*} \delta(x_1, y) dy - a_2$ and $x_2(x_1^*, a_1, a_2)$ is defined by $\int_{y < x_1^*} \delta(x_2, y) dy - a_1 = \gamma \int_{y > 1-x_1^*} \delta(x_2, y) dy - a_2$. The $T_1^r(x, T_1^*, a_1, a_2)$ function can be fully described by x_1 and x_2 . The exact expressions for $x_1(x_1^*, a_1, a_2)$ and $x_2(x_1^*, a_1, a_2)$ are complicated, but the derivatives $\frac{\partial x_1(x_1^*, a_1, a_2)}{\partial a_1}$ and $\frac{\partial x_1(x_1^*, a_1, a_2)}{\partial x_1^*}$ at $x_1^* = \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma + 3)}$ are easy to obtain from implicit function theorems. For example,

$$\begin{aligned} & \frac{\partial x_1(x_1^*, a_1, a_2)}{\partial a_1} \\ &= - \frac{\partial (\gamma \int_{y < x_1^*} \delta(x_1, y) dy - \int_{y > 1-x_1^*} \delta(x_1, y) dy - a_1 + a_2) / \partial a_1}{\partial (\gamma \int_{y < x_1^*} \delta(x_1, y) dy - \int_{y > 1-x_1^*} \delta(x_1, y) dy - a_1 + a_2) / \partial x_1} \\ &= - \frac{1}{\beta(1 - x_1^*) + \gamma\beta x_1^*}. \end{aligned}$$

Firm profits are:

$$\begin{aligned}\Pi_1(a_1, a_2, x_1^*) &= (x_1(x_1^*, a_1, a_2) + \frac{x_2(x_1^*, a_1, a_2) - x_1(x_1^*, a_1, a_2)}{2})a_1 \\ \Pi_2(a_1, a_2, x_1^*) &= (1 - x_2(x_1^*, a_1, a_2) + \frac{x_2(x_1^*, a_1, a_2) - x_1(x_1^*, a_1, a_2)}{2})a_2.\end{aligned}\quad (16)$$

The best responses are characterized by the first order conditions:

$$\begin{cases} \frac{\partial \Pi_1(a_1, a_2, x_1^*)}{\partial a_1} = 0 \\ \frac{\partial \Pi_2(a_1, a_2, x_1^*)}{\partial a_2} = 0. \end{cases}\quad (17)$$

The exact expressions of $\frac{\partial \Pi_1(a_1, a_2, x_1^*)}{\partial a_1}$ and $\frac{\partial \Pi_2(a_1, a_2, x_1^*)}{\partial a_2}$ can be obtained by plugging in $\frac{\partial x_1(x_1^*, a_1, a_2)}{\partial a_1} = -\frac{1}{\beta(1-x_1^*) + \gamma\beta x_1^*}$, $\frac{\partial x_1(x_1^*, a_1, a_2)}{\partial a_2} = \frac{1}{\beta(1-x_1^*) + \gamma\beta x_1^*}$, $\frac{\partial x_2(x_1^*, a_1, a_2)}{\partial a_1} = \frac{1}{\beta x_1^* + \gamma\beta(1-x_1^*)}$, $\frac{\partial x_2(x_1^*, a_1, a_2)}{\partial a_2} = -\frac{1}{\beta x_1^* + \gamma\beta(1-x_1^*)}$ at $x_1 = x_1^*$ and $x_2 = 1 - x_1^*$. We verify that the advertising levels $a_1^* = a_2^* = \frac{\gamma\beta + \beta + \alpha\gamma^2 - \alpha}{2(\gamma + 3)}$ satisfy the first order conditions. It is easy to see that the sites do not have incentives to deviate their advertising levels far from $a_1^* = a_2^*$ to qualitatively change the demand schedule, such that no consumer will single-home in community 1 or 2. Thus, the advertising levels are also global best response to each other.

Step 2: It is easy to verify that the market expectation is self fulfilling, by observing that the advertising levels and market expectation satisfy the following system of equations:

$$\begin{cases} x_1(x_1^*, a_1^*, a_2^*) = x_1^* \\ x_2(x_1^*, a_1^*, a_2^*) = 1 - x_1^* \end{cases}\quad (18)$$

Step 3: Finally we provide conditions under which the equilibrium is also stable according to Definitions 1 and 2. Note that the marginal consumers in this case consist of the consumers located at x_1^* and $1 - x_1^*$. Any marginal perturbation of $T_1'(x)$ of $T_1^*(x)$ can be described by x_1' and x_2' such that $|x_1' - x_1^*| < \varepsilon$ and $|x_2' - (1 - x_1^*)| < \varepsilon$. Thus the Γ mapping is reduced to a function $R^2 \rightarrow R^2$, which defines $(x_1', x_2') \rightarrow (\Gamma_1(x_1', x_2'), \Gamma_2(x_1', x_2'))$. The stability condition can be inferred from the eigenvalues of the Jacobian matrix:

$$\left| \begin{pmatrix} \frac{\partial \Gamma_1(x'_1, x'_2)}{\partial x'_1} & \frac{\partial \Gamma_1(x'_1, x'_2)}{\partial x'_2} \\ \frac{\partial \Gamma_2(x'_1, x'_2)}{\partial x'_1} & \frac{\partial \Gamma_2(x'_1, x'_2)}{\partial x'_2} \end{pmatrix} - \sigma I \right|_{x'_1=x'_1, x'_2=1-x'_1} = 0. \quad (19)$$

The partial derivatives are obtained by the total derivative formula. In words, when market expectation changes, the realized time allocation pattern changes for two reasons. First, when advertising levels are fixed, consumers change their time allocation behavior due to the change in their valuation of the sites' content. Second, sites change their advertising level, which leads to further change in consumer time allocation decision. For example,

$$\frac{\partial \Gamma_1(x'_1, x'_2)}{\partial x'_1} = \frac{\partial x_1(x'_1, a_1, a_2)}{\partial x'_1} + \frac{\partial x_1(x'_1, a_1, a_2)}{\partial a_1} \frac{\partial a_1}{\partial x'_1} + \frac{\partial x_1(x'_1, a_1, a_2)}{\partial a_2} \frac{\partial a_2}{\partial x'_1},$$

where derivatives $\frac{\partial a_1}{\partial x'_1}$ and $\frac{\partial a_2}{\partial x'_1}$ describe how the equilibrium advertising levels change with x'_1 . All the derivatives are evaluated at $x'_1 = \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}$ and $a_1^* = a_2^* = \frac{\gamma\beta + \beta + \alpha\gamma^2 - \alpha}{2(\gamma+3)}$. $\frac{\partial x_1(x'_1, a_1, a_2)}{\partial x'_1}$ and $\frac{\partial x_1(x'_1, a_1, a_2)}{\partial a_1}$ are evaluated as before based on the implicit function theorem and the implicit definition of $x_1(x'_1, a_1, a_2)$. $\frac{\partial a_1}{\partial x'_1}$ and $\frac{\partial a_2}{\partial x'_1}$ are obtained based on the implicit function theorem from the first order conditions (17).

The stability condition is met when both eigenvalues σ are bounded by 1 in absolute values. This leads to the condition $\alpha < \frac{5\beta + 7\beta\gamma}{8\gamma - 5\gamma^2 + 5}$. Finally, when $\alpha > \frac{\beta}{4}$, the consumers always derive positive network utility from the community they join.

□

PROOF OF PROPOSITION 3: When $a_1 = a_2 \equiv 0$, the fulfilled expectation equilibrium must satisfy $\forall x, T_1^*(x) = T_1^r(x, T_1^*, 0, 0)$. Clearly, the same equilibrium time allocation pattern described in Proposition 2 is self-fulfilling:

$$T_1^*(x) = \begin{cases} 2 & \text{if } x < \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}, \\ 1 & \text{if } \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)} \leq x \leq 1 - \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}, \\ 0 & \text{if } x > 1 - \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}. \end{cases} \quad (20)$$

This is due to the fact that when $a_1 = a_2$, the time allocation decision of a consumer depends only on her expectation of other consumers' time allocation pattern. Thus $T_1^*(x)$ is self-fulfilling under $a_1 = a_2 = 0$ if and only if it is self-fulfilling under $a_1^* = a_2^* = \frac{\gamma\beta + \beta + \alpha\gamma^2 - \alpha}{2(\gamma+3)}$.

The stability of the equilibrium can be determined by examining the eigenvalues of the Jacobian matrix, as in the proof for Proposition 2. Without competitive interaction, we have $\frac{\partial \Gamma_1(x_1', x_2')}{\partial x_1} = \frac{\partial x_1(x_1', a_1, a_2)}{\partial x_1}$. This leads to the conditions given in Proposition 3.

Finally, the stability conditions is equivalent to $\gamma(\alpha - \beta x_1^*) + \alpha - \beta(1 - x_1^*) < 0$ where $x_1^* = \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma+3)}$. Since $\alpha - \beta x_1^* > \alpha - \beta(1 - x_1^*)$, it is a necessary condition that $\alpha - \beta(1 - x_1^*) < 0$.

Note that

$$\int \int_{(x,y) \in \{(x,y) | t_{xy}=0\}} \delta(x,y) dy dx = (1 - x_1^*)(\alpha - \beta(1 - x_1^*)).$$

Thus, a necessary condition for the spontaneous differentiation outcome to be stable is

$$\int \int_{(x,y) \in \{(x,y) | t_{xy}=0\}} \delta(x,y) dy dx < 0$$

□

PROOF OF PROPOSITION 4: The proof of Proposition 4 follows the exact same logic as Proposition 2. The derivation is algebraically tedious and is done with the help of mathematical software. Consider any symmetric market expectation in which the consumers located in the middle of the linear city will single-home in site 1 and the consumers located at the two ends of the linear city will single-home in site 2. Observe that given this market expectation and any advertising levels, the consumer behavior is monotonic: those located at the ends of the linear city are strictly more

likely to single-home in community 2 while those located closer to $x = \frac{1}{2}$ are strictly more likely to single-home in community 1. We derive demand schedules as before. We show that the advertising levels are locally best response to each other by verifying that they simultaneously solve the two first order conditions.

We provide the analytical expressions for $A(\alpha, \beta, \gamma)$, $B(\alpha, \beta, \gamma)$ and $\lambda(\gamma)$. A and B are described by the following general expressions:

$$\begin{aligned} A(\alpha, \beta, \gamma) &= \frac{\lambda_1(\gamma)\alpha + \lambda_2(\gamma)\beta + \lambda_3(\gamma)\sqrt{\lambda_4(\gamma)\alpha^2 + \lambda_5(\gamma)\beta^2 + \lambda_6(\gamma)\alpha\beta}}{\lambda_7(\gamma)\beta} \\ B(\alpha, \beta, \gamma) &= \frac{\lambda_8(\gamma)\alpha + \lambda_9(\gamma)\beta + \lambda_{10}(\gamma)\sqrt{\lambda_4(\gamma)\alpha^2 + \lambda_5(\gamma)\beta^2 + \lambda_6(\gamma)\alpha\beta}}{\lambda_7(\gamma)\beta} \end{aligned} \quad (21)$$

where $\lambda_1 - \lambda_{10}$ are defined as:

$$\begin{aligned} \lambda_1(\gamma) &= \gamma^5 + 3\gamma^4 + \gamma^3 - 21\gamma^2 + 18\gamma + 6 \\ \lambda_2(\gamma) &= 6\gamma^2 - 3\gamma^4 - 3/2\gamma^3 - 1/2\gamma^5 - 10\gamma - 3 \\ \lambda_3(\gamma) &= 3/2\gamma - 1 + 1/2\gamma^2 \\ \lambda_4(\gamma) &= 4\gamma^6 - 8\gamma^5 + 40\gamma^4 - 112\gamma^3 + 116\gamma^2 + 24\gamma \\ \lambda_5(\gamma) &= \gamma^6 + 6\gamma^5 + \gamma^4 - 14\gamma^3 + 32\gamma^2 + 6\gamma \\ \lambda_6(\gamma) &= -\gamma^6 - 16\gamma^5 + 12\gamma^4 - 96\gamma^3 + 120\gamma^2 + 24\gamma \\ \lambda_7(\gamma) &= \gamma^5 + 6\gamma^4 - 7\gamma^3 - 6\gamma^2 + 28\gamma + 6 \\ \lambda_8(\gamma) &= 2\gamma^5 + 4\gamma^4 - 18\gamma^3 + 38\gamma^2 - 8\gamma - 2 \\ \lambda_9(\gamma) &= 2\gamma^4 + 4\gamma^3 - 14\gamma^2 + 10\gamma + 2 \\ \lambda_{10}(\gamma) &= -2\gamma + 3 + \gamma^{-1} \end{aligned}$$

The condition $\alpha > \frac{\beta}{2}$ is required such that in equilibrium, all consumers derive positive network utility from the communities they join.

□

PROOF OF COROLLARY 1: From Proposition 2, the percentage of multi-homing consumers is measured by $m = 1 - 2x_1^*$ where $x_1^* = \frac{2(\beta - \alpha + \alpha\gamma)}{\beta(\gamma + 3)}$. Taking derivatives of m with respect to β and γ , we have $\frac{\partial m}{\partial \beta} = -\frac{4\alpha(1-\gamma)}{\beta^2(\gamma+3)} < 0$ and $\frac{\partial m}{\partial \gamma} = -\frac{16\alpha - 4\beta}{\beta(\gamma+3)^2} < 0$. Thus, more consumers multi-home when γ and β are smaller.

□

PROOF OF COROLLARY 2: From Proposition 2, firm profits are $\Pi = \frac{\beta\gamma + \beta + \alpha\gamma^2 - \alpha}{2(\gamma+3)}$. Taking derivatives of Π with respect to β and γ , we have $\frac{\partial \Pi}{\partial \beta} = \frac{1+\gamma}{2(\gamma+3)} > 0$ and $\frac{\partial \Pi}{\partial \gamma} = \frac{\alpha}{2} - \frac{4\alpha - \beta}{(\gamma+3)^2} > 0$. Thus, profits increase when γ and β are larger.

□

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